

# SOLUTIONS MANUAL

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## ENGINEERING MECHANICS *of* SOLIDS SECOND EDITION

EGOR P. POPOV



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EGOR P. POPOV

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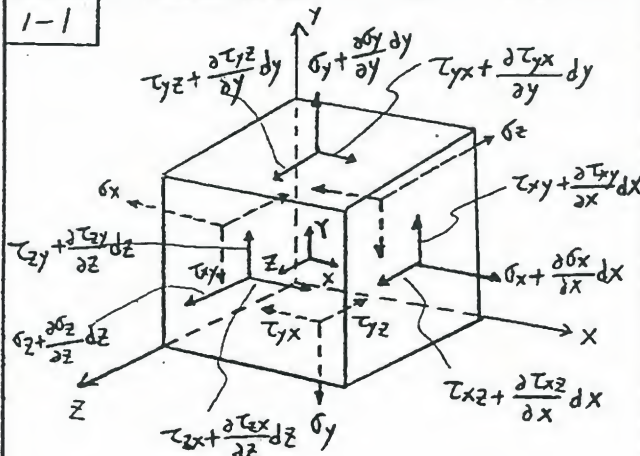
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1-1



$$\begin{aligned} \Sigma F_x = & (\sigma_x + \frac{\partial \sigma_x}{\partial x} dx) dy \cdot dz + \\ & (\tau_{yx} + \frac{\partial \tau_{yx}}{\partial y} dy) dx \cdot dz + \\ & (\tau_{zx} + \frac{\partial \tau_{zx}}{\partial z} dz) dx \cdot dy - \\ & \sigma_x \cdot dy \cdot dz - \tau_{yx} \cdot dx \cdot dz - \\ & \tau_{zx} \cdot dx \cdot dy + x \end{aligned}$$

$$= 0$$

$$\begin{aligned} \frac{\partial \sigma_x}{\partial x} dx \cdot dy \cdot dz + \frac{\partial \tau_{yx}}{\partial y} dx \cdot dy \cdot dz + \frac{\partial \tau_{zx}}{\partial z} dx \cdot dy \cdot dz \\ + x = 0 \end{aligned}$$

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} + x = 0$$

1-2

$$\begin{aligned} \Sigma F_r = & (\sigma_r + \frac{\partial \sigma_r}{\partial r} dr)(r+dr) d\theta + \\ & (\tau_{\theta r} + \frac{\partial \tau_{\theta r}}{\partial \theta} d\theta) dr - \\ & \tau_{\theta r} dr - \sigma_r r d\theta - (\sigma_\theta + \frac{\partial \sigma_\theta}{\partial \theta} d\theta) dr \frac{d\theta}{2} \\ & - \sigma_\theta dr \frac{d\theta}{2} = 0 \end{aligned}$$

$$\begin{aligned} \sigma_r dr d\theta + r \frac{\partial \sigma_r}{\partial r} dr d\theta + \frac{\partial \sigma_r}{\partial r} (dr)^2 d\theta + \frac{\partial \tau_{\theta r}}{\partial \theta} dr d\theta \\ - \sigma_\theta dr d\theta - \frac{1}{2} \frac{\partial \sigma_\theta}{\partial \theta} (d\theta)^2 dr = 0 \end{aligned}$$

$$\sigma_r + r \frac{\partial \sigma_r}{\partial r} + \frac{\partial \sigma_r}{\partial r} r + \frac{\partial \tau_{\theta r}}{\partial \theta} - \sigma_\theta - \frac{1}{2} \frac{\partial \sigma_\theta}{\partial \theta} d\theta = 0$$

$$\rightarrow \frac{\partial \sigma_r}{\partial r} + \frac{1}{r} \frac{\partial \tau_{\theta r}}{\partial \theta} + \frac{\sigma_r - \sigma_\theta}{r} = 0$$

$$\begin{aligned} \Sigma F_\theta = & (\tau_{\theta r} + \frac{\partial \tau_{\theta r}}{\partial r} dr)(r+dr) d\theta + \\ & (\sigma_\theta + \frac{\partial \sigma_\theta}{\partial \theta} d\theta) dr + (\tau_{r\theta} + \frac{\partial \tau_{r\theta}}{\partial \theta} d\theta) dr \frac{d\theta}{2} \\ & + \tau_{\theta r} dr \frac{d\theta}{2} - \tau_{r\theta} r d\theta - \sigma_\theta dr = 0 \end{aligned}$$

$$\begin{aligned} \tau_{\theta r} dr d\theta + \frac{\partial \tau_{\theta r}}{\partial r} r dr d\theta + \frac{\partial \tau_{\theta r}}{\partial r} (dr)^2 d\theta + \\ \frac{\partial \sigma_\theta}{\partial \theta} dr d\theta + \tau_{\theta r} dr d\theta + \frac{1}{2} \frac{\partial \tau_{r\theta}}{\partial \theta} (d\theta)^2 dr = 0 \end{aligned}$$

$$\begin{aligned} \tau_{\theta r} + \frac{\partial \tau_{\theta r}}{\partial r} r + \frac{\partial \tau_{\theta r}}{\partial r} r + \frac{\partial \sigma_\theta}{\partial \theta} + \tau_{\theta r} + \\ \frac{1}{2} \frac{\partial \tau_{r\theta}}{\partial \theta} d\theta = 0 \end{aligned}$$

$$\rightarrow \frac{\partial \tau_{\theta r}}{\partial r} + \frac{1}{r} \frac{\partial \sigma_\theta}{\partial \theta} + \frac{2\tau_{\theta r}}{r} = 0$$

1-3

$$\sigma = \frac{P}{A} = \frac{110}{7.08} = 15.54 \text{ ksi}$$

$$\sigma = \frac{P}{A} = \frac{110}{6.09} = 18.06 \text{ ksi}$$

1-4

$$P = 50 \times 40 = 2000 \text{ lb}$$

$$\sigma_1 = \frac{2000}{5.5^2} = 66.1 \text{ psi}$$

$$\sigma_2 = \frac{2000}{6^2} = 55.6 \text{ psi}$$

$$\sigma_3 = \frac{2000}{16^2} = 7.8 \text{ psi}$$

1-5

magnitude of the applied force:

$$\begin{aligned} P &= \sigma A \\ &= 150 \times (30 \times 10 + 30 \times 12) \\ &= 150 \times 660 \\ &= 99000 \text{ N} \end{aligned}$$

1-5

The location of stress resultant :  
(at the centroid)

$$\frac{1}{660} \left[ 30 \times 10 \times \frac{10}{2} + 30 \times 12 \times \left( 10 + \frac{30}{2} \right) \right]$$

$$= 15.91 \text{ mm below the top}$$

1-6

$$A = 79.5 \times 9.5 + 102 \times 9.5 = 1724.25 \text{ mm}^2$$

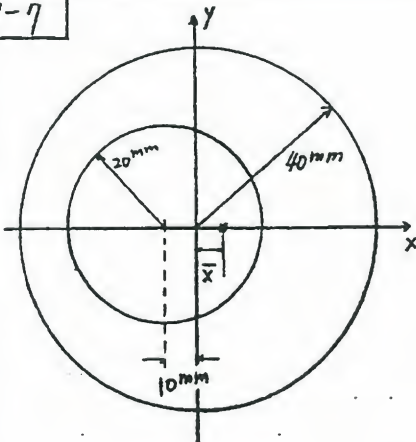
$$\bar{x} = \frac{1}{2} \times \frac{102 \times 9.5^2}{1724.25} + \frac{79.5 \times 9.5 \times \left( \frac{1}{2} \times 79.5 + 9.5 \right)}{1724.25}$$

$$= 24.24 \text{ mm from the left side}$$

$$\bar{y} = \frac{1}{2} \times \frac{102^2 \times 9.5}{1724.25} + \frac{1}{2} \times \frac{9.5^2 \times 79.5}{1724.25}$$

$$= 30.74 \text{ mm below the top}$$

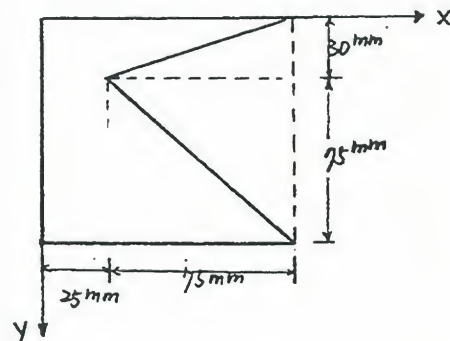
1-7



$$\bar{x} = \frac{\pi 40^2 \times 0 + (-\pi 20^2) \cdot (-10)}{\pi 40^2 + (-\pi 20^2)}$$

$$= 2 \text{ mm}$$

1-8



$$A = 100 \times 105 - \frac{1}{2} \times 105 \times 75 = 6562.5 \text{ mm}^2$$

$$\bar{x} = \frac{1}{2} \times \frac{100^2 \times 105}{6562.5} - \frac{1}{2} \times \frac{105 \times 75 \times (50 + 25)}{6562.5}$$

$$= 35 \text{ mm}$$

$$\bar{y} = \frac{1}{2} \times \frac{100 \times 105^2}{6562.5} - \frac{1}{2} \times \frac{30 \times 75 \times 20}{6562.5} - \frac{1}{2} \times \frac{75^2 \times (30 + 25)}{6562.5}$$

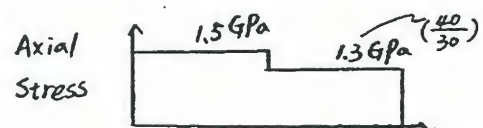
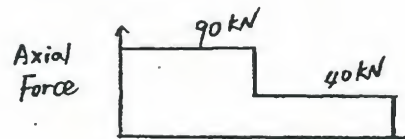
$$= 57 \text{ mm}$$

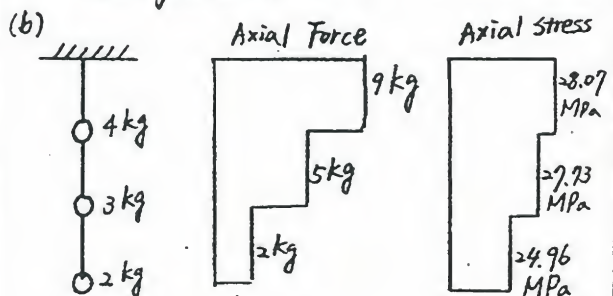
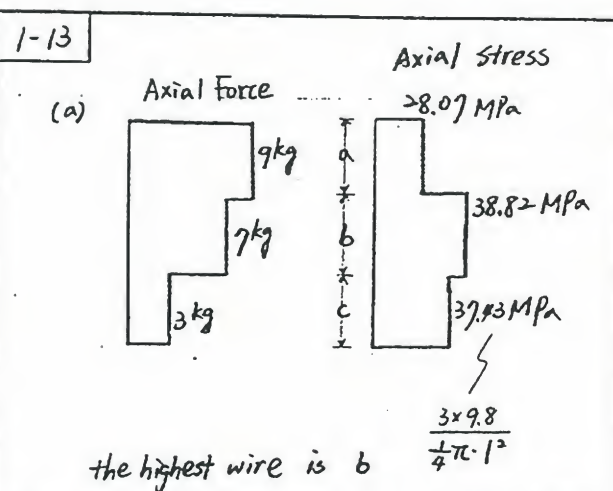
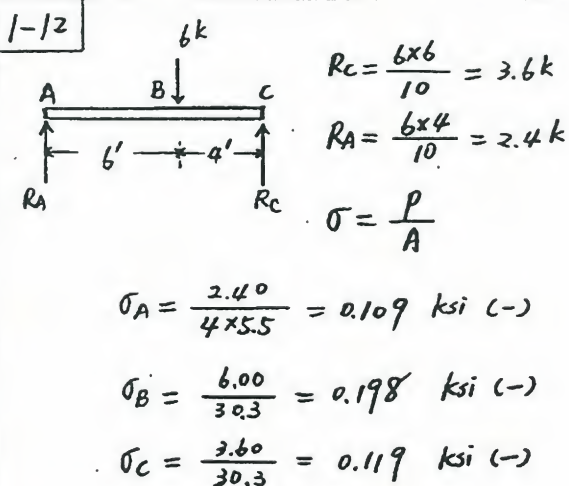
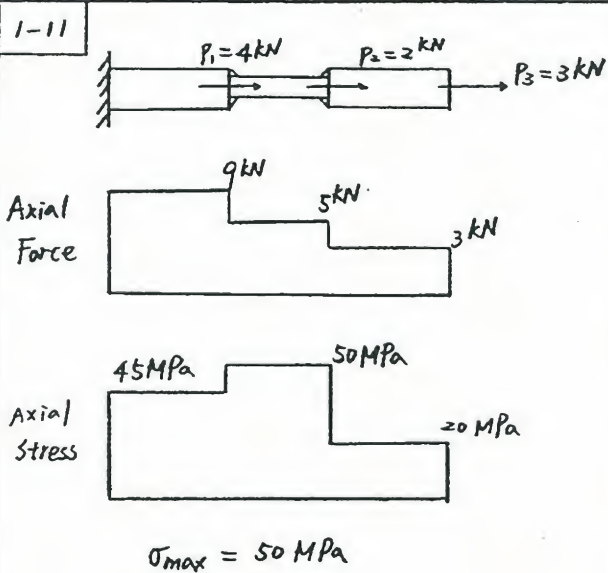
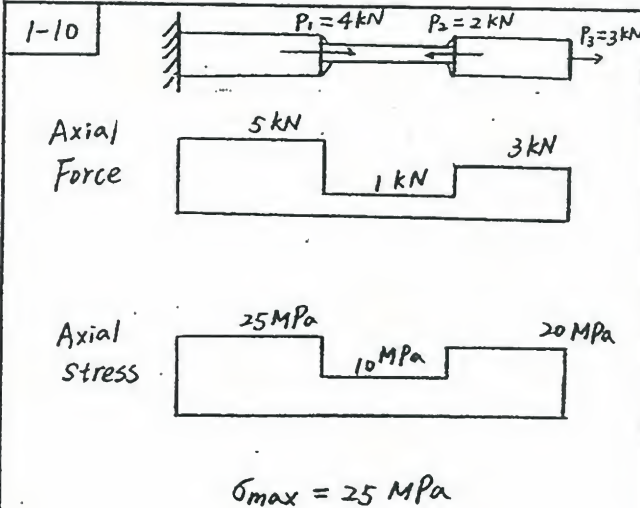
1-9

$$(a) \sigma_{\max} = \frac{P_1 + P_2}{A_1} = \frac{50 + 40}{60}$$

$$= 1.5 \text{ GPa}$$

(b)





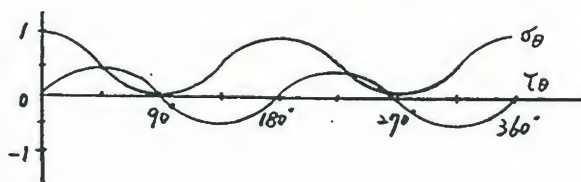
1-14

$$\sigma_\theta = \frac{P}{A} \cos^2 \theta$$

$\sigma_\theta$	$\theta^\circ$	0	45	90	135	180	225	270	315	360
$\cos^2 \theta$		1	.5	0	.5	1	.5	0	.5	1

$$\tau_\theta = \frac{P}{A} \sin \theta \cos \theta$$

$\tau_\theta$	$\theta^\circ$	0	45	90	135	180	225	270	315	360
$\sin \theta \cos \theta$		0	.5	0	-.5	0	.5	0	-.5	0



Maxima  $\left\{ \begin{array}{l} \sigma = 0^\circ, 180^\circ \\ \tau = 45^\circ, 225^\circ \end{array} \right.$

minima  $\left\{ \begin{array}{l} \sigma = 90^\circ, 270^\circ \\ \tau = 135^\circ, 315^\circ \end{array} \right.$



1-15

$$\sigma_\theta = \tau_\theta$$

$$\left| \frac{P}{A} \cos^2 \theta \right| = \left| \frac{P}{A} \sin \theta \cos \theta \right|$$

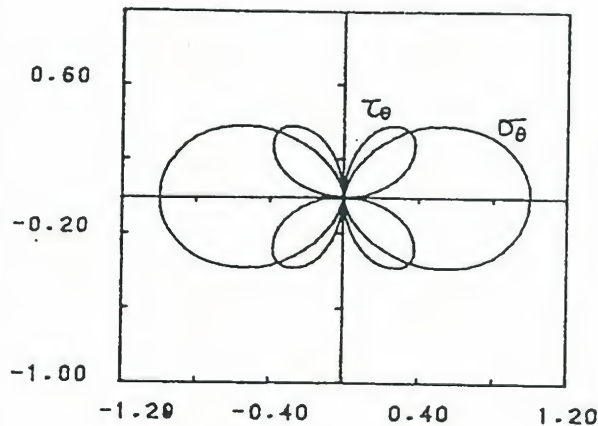
$$|\cos \theta| = |\sin \theta|$$

$$|\tan \theta| = 1$$

$$\theta = \pm \tan^{-1}(1) \rightarrow \theta = \pm 45^\circ$$

1-16

$$\sigma_\theta = \frac{P}{A} \cos^2 \theta \quad \tau_\theta = \frac{P}{A} \sin \theta \cos \theta$$



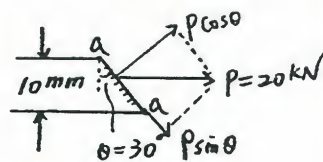
$$\text{maxima} \begin{cases} \sigma = 0^\circ, 180^\circ \\ \tau = 45^\circ, 225^\circ \end{cases}$$

$$\text{minima} \begin{cases} \sigma = 90^\circ, 270^\circ \\ \tau = 135^\circ, 315^\circ \end{cases}$$

1-17

A)

section a-a



$$P \cos \theta = 17.32 \text{ kN}$$

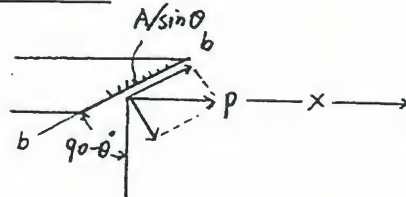
$$\text{Area} = A / \cos 30^\circ = 100 / \cos 30^\circ = 115.47 \text{ mm}^2$$

$$\sigma = \frac{P}{A} = \frac{P \cos \theta}{115.47} = \frac{17.32 \times 10^3}{115.47} = 150 \text{ MPa}$$

$$\tau = \frac{P}{A} = \frac{P \sin \theta}{115.47} = \frac{20 (\sin 30^\circ) \times 10^3}{115.47}$$

$$= 86.6 \text{ MPa}$$

section b-b



$$\text{Area} = \frac{A}{\sin \theta} = \frac{100}{\sin 30^\circ} = 200 \text{ mm}^2$$

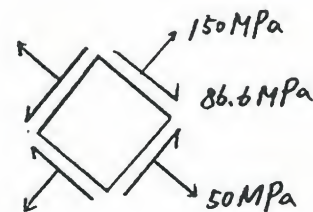
$$\sigma = \frac{P}{A} = \frac{20 \sin 30^\circ \times 10^3}{200} = 50 \text{ MPa}$$

$$\tau = \frac{P}{A} = \frac{20 \cos 30^\circ \times 10^3}{200} = 86.6 \text{ MPa}$$

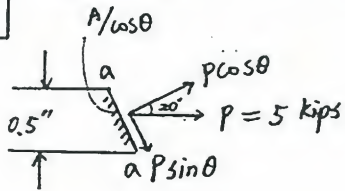
$$\begin{aligned} B) \quad \sigma_{aa} &= \frac{P}{A} \cos^2 \theta = \frac{20 \times 10^3}{100} \cos^2 (30^\circ) \\ &= 150 \text{ MPa} \end{aligned}$$

$$\begin{aligned} \tau_{a-a, b-b} &= \frac{P}{A} \cos \theta \sin \theta \\ &= \frac{20 \times 10^3}{100} (\cos 30^\circ) (\sin 30^\circ) \\ &= 86.6 \text{ MPa} \end{aligned}$$

$$\sigma_{b-b} = \frac{20 \times 10^3}{100} \cos^2 (60^\circ) = 50 \text{ MPa}$$



1-18



(A) section a-a

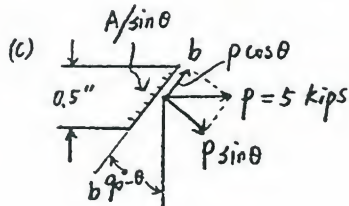
$$A = \frac{A}{\cos \theta} = \frac{0.5^2}{\cos 20^\circ} = 0.266 \text{ in}^2$$

$$\sigma = \frac{P}{A} = \frac{P \cos \theta}{0.266} = \frac{5 \times \cos 20^\circ}{0.266} = 17.66 \text{ ksi}$$

$$\tau = \frac{P}{A} = \frac{P \sin \theta}{0.266} = \frac{5 \times \sin 20^\circ}{0.266} = 6.43 \text{ ksi}$$

$$(B) \sigma = \frac{P}{A} \cos^2 \theta = \frac{5}{0.5^2} \times \cos^2 20^\circ = 17.66 \text{ ksi (eq. 1-7a)}$$

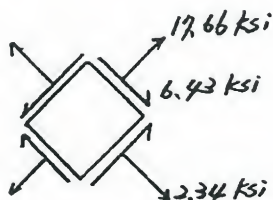
$$\tau = \frac{P}{A} \sin \theta \cos \theta = \frac{5}{0.5^2} \sin 20^\circ \cos 20^\circ = 6.43 \text{ ksi (eq. 1-7b)}$$



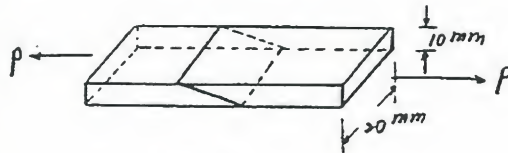
section b-b

$$\sigma = \frac{5}{0.5^2} \times \cos^2 70^\circ = 2.34 \text{ ksi}$$

$$\tau = \frac{5}{0.5^2} \times \sin 70^\circ \cos 70^\circ = 6.43 \text{ ksi}$$



1-19



$$\tau_\theta = \frac{P \sin \theta}{A / \cos \theta} = \frac{P}{A} \cos \theta \sin \theta$$

$$\tau = 10 = \frac{P (\cos 20^\circ) (\sin 20^\circ)}{20 \times 10}$$

$$P = \frac{10 \times 20 \times 10}{(\cos 20^\circ) (\sin 20^\circ)} = 6223 \text{ N} = 6.2 \text{ kN}$$

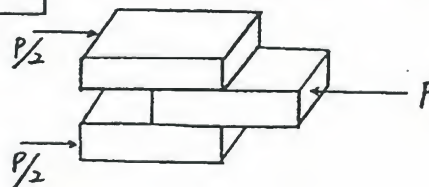
1-20

$$A = \frac{\pi}{4} D^2 = \frac{\pi}{4} \times 19^2 = 283.53 \text{ mm}^2$$

$$\sigma = \frac{P}{A} \cos^2 \theta = \frac{100 \times 10^3}{283.53} \times \cos^2 60^\circ = 88.17 \text{ MPa}$$

$$\tau = \frac{P}{A} \sin \theta \cos \theta = \frac{100 \times 10^3}{283.53} \times \sin 60^\circ \cos 60^\circ = 152.72 \text{ MPa}$$

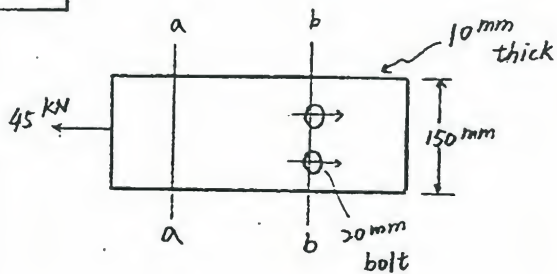
1-21



$$A (\text{shear Area}) = 40 \times 80 = 3200 \text{ mm}^2 = 3.2 \times 10^{-3} \text{ m}^2$$

$$\tau = \frac{20/2 \text{ k}}{3.2 \times 10^{-3} \text{ m}^2} = 3.125 \text{ MPa}$$

1-22



$$(a) \sigma_{aa} = \frac{P}{A} = \frac{45 \times 10^3}{10 \times 150} = 30 \text{ MPa}$$

$$(b) \sigma_{bb} = \frac{P}{A} = \frac{45 \times 10^3}{10(150-40)} = 40.91 \text{ MPa}$$

$$(c) \tau = \frac{V}{A} = \frac{45 \times 10^3}{2(\pi \times 10^2)} = 71.62 \text{ MPa}$$

$$(d) \sigma_{br} = \frac{P}{A} = \frac{45 \times 10^3}{2 \times (20 \times 10)} = 112.5 \text{ MPa}$$

1-23

Maximum normal stress:

$$\frac{70 \times 10^3}{10 \times (60-20)} = 175 \text{ MPa}$$

Bearing stress:

$$\left\{ \begin{array}{l} \frac{P}{td} = \frac{70 \times 10^3}{20 \times 20} = 175 \text{ MPa} \\ \text{for the middle plate} \end{array} \right.$$

$$\left\{ \begin{array}{l} \frac{P/2}{t_1 d} = \frac{70 \times 10^3/2}{20 \times 20} = 87.5 \text{ MPa} \\ \text{for the outer plate} \end{array} \right.$$

Shear stress:

$$\frac{P/2}{\frac{1}{4} \pi d^2} = \frac{70 \times 10^3/2}{\frac{1}{4} \pi \cdot 20^2} = 111.41 \text{ MPa}$$

1-24

$$T = P \cdot R = 10 \times 185.75 = 1857.5 \text{ N-m}$$

$$P' = \frac{T}{r} = \frac{1857.5}{40} = 46.44 \text{ kN}$$

$$\tau = \frac{P'}{A} = \frac{46.44 \times 10^3}{6 \times 4} = 1935 \text{ MPa}$$

1-25

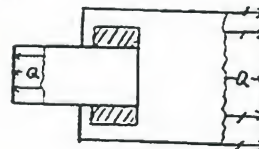
shear stress:

$$\frac{\text{shear force}}{\text{bolt area}} = \frac{16.5}{\frac{1}{4} \pi \cdot 1^2} = 21 \text{ ksi}$$

tension stress in the material:

$$\frac{\text{tension force}}{\text{material area}} = \frac{64}{\frac{1}{4} \pi \cdot 3^2} = 9.05 \text{ ksi}$$

1-26

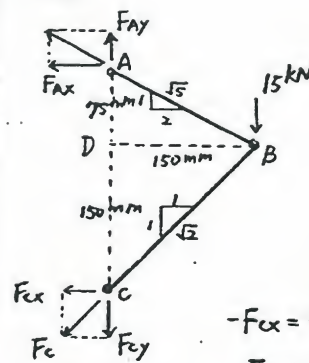


$$A = \frac{W}{\sqrt{2}} \text{ in}^2/\text{in}$$

$$\tau = \frac{Q}{2A} = \frac{Q}{\sqrt{2}W} = 21$$

$$Q = 29.7 W \text{ kips/in}$$

1-27



$$\sum F_x = 0 \rightarrow (+)$$

$$-F_{Ax} - F_{Cx} = 0$$

$$\sum F_y = 0 \uparrow (+)$$

$$-F_{Cy} + F_{Ay} - 15 = 0$$

$$F_{Cy} = -\frac{2}{3} \times 15 = -10 \text{ kN}$$

$$F_{Ay} = \frac{1}{3} \times 15 = 5 \text{ kN}$$

$$-F_{Cx} = F_{Ax} = 10 \text{ kN}$$

$$F_C = -10\sqrt{2} = -14.14 \text{ kN (comp.)}$$



1-27

$$F_A = 5\sqrt{5} = 11.18 \text{ kN (tension)}$$

tensile stress in main bar AB:

$$\sigma_{AB} = \frac{F_A}{A} = \frac{11.18 \times 10^3}{6 \times 15} = 124.23 \text{ MPa}$$

tensile stress in clevis of bar AB:

$$(\sigma_{AB})_{\text{clevis}} = \frac{F_A}{A_{\text{net}}} = \frac{11.18 \times 10^3}{25 \times (25-10)} = 74.54 \text{ MPa}$$

bearing between pin c &amp; the clevis:

$$\sigma_b = \frac{F_c}{A_{\text{bearing}}} = \frac{14.14 \times 10^3}{10 \times 5 \times 2} = 141.42 \text{ MPa}$$

bearing between pin c &amp; the main plate:

$$\sigma_b = \frac{F_c}{A} = \frac{14.14 \times 10^3}{10 \times 6} = 235.70 \text{ MPa}$$

double shear in pin c:

$$\tau = \frac{F_c}{A} = \frac{14.14 \times 10^3}{2\pi (10/2)^2} = 90.03 \text{ MPa}$$

1-28

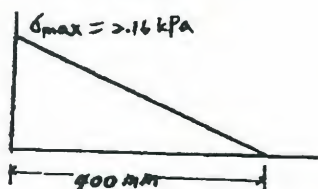
$$\text{Area} = 2\pi R l = 2\pi \times (15 \times 10^{-3}) \times 0.4 = 0.038 \text{ m}^2$$

$$P = 10 \times 0.038 = 0.38 \text{ N}$$

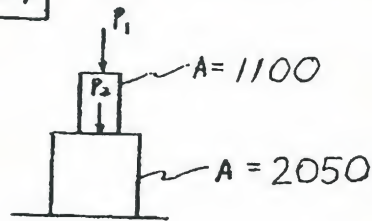
Cross section area:

$$\pi R^2 - \pi r^2 = \pi (15^2 - 13^2) = 175.93 \text{ mm}^2$$

$$\sigma_{\text{max}} = \frac{0.38}{175.93} = 2.16 \text{ kPa}$$



1-29



$$(\omega) P_1 = A \sigma$$

$$= (1100)(100)/1000 = 110 \text{ kN}$$

$$P_1 + P_2 = (2050)(100)/1000 = 205$$

$$P_1 + 200 = 205$$

$$P_1 = 5 \text{ kN}$$

$$\Rightarrow P_1 = 5 \text{ kN}$$

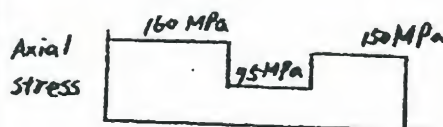
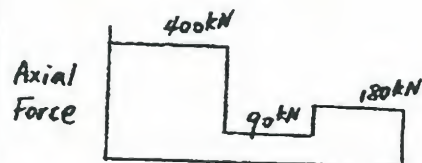
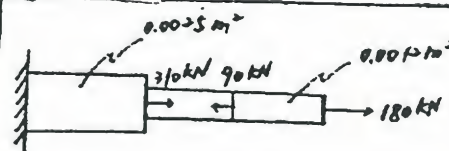
(b)

$$P_1 + 65 = 205$$

$$P_1 = 140$$

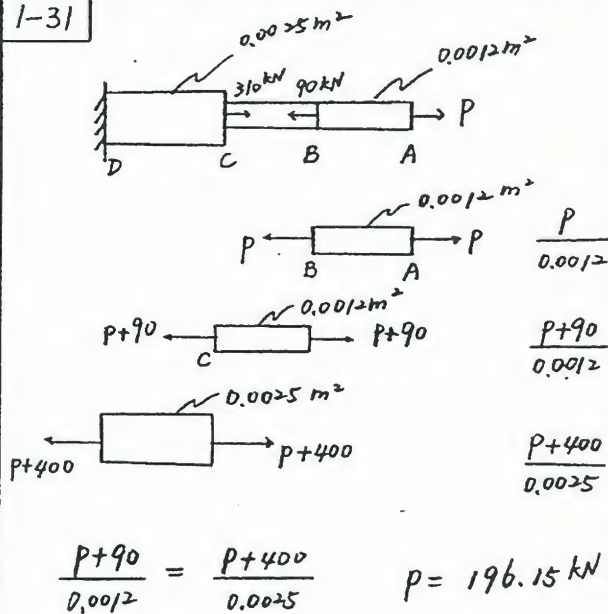
$$\Rightarrow P_1 = 110 \text{ kN governs}$$

1-30



$$\sigma_{\text{max}} = 160 \text{ MPa}$$

1-31



$$F_{AB} = 75 \cdot \frac{\pi}{4} \cdot 3^2 = 530.1 \text{ N}$$

$$\sum M_A = 0, \quad -\frac{4}{5}P + \frac{9}{5}P = 530.1 \text{ N}$$

$$\frac{3}{5}P = 318.1 \text{ N}, \quad \frac{4}{5}P = 424.1 \text{ N}$$

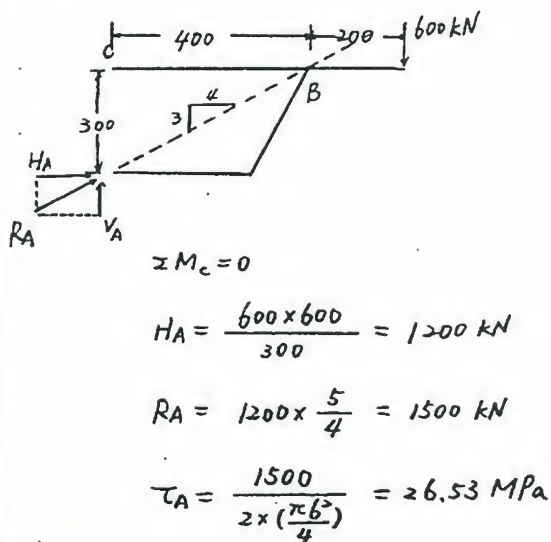
$$R_A = \sqrt{318.1^2 + 424.1^2} = 1005.8 \text{ N}$$

$$R_B = \sqrt{424.1^2 + 848.2^2} = 948.3 \text{ N}$$

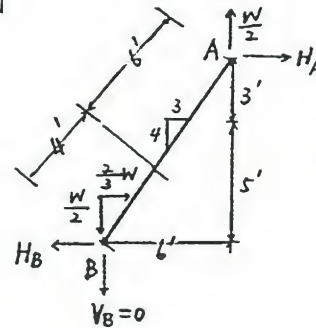
$$\tau_A = \frac{V_A}{A} = \frac{1005.8}{0.022 \times 2} = 23051 \text{ MPa} = 23.05 \text{ GPa}$$

$$\tau_B = \frac{V_B}{A} = \frac{948.3}{0.022 \times 2} = 21552 \text{ MPa} = 21.55 \text{ GPa}$$

1-32



1-34

(From the symmetry of the structure,  $V_B = 0$ )

$$H_B = \frac{5}{8}W = \frac{5}{8} \times 15 = 9.375 \text{ k}$$

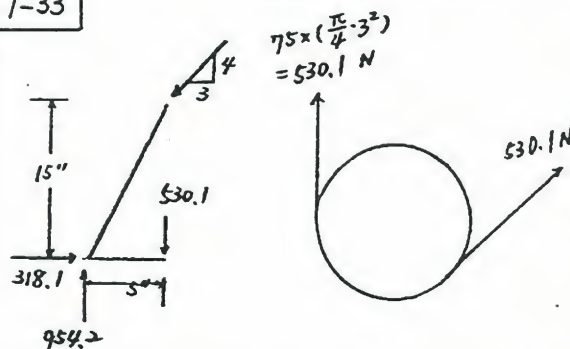
$$\tau_B = \frac{9.375}{2 \times \frac{\pi 1^2}{4}} = 5.97 \text{ ksi}$$

$$H_A = H_B - \frac{2}{3}W = 9.375 - \frac{2}{3} \times 15 = -0.625 \text{ k}$$

$$R_A = \sqrt{0.625^2 + 2.5^2} = 2.53 \text{ k}$$

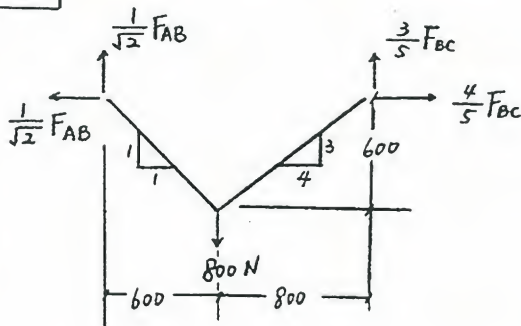
$$\tau_A = \frac{2.53}{2 \times \frac{\pi 1^2}{4}} = 4.79 \text{ ksi}$$

1-33





1-35



$$\sum F_x = 0$$

$$F_{AB} \frac{1}{\sqrt{2}} = F_{BC} \frac{4}{5} \Rightarrow F_{BC} = 0.8839 F_{AB}$$

$$\sum F_y = 0$$

$$F_{AB} \frac{1}{\sqrt{2}} + F_{BC} \frac{3}{5} = 800$$

$$\Rightarrow F_{AB} = 646.49 \text{ N}$$

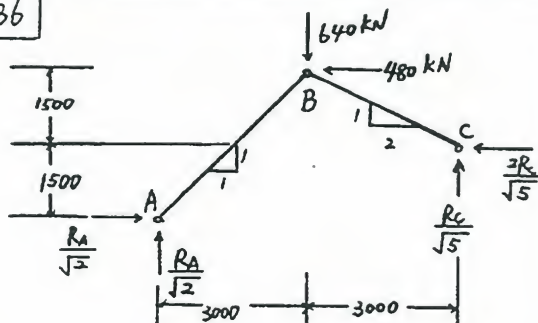
$$F_{BC} = 571.43 \text{ N}$$

$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{646.49}{\pi \left(\frac{2.68}{2}\right)^2} = 114.60 \text{ MPa}$$

$$\sigma_{BC} = \frac{F_{BC}}{A_{BC}} = \frac{571.43}{\pi \left(\frac{2.52}{2}\right)^2} = 114.57 \text{ MPa}$$

$$114.60 \approx 114.57 \Rightarrow \text{Good Design}$$

1-36



$$\sum M_A = 0$$

$$480 \times 3000 - 640 \times 3000 + R_C \times \frac{2}{\sqrt{5}} \times 1500 + R_C \frac{1}{\sqrt{5}} \times 6000 = 0$$

$$R_C = -119.26 \text{ kN}$$

$$\sum F_x = 0$$

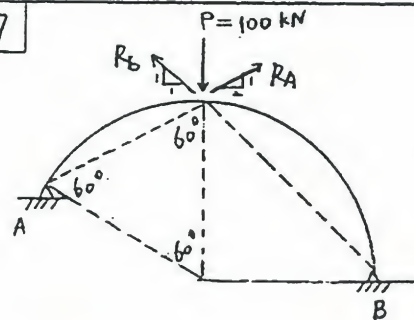
$$\frac{R_A}{\sqrt{2}} + 119.26 \times \frac{2}{\sqrt{5}} - 480 = 0$$

$$R_A = 373.33 \text{ kN}$$

$$\sigma_A = \frac{373.33 \times 10^3}{\frac{1}{4} \pi \times 102^2} = 45.69 \text{ MPa (comp.)}$$

$$\sigma_C = \frac{119.26 \times 10^3}{\frac{1}{4} \pi \times 102^2} = 14.6 \text{ MPa (tension)}$$

1-37



$$R_{Ax} = R_{Bx}$$

$$R_{Ay} = \frac{1}{3} \times 100 = 33.3 \text{ kN}$$

$$R_{By} = \frac{2}{3} \times 100 = 66.7 \text{ kN}$$

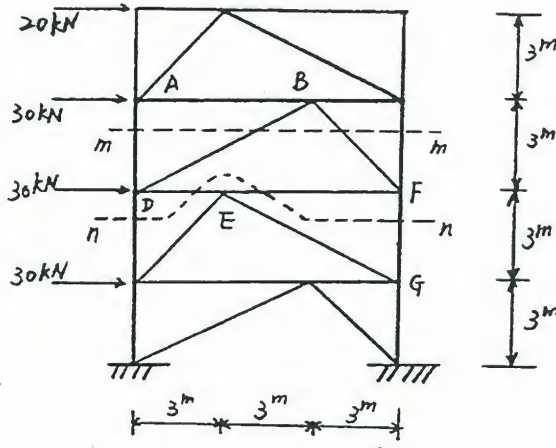
$$R_A = \sqrt{3} R_{Ay} = 33.3 \sqrt{3} = 57.73 \text{ kN}$$

$$R_B = \sqrt{2} R_{By} = 66.7 \sqrt{2} = 94.28 \text{ kN}$$

$$\tau_A = \frac{V_A}{A} = \frac{57.73 \times 10^3}{\frac{\pi}{4} \times 20^2} = 183.8 \text{ MPa}$$

$$\tau_B = \frac{V_B}{A} = \frac{94.28 \times 10^3}{\frac{\pi}{4} \times 20^2} = 300.0 \text{ MPa}$$

1-38



section m-m

$$\sum F_H = 0$$

$$-\frac{2}{\sqrt{5}} P_{BD} + \frac{1}{\sqrt{2}} P_{BF} + 50 = 0$$

joint B,  $\sum F_v = 0$

$$\frac{1}{\sqrt{5}} P_{BD} + \frac{1}{\sqrt{2}} P_{BF} = 0$$

$$50 = \frac{3}{\sqrt{5}} P_{BD} \quad P_{BD} = 37.27 \text{ kN}$$

joint D,  $\sum F_H = 0$

$$P_{DE} + \frac{2}{\sqrt{5}} P_{BD} + 30 = 0$$

$$P_{DE} = -30 - \frac{2}{\sqrt{5}} \times 37.27 = -63.34 \text{ kN}$$

section n-n

$$+\circlearrowleft \sum M_D = 0$$

$$-30 \times 3 - 20 \times 6 - P_{FG} \times 9 = 0$$

$$P_{DG} = -32.89 \text{ kN}$$

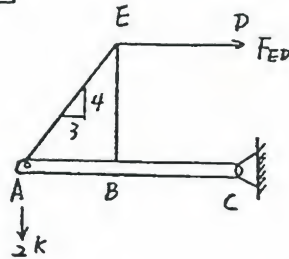
stress:

$$\sigma_{BD} = \frac{37.27 \times 10^3}{160} = 232.94 \text{ MPa (T)}$$

$$\sigma_{DE} = \frac{63.34 \times 10^3}{130} = 487.23 \text{ MPa (C)}$$

$$\sigma_{FG} = \frac{32.89 \times 10^3}{400} = 82.23 \text{ MPa (C)}$$

1-39



$$\sum M_C = 0 \Rightarrow F_{ED}(4) - 2(6) = 0$$

$$F_{ED} = \frac{12}{4} = 3 \text{ k}$$

$$\sum F_H = 0 \Rightarrow F_{AE} \frac{3}{5} - 3 = 0$$

$$F_{AE} = 5 \text{ k}$$

Force per bar = 2.5 k

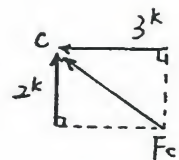
$$\sigma = \frac{2.5 \times 10^3}{6 \times 25} = 16.67 \text{ MPa}$$

Double shear at C

$$\sum F_H = 0 \Rightarrow F_{HC} = 3 \text{ k}$$

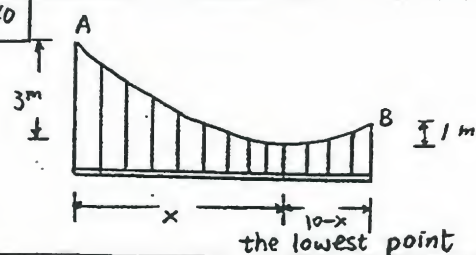
$$\sum F_v = 0 \Rightarrow F_{VC} = 2 \text{ k}$$

$$F_C = \sqrt{3^2 + 2^2} = 3.61 \text{ k}$$

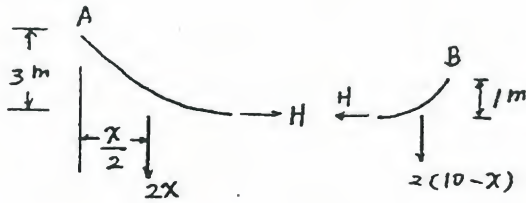


$$\text{Pin shear} = \frac{F}{A} = \left( \frac{3.61}{\pi \frac{19^2}{4}} \right) \frac{1}{2} = 6.36 \text{ MPa}$$

1-40



1-40



$$\sum M_A = 0$$

$$3 \cdot H = 2x \cdot \frac{x}{2} \quad H = \frac{1}{3}x^2$$

$$\sum M_B = 0$$

$$\frac{2(10-x)^2}{2} = H \quad H = (10-x)^2$$

$$\rightarrow \frac{1}{3}x^2 = (10-x)^2$$

$$x = 6.34 \text{ m}, \quad H = 13.40 \text{ kN}$$

at support

$$F_v = 2x = 12.68 \text{ kN}$$

$$R = \sqrt{13.4^2 + 12.68^2} = 18.45 \text{ kN}$$

$$A = \frac{R}{\sigma_y} \times 2 = \frac{18.45 \times 10^3}{1000} \times 2$$

$$= 36.9 \text{ mm}^2$$

1-41

$$F_1 = (R_1 + R_2 + R_3) m \omega^2$$

$$= (0.6 + 1.2 + 1.8)(0.5)(4 \times 2\pi)^2$$

$$= 1136.98 \text{ N}$$

$$F_2 = (R_2 + R_3) m \omega^2$$

$$= (1.2 + 1.8)(0.5)(4 \times 2\pi)^2$$

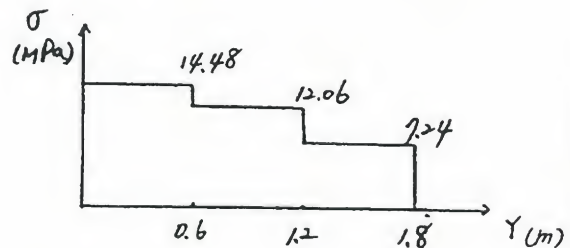
$$= 947.48 \text{ N}$$

$$F_3 = R_3 m \omega^2 = (1.8)(0.5)(4 \times 2\pi)^2 = 568.49 \text{ N}$$

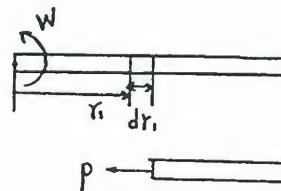
$$\sigma_1 = \frac{1136.98}{\pi(5)^2} = 14.48 \text{ MPa}$$

$$\sigma_2 = \frac{947.48}{\pi(5)^2} = 12.06 \text{ MPa}$$

$$\sigma_3 = \frac{568.49}{\pi(5)^2} = 7.24 \text{ MPa}$$



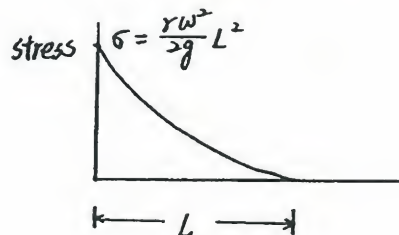
1-42



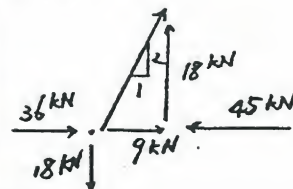
$$P = \int_r^L \left( \frac{\gamma A dr}{g} \right) \omega^2 dr$$

$$= \frac{\gamma A \omega^2}{2g} (L^2 - r^2)$$

$$\sigma = \frac{P}{A} = \frac{r \omega^2}{2g} (L^2 - r^2)$$



1-43



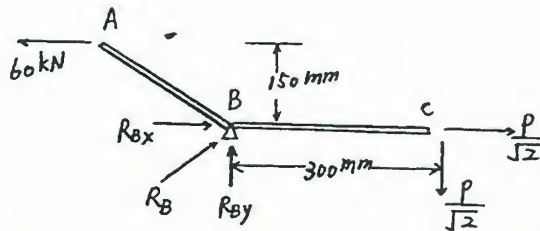
$$\frac{45 \times 10^3}{2 \left( \frac{\pi}{4} d^2 \right)} \leq 20$$

$$d \geq 37.85 \text{ mm}$$

$$d = 40 \text{ mm}$$



1-44



$$\sum M_B = 0 \quad \frac{P}{\sqrt{2}} \times 300 = 60 \times 150$$

$$\frac{P}{\sqrt{2}} = 30 \text{ kN}$$

$$\sum F_H = 0 \quad 30 + R_{Bx} - 60 = 0$$

$$R_{Bx} = 30 \text{ kN}$$

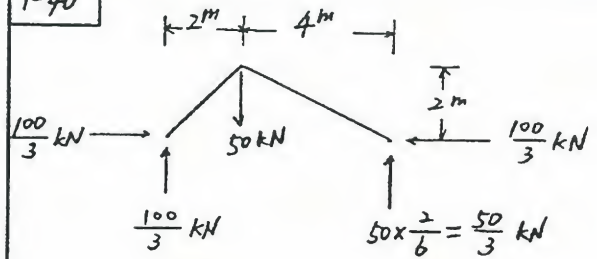
$$\sum F_V = 0 \quad R_{By} = 30 \text{ kN}$$

$$\therefore R_B = 30\sqrt{2} = 42.43 \text{ kN}$$

$$\tau = \frac{R_B}{2 \times \frac{\pi}{4} D^2}$$

$$D = \sqrt{\frac{R_B}{\frac{\pi}{2} \tau}} = \sqrt{\frac{42.43 \times 10^3}{\frac{\pi}{2} \times 100}} = 16.44 \text{ mm}$$

1-46

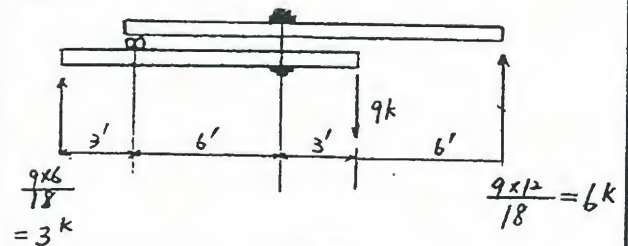


$$\frac{P}{A} = \frac{\tau}{F.S.}$$

$$\frac{100}{3} \times \frac{1}{200 a} = \frac{3.5}{5}$$

$$a = b = 23.8 \text{ mm}$$

1-47



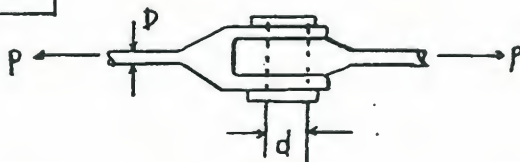
$$\text{tension} = \frac{6 \times 15}{6} = 15 \text{ k}$$

$$A = \frac{P}{\sigma} = \frac{15}{18} = 0.833 \text{ in}^2$$

use  $1 \frac{1}{4}$  inch bolt

$$A_{br} = \frac{15}{0.5} = 30 \text{ in}^2$$

1-45



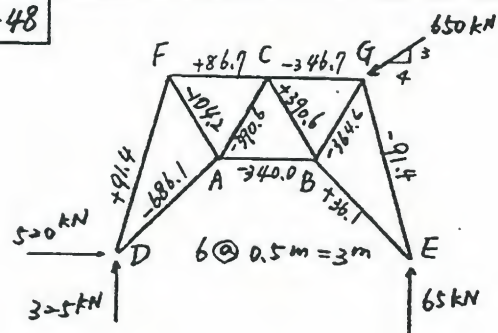
$$\sigma_a = P \cdot \frac{4}{\pi D^2}$$

$$\tau_a = \frac{P}{2} \cdot \frac{4}{\pi d^2}$$

$$\tau_a = \frac{\sigma_a}{2}$$

$$P \cdot \frac{4}{\pi D^2} = 2 \cdot \frac{P}{2} \cdot \frac{4}{\pi d^2} \quad \therefore D = d$$

1-48



$$\sigma_{All} = 140 \text{ MPa}$$

Tension members:  $\overline{DF}$ ,  $\overline{FC}$ ,  $\overline{BE}$ ,  $\overline{CB}$

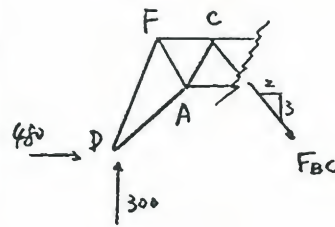
$$A_{DF} = \frac{P}{\sigma} = \frac{91.4 \times 10^3 \text{ N}}{140 \times 10^6 \text{ N/m}^2} = 652.8 \approx 653 \text{ mm}^2$$

1-48

$$A_{FC} = \frac{P}{\sigma} = \frac{86.7 \times 10^3}{140} = 619.3 \approx 620 \text{ mm}^2$$

$$A_{BC} = \frac{390.6 \times 10^3}{140} = 2790 \text{ mm}^2$$

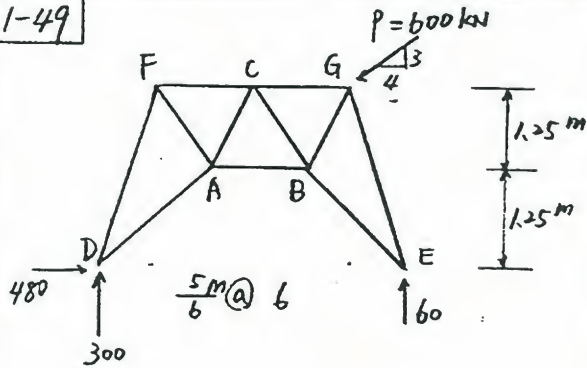
$$A_{BE} = \frac{36.1 \times 10^3}{140} = 257.9 \approx 258 \text{ mm}^2$$



$$\begin{aligned} F_{BC} &= 300 \times \frac{\sqrt{13}}{3} \\ &= 100\sqrt{13} \\ &= 360.56 \text{ kN (tension)} \end{aligned}$$

$$A_{BC} = \frac{F_{BC}}{\sigma_{allow}} = \frac{360.56 \times 10^3}{140} = 2575.39 \approx 2576 \text{ mm}^2$$

1-49

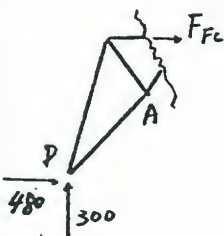


$$\sum F_x = 0 \quad R_{Dx} = 480 \text{ kN} (\rightarrow)$$

$$\sum M_D = 0 \quad (+) \quad 360 \times \frac{25}{6} - 480 \times 2.5 - R_E \times 5 = 0$$

$$R_E = 60 \text{ kN} (\uparrow)$$

$$\sum F_y = 0 \quad R_{Dy} = 300 \text{ kN} (\uparrow)$$



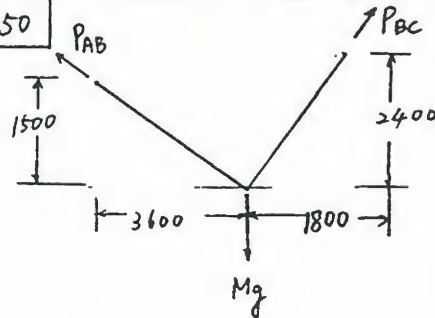
$$\sum M_A = 0 \quad (+)$$

$$F_{FC} \times 1.25 + 300 \times \frac{10}{6} - 480 \times 1.25 = 0$$

$$F_{FC} = 80 \text{ kN (tension)}$$

$$\begin{aligned} A_{FC} &= \frac{F_{FC}}{\sigma_{allow}} = \frac{80 \times 10^3}{140} \\ &= 571.43 \\ &\approx 572 \text{ mm}^2 \end{aligned}$$

1-50



$$\frac{12}{13} P_{AB} = \frac{3}{5} P_{BC}$$

$$\frac{5}{12} P_{AB} + \frac{4}{5} P_{BC} = M_g$$

$$P_{AB} = \frac{13}{21} M_g, \quad P_{BC} = \frac{20}{21} M_g$$

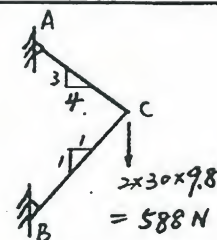
$$\sigma_{AB} = \frac{P_{AB}}{A_{AB}} = \frac{13}{21} \cdot \frac{M_g}{200} = 3.1 M_g \text{ kPa (control)}$$

$$\sigma_{BC} = \frac{20}{21} \cdot \frac{M_g}{400} = 2.38 M_g \text{ kPa}$$

$$\sigma_w = \frac{\sigma_u}{F.S.} = \frac{800}{2} = 3.1 M_g$$

$$M = 13.169 \text{ kJ}$$

1-51



$$\begin{aligned} &2 \times 30 \times 9.8 \\ &= 588 \text{ N} \end{aligned}$$





1-54

$$\sum M_J = 0$$

$$P_{AD} \times 9 + 22.36 \times \frac{1}{\sqrt{5}} \times 3 = 23.57 \times \frac{1}{\sqrt{2}} \times 3$$

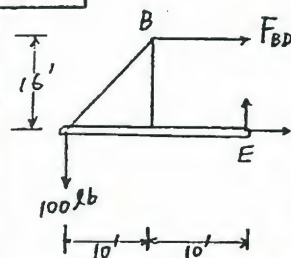
$$P_{AD} = 2.22 \text{ kN}$$

$$A_{AB} = \frac{P_{AB}}{\sigma_{all}} = \frac{36.88 \times 10^3}{85} \approx 434 \text{ mm}^2$$

$$A_{AD} = \frac{P_{AD}}{\sigma_{all}} = \frac{2.22 \times 10^3}{120} \approx 18 \text{ mm}^2$$

$$A_{BF} = \frac{P_{BF}}{\sigma_{all}} = \frac{23.57 \times 10^3}{85} \approx 277 \text{ mm}^2$$

1-55



$$\sum M_E = 0$$

$$F_{BD} \times 16 = 100 \times 20$$

$$\Rightarrow F_{BD} = 125 \text{ lbf}$$

$$\text{Joint B, } \sum F_h = 0$$

$$F_{AB} \times \frac{10}{\sqrt{10^2 + 16^2}} = 125$$

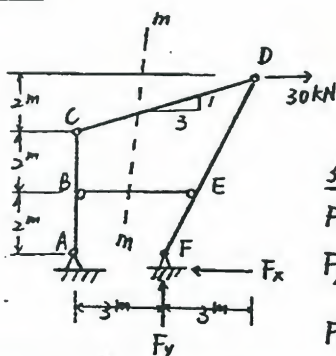
$$\Rightarrow F_{AB} = 235.8 \text{ lbf}$$

$$A_{req} = \frac{F_{AB}}{\sigma_w} = \frac{235.8}{40 \times 10^3} = 5.90 \times 10^{-3} \text{ in}^2$$

$$D = \sqrt{\frac{4 A_{req}}{\pi}} = \sqrt{\frac{4 \times 5.90 \times 10^{-3}}{\pi}}$$

$$= 0.087 \text{ in}$$

1-56



$$\sum M_A = 0$$

$$F_y \times 3 = 30 \times 6$$

$$F_y = 60 \text{ kN}$$

$$\text{section m-m}$$

$$F_v = 0$$

$$F_y = F_{CD} \times \frac{1}{\sqrt{10}}$$

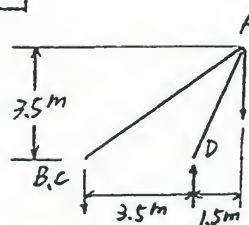
$$F_{CD} = 189.74 \text{ kN}$$

$$\sigma_w = \frac{\sigma_u \times 0.8}{F.S} = \frac{1250 \times 0.8}{1.5} = 666.67 \text{ MPa}$$

$$A = \frac{F_{CD}}{\sigma_w} = \frac{189.74 \times 10^3}{666.67} = 284.61 \text{ mm}^2$$

$$D = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4 \times 284.61}{\pi}} = 17.8 \text{ mm}$$

1-57



$$\sum M_D = 0$$

$$V_{AB} = \frac{200 \times 1.5}{2 \times 3.5}$$

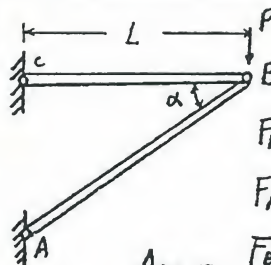
$$= 42.86 \text{ kN}$$

$$R_{AB} = R_{AC} = V_{AB} \frac{\sqrt{(3.5)^2 + [(5)^2 + (2.5)^2]}}{3.5}$$

$$= 42.86 \times \frac{6.6}{3.5} = 80.82 \text{ kN}$$

$$A = \frac{\pi d^2}{4} = \frac{R}{\sigma_{all}} = \frac{80.82}{12}, d = 2.93 \text{ cm}$$

1-58



$$F_{BC} = P \cot \alpha$$

$$F_{AB} = \frac{P}{\sin \alpha}$$

$$A_{BC} = \frac{F_{BC}}{\sigma} = \frac{P}{\sigma} \cot \alpha$$

$$A_{AB} = \frac{F_{AB}}{\sigma} = \frac{P}{\sigma} \frac{1}{\sin \alpha}$$

$$V = L A_{BC} + \frac{L}{\cos \alpha} A_{AB}$$

$$= \frac{PL}{\sigma} \left( \cot \alpha + \frac{1}{\sin \alpha \cos \alpha} \right)$$

$$\frac{\partial V}{\partial \alpha} = 0, \cos^2 \alpha = \frac{1}{3} \therefore \alpha = 54.7^\circ$$

1-59

$$(1) \quad Z = \mu_z - \sigma_z \quad W = \frac{Z - \mu_z}{\sigma_z} = \frac{(\mu_z - \sigma_z) - \mu_z}{\sigma_z} = -1$$

$$Z = \mu_z + \sigma_z \quad W = \frac{(\mu_z + \sigma_z) - \mu_z}{\sigma_z} = 1$$

$$\therefore P(\mu_z - \sigma_z < Z < \mu_z + \sigma_z)$$

$$= P(-1 < W < 1)$$

$$= P(W < 1) - P(W < -1)$$

$$= 0.8413 - 0.1587^*$$

$$= 0.6827$$

$$(2) \quad Z = \mu_z - 2\sigma_z \quad W = \frac{(\mu_z - 2\sigma_z) - \mu_z}{\sigma_z} = -2$$

$$Z = \mu_z + 2\sigma_z \quad W = 2$$

$$\therefore P(\mu_z - 2\sigma_z < Z < \mu_z + 2\sigma_z)$$

$$= P(-2 < W < 2)$$

$$= P(W < 2) - P(W < -2)$$

$$= 0.9772 - 0.0228^*$$

$$= 0.9545$$

\* See "Probability and Statistics for Engineering and Scientists" 5ed. by R.E. Walpole and R.H. Myers. <Table A.3>

1-60

from Fig. 1-26 (c)

strength (unit: $10^3$ psi)	freq.	$X_i$	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
20-22	1	2100	-1840	3385600
22-24	1	2300	-1640	2689600
24-26	2	2500	-1440	2073600
26-28	3	2700	-1240	1537600
28-30	6	2900	-1040	1081600
30-32	11	3100	-840	705600
32-34	12	3300	-640	409600
34-36	17	3500	-440	193600
36-38	21	3700	-240	57600
38-40	22	3900	-40	1600
40-42	20	4100	160	25600
42-44	17	4300	360	129600
44-46	13	4500	560	313600
46-48	9	4700	760	577600
48-50	11	4900	960	921600
50-52	7	5100	1160	1345600
52-54	1	5300	1360	1849600
54-56	1	5500	1560	2433600
56-58	0	5700	1760	0
58-60	0	5900	1960	0
60-62	1	6100	2160	4665600
Total	176			19033600

$$S = \sqrt{\frac{1}{n} (x_i - \bar{x})^2}$$

$$= \sqrt{\frac{19033600}{176}}$$

$$= 670 \text{ psi}$$



2-1

$$E = \frac{\sigma}{\epsilon} = \frac{P/A}{\Delta/L} = \frac{(29500)/\frac{\pi}{4}(0.013)^2}{(2.2 \times 10^{-4})/0.2} = 202 \times 10^9 \frac{N}{m^2}$$

2-2

$$A = \max \text{ of } \begin{cases} \frac{P}{\sigma} = \frac{5 \times 10^3}{150} = 33.33 \text{ mm}^2 \\ \frac{PL}{\Delta E} = \frac{(5 \times 10^3)(10^4)}{(3)(2 \times 10^5)} = 83.33 \text{ mm}^2 \end{cases}$$

$$\frac{\pi}{4} d^2 = 83.33 \rightarrow d = 10.3 \text{ mm}$$

Use 11 mm rod

2-3

$$(a) \frac{PL_1}{AE_{st}} = \frac{PL_2}{AE_{Al}}$$

$$L_1 = \frac{E_{st}}{E_{Al}} L_2 = \frac{200}{70} L_2$$

$$L_2 = \frac{900}{(1 + \frac{200}{70})} = 233 \text{ mm}, L_1 = 667 \text{ mm}$$

$$(b) \Delta = \sum \frac{PL}{EA} = \frac{1.2 \times 10^5}{\pi (25)^2} \left( \frac{667}{2 \times 10^5} + \frac{233}{7 \times 10^4} \right)$$

$$\Delta = 0.407 \text{ mm}$$

2-4

$$\epsilon = \frac{\Delta}{L} = \text{CONSTANT}$$

$$\frac{0.035}{200} = \frac{600}{L}, L = 3429 \text{ m}$$

2-5

$$\bar{\sigma} = \sigma (1 + \epsilon)$$

$$A = A_0 / (1 + \epsilon)$$

	compression					tension				
strain (%)	-0.1	-0.2	-1	-2	-4	0.1	0.2	1	2	4
eng. stress	18	36	72	100	128	18	36	72	85	92
true stress	18.0	35.9	71.3	98.0	122.9	18.0	36.1	72.7	86.7	95.7
true area (A)	1.00	1.00	1.01	1.02	1.04	1.00	1.00	0.99	0.98	0.96

2-6

$$R_o^2 - (R_o - 300)^2 = (25000)^2$$

$$R_o = 1041816.667 \text{ mm}$$

$$\theta_o = 2 \sin^{-1} \frac{25000}{R_o}$$

$$L_o \approx R_o \theta = (1041816.667) \left( \frac{\pi}{180} \times \theta_o \right) = 50004.8 \text{ mm}$$

$$L = L_o [1 + (16 \times 10^{-6})(70)] = 50060.8 \text{ mm}$$

$$2R \sin^{-1} \frac{25000}{R} = 50060.8 \rightarrow R = 293163$$

$$-(293163)^2 - (293163 - S)^2 = (25000)^2$$

$$S = 1068 \text{ mm}$$

Approximate by parabola,  $y = \frac{4H}{L^2} (x-L)x$ 

$$S \approx \int_0^L \sqrt{1 + \left( \frac{dy}{dx} \right)^2} dx$$

$$2-7 \quad \epsilon_z = \frac{P}{AE} = \frac{200 \times 10^3}{\frac{\pi}{4} (60)^2 (85 \times 10^3)} = 8.322 \times 10^{-4}$$

$$\frac{\Delta d}{60} = \nu \epsilon_z \rightarrow \Delta d = 60(0.3)(8.322 \times 10^{-4}) = 0.015 \text{ mm}$$

2-8

$$P_{st} = \epsilon E A / \nu = \alpha \Delta T E A / \nu$$

$$= (11.7 \times 10^{-6})(200)(200) \left( \frac{\pi}{4} \times 20^2 \right) / 0.3 = 490 \text{ kN}$$

$$P_{al} = (23.4 \times 10^{-6})(200)(70) \left( \frac{\pi}{4} \times 20^2 \right) / 0.35 = 294 \text{ kN}$$

2-9

$$(a) F = \frac{5(9.81)}{\cos 30^\circ} = 56.64$$

$$\frac{F}{\sigma_{all}} = \frac{\pi}{4} d^2 \rightarrow d = \sqrt{\frac{4(56.64)}{\pi (300)}} = 0.49 \text{ mm}$$

$$(b) \Delta = \frac{\sigma}{E} L_o = \frac{300}{(200 \times 10^3)} (1.5 \times 10^3) = 2.25 \text{ mm}$$

2-10

$$U_{T, \text{tool steel}} \approx 11 \text{ ksi}$$

$$U_{T, \text{Low-alloy high-strength steel}} \approx 17 \text{ ksi}$$

$$U_{T, \text{Low-carbon steel}} \approx 18 \text{ ksi}$$

2-11

$$\dot{\sigma} + \frac{E}{\eta} \sigma = E \dot{\epsilon}$$

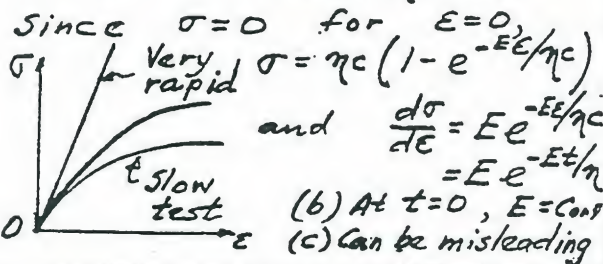
But  $\frac{d\epsilon}{dt} = \text{Const} = c = \dot{\epsilon}$

or  $\epsilon = ct$  or  $t = \frac{\epsilon}{c}$

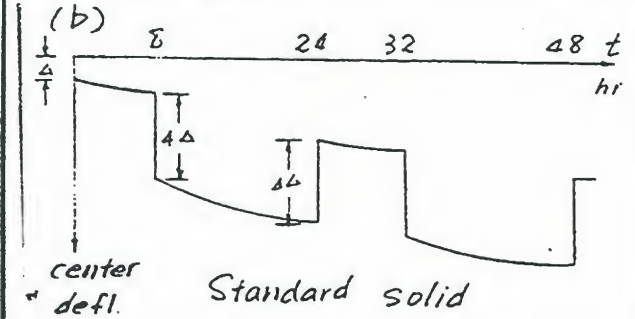
Hence  $\dot{\sigma} + \frac{E}{\eta} \sigma = Ec$

Solution:  $\sigma = Ae^{-\frac{E}{\eta}t} + \eta c$

or  $\sigma = Ae^{-\frac{E\epsilon}{\eta c}} + \eta c$



(b)



2-14

$$\dot{\epsilon} = B \sigma^n$$

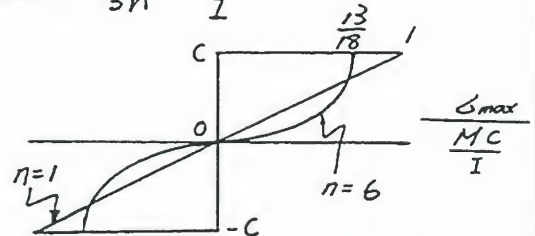
$$\frac{\dot{\sigma}}{\sigma^n} = \frac{\dot{\epsilon}}{\epsilon_{max}} = \frac{y}{c} \rightarrow \sigma = \left(\frac{y}{c}\right)^{\frac{1}{n}} \sigma_{max}$$

$$M = 2 \int_0^c \sigma y dA = \frac{2b \sigma_{max}}{c^{\frac{1}{n}}} \int_0^c y^{\frac{n+1}{n}} dy$$

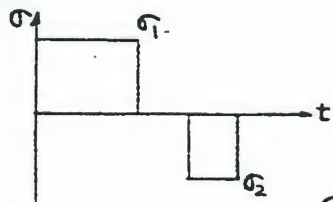
$$M = \frac{2b \sigma_{max}}{c^{\frac{1}{n}}} \frac{n}{2n+1} c^{\frac{2n+1}{n}}$$

$$M = \frac{n}{2n+1} 2b c^2 \sigma_{max} = \frac{3n}{2n+1} \frac{I \sigma_{max}}{c}$$

$$\sigma_{max} = \frac{2n+1}{3n} \frac{MC}{I}$$



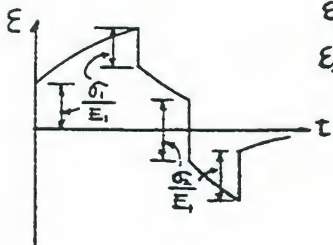
2-12



$$\epsilon = \epsilon_1 + \epsilon_2$$

$$\epsilon_1: \text{due to } E_1 \text{ and } \eta_1$$

$$\epsilon_2: \text{due to } E_2 \text{ and } \eta_2$$



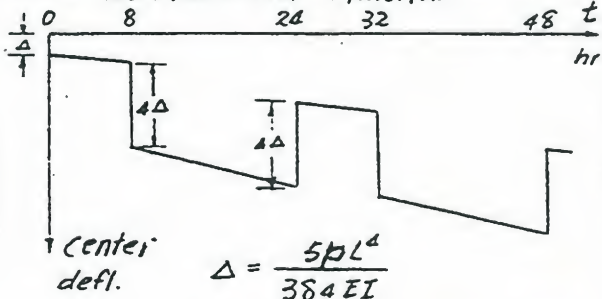
2-15

From Fig. 2-5, the ultimate stress for steel is 186 MPa,

for aluminum is 150 MPa

2-13

(a) Maxwellian Material



$$\Delta = \frac{5pL^4}{384EI}$$

$p = \text{empty wt./unit length}$

2-16

(a) from Fig. 2-5

$$\text{Al: } \frac{2 \times 10^6}{5.3 \times 10^6} + \frac{4 \times 10^5}{8.7 \times 10^5} = 0.84 < 1 \text{ (O.K.)}$$

$$\text{St: } \frac{2 \times 10^6}{1.1 \times 10^6} + \frac{4 \times 10^5}{2 \times 10^5} = 3.82 > 1 \text{ (N.G.)}$$

(b)

$$\text{Al: } \frac{5 \times 10^6}{1.8 \times 10^7} + \frac{4 \times 10^8}{5 \times 10^8} = 1.08 > 1 \text{ (N.G.)}$$

$$\text{St: } 0 + 0 < 1 \text{ (O.K.)}$$



3-1

$$\sigma = \frac{P}{A} = \frac{10 \times 10^3}{\pi/4 d^2} \leq 100$$

$$d \geq 11.28 \text{ mm}$$

$$\epsilon = \frac{\sigma}{E} = \frac{\frac{10 \times 10^3}{\pi/4 d^2}}{45 \times 10^3} \leq 0.1\%$$

$$d \geq 16.82 \text{ mm (governs)}$$

magnesium alloy elastic modulus:  
45 GPa for tension or compression.

3-2

$$(a) A_{req} = \frac{5 \times 10^3}{150} = 33.33 \text{ mm}^2$$

$$A_{req} = \frac{PL}{\Delta E} = \frac{5 \times 10^3 \times 10 \times 10^3}{4 \times 210 \times 10^3} = 59.52 \text{ mm}^2$$

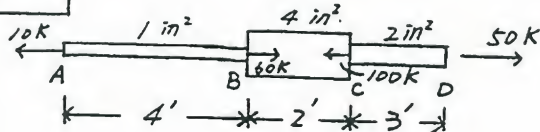
(governs)

→ stiffness control.

$$(b) P = K \Delta$$

$$\Rightarrow K = \frac{P}{\Delta} = \frac{5}{4} = 1.25 \text{ kN/mm}$$

3-3



$$(a) \Sigma F = 0, \quad P_2 = 60 \text{ k}$$

$$(b) \Delta = \left( \frac{PL}{AE} \right)_{AB} + \left( \frac{PL}{AE} \right)_{BC} + \left( \frac{PL}{AE} \right)_{CD}$$

$$= \frac{10(4 \times 12)}{1(30 \times 10^3)} + \frac{(-60)(2 \times 12)}{4(30 \times 10^3)} + \frac{50(3 \times 12)}{2(30 \times 10^3)}$$

$$= 0.016 - 0.010 + 0.03$$

$$= 0.036 \text{ in}$$

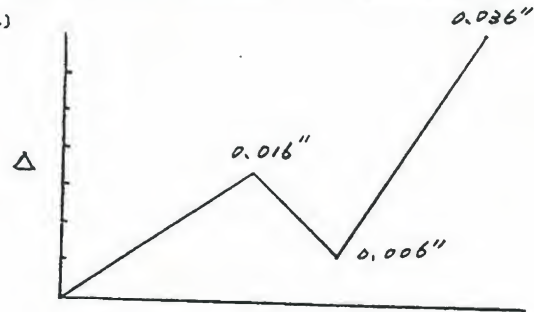
$$(d) f_{eff} = \Sigma \frac{L_i}{EA_i}$$

$$= \frac{1}{30 \times 10^3} \left( \frac{4 \times 12}{1} + \frac{2 \times 12}{4} + \frac{3 \times 12}{2} \right)$$

$$= 2.4 \times 10^{-3} \text{ in/k}$$

$$K_{eff} = \frac{1}{2.4 \times 10^{-3}} = 416.67 \text{ k/in}$$

(c)



3-4

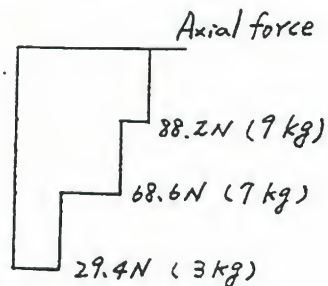
$$\Delta = \frac{PL}{AE} = \frac{mgL}{AE}$$

$$f = \frac{1}{2\pi} \sqrt{\frac{g}{\Delta}} = \frac{1}{2\pi} \sqrt{\frac{AE}{mL}}$$

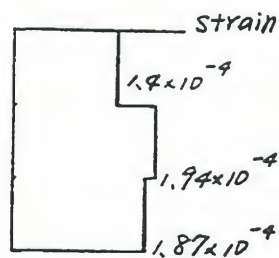
$$= \frac{1}{2\pi} \sqrt{\frac{\pi \times 20^2 \times 180 \times 10^3}{2 \times 0.4}} = 1.3 \text{ kHz}$$

3-5

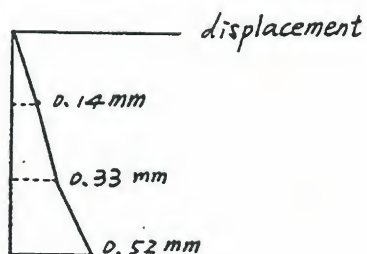
(a)



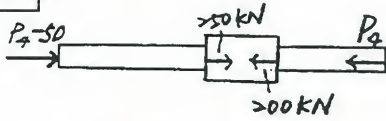
(b)



(c)



3-6

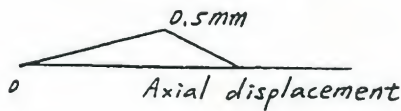
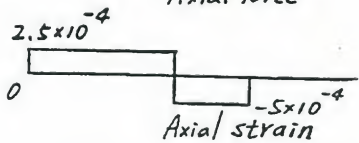
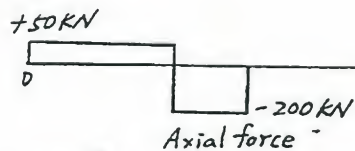


$$\Delta = \sum \left( \frac{PL}{EA} \right)_i$$

$$= - \frac{(P_4 - 50) \times 2000}{1000 \times E} - \frac{(P_4 + 200) \times 1000}{2000 \times E} - \frac{P_4 \times 1500}{1000 \times E}$$

$$= 0$$

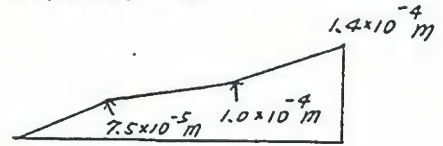
$$\rightarrow P_4 = 0$$



$$\Delta_3 = \Delta_1 + \Delta_2 + \Delta_3$$

$$= 1.0 \times 10^{-4} + (1 \times 10^{-4})(0.4)$$

$$= 1.4 \times 10^{-4} \text{ m}$$



Axial displacement

$$K = \frac{P}{\Delta} \quad (\text{use unit Force})$$

$$\Delta = \frac{P}{E} \left( \frac{L_1}{A_1} + \frac{L_2}{A_2} + \frac{L_3}{A_3} \right)$$

$$= \frac{1}{200 \times 10^6} \left( \frac{0.6}{2 \times 10^{-4}} + \frac{0.5}{1 \times 10^{-4}} + \frac{0.4}{1.5 \times 10^{-4}} \right)$$

$$= 5.33 \times 10^{-5}$$

$$\Rightarrow K = \frac{P}{\Delta} = \frac{1}{5.33 \times 10^{-5}} = 1.875 \times 10^4 \text{ kN/m}$$

3-8

$$(a) \sigma_{AL} = E \epsilon = 70 \times 10^3 \times 873 \times 10^{-6}$$

$$= 61.1 \text{ MPa} = \sigma_{st}$$

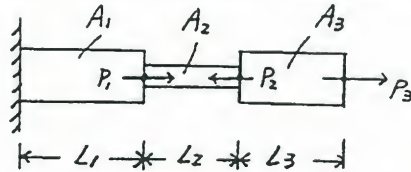
$$P = \frac{\pi}{4} d^2 \sigma = \frac{\pi}{4} \times 50^2 \times 61.1 = 120 \text{ kN}$$

$$(b) \epsilon_{st} = \frac{\sigma_{st}}{E_{st}} = \frac{61.1}{200 \times 10^3} = 305.5 \times 10^{-6}$$

$$\Delta = [(305.5 \times 1500) + (873 \times 500)] \times 10^{-6}$$

$$= 0.89 \text{ mm}$$

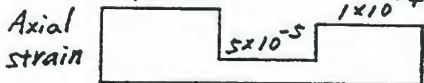
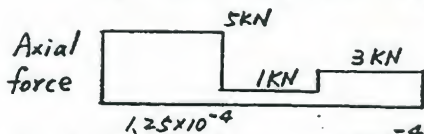
3-7



$$P_1 = 4 \text{ kN} \quad L_1 = 0.6 \text{ m} \quad A_1 = 2 \times 10^{-4} \text{ m}^2$$

$$P_2 = 2 \text{ kN} \quad L_2 = 0.5 \text{ m} \quad A_2 = 1 \times 10^{-4} \text{ m}^2$$

$$P_3 = 3 \text{ kN} \quad L_3 = 0.4 \text{ m} \quad A_3 = 1.5 \times 10^{-4} \text{ m}^2$$



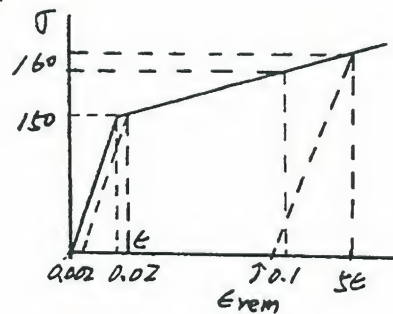
$$\Delta = \frac{PL}{AE} = \epsilon L$$

$$\Delta_1 = 1.25 \times 10^{-4} (0.6) = 7.5 \times 10^{-5} \text{ m}$$

$$\Delta_2 = \Delta_1 + \Delta_2 = (7.5 \times 10^{-5}) + (5 \times 10^{-5})(0.5)$$

$$= 1.0 \times 10^{-4} \text{ m}$$

3-9



$$125(0.02 + \epsilon) = 7500 \times \epsilon$$

$$x = 0.00034$$

$$\epsilon = 0.0034 + 0.22 = 0.2234$$

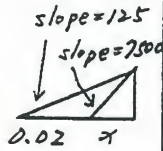
$$5\epsilon = 0.1117$$

$$\sigma = 150 + (0.1117 - 0.02) \times 125 = 161.46 \text{ ksi}$$

$$F = 161.46 \times 5 \times 0.2 = 32.29 \text{ kN}$$

$$\epsilon_{rem} = 0.1117 - \frac{161.46 \times 5}{7500} = 0.09$$

$$\Delta_{rem} = 0.09 \times 600 = 54 \text{ mm}$$

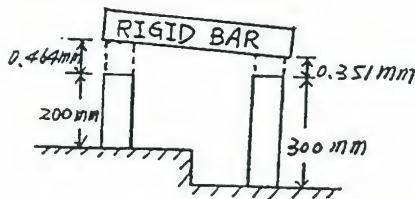


3-10

$$(a) \Delta_{AL} = \alpha L \Delta T = 23.2 \times 10^{-6} \times 200 \times 100 = 0.464 \text{ mm}$$

$$\Delta_{st} = \alpha_{st} L_{st} \Delta T = 11.7 \times 10^{-6} \times 300 \times 100 = 0.351 \text{ mm}$$

$$\text{Inclination} \approx \frac{0.464 - 0.351}{400} = 2.83 \times 10^{-4} \text{ rad}$$



$$(b) \Delta = \frac{PL}{EA} = \frac{\sigma L}{E} \Rightarrow \sigma = \frac{E \Delta}{L}$$

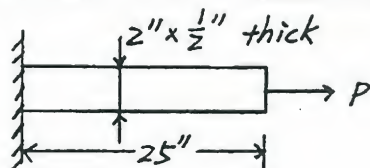
$$\sigma_{AL} = \frac{\Delta_{AL} E_{AL}}{L} = \frac{0.464 \times 75 \times 10^3}{200}$$

$$= 174 \text{ MPa}$$

$$\sigma_{st} = \frac{\Delta_{st} E_{st}}{L_{st}} = \frac{0.351 \times 200 \times 10^3}{300}$$

$$= 234 \text{ MPa}$$

3-11



$$\Delta_x = 0.5 \times 10^{-3} \text{ in}$$

$$\epsilon_x = \frac{-0.3 \times 10^{-3}}{2} = -0.15 \times 10^{-3}$$

$$\nu = \frac{0.15 \times 10^{-3}}{\epsilon_a} = 0.25$$

$$\epsilon_a = 6 \times 10^{-4}$$

$$\sigma = E \epsilon_a = (30 \times 10^3) \times 6 \times 10^{-4} = 18 \text{ ksi}$$

$$P = 18 \times 2 \times \frac{1}{2} = 18 \text{ k}$$

$$\Delta = 25 \times 6 \times 10^{-4} = 0.015 \text{ in}$$

3-12

$$\epsilon_x = \frac{-16 \times 10^{-3}}{180} = -8.9 \times 10^{-5}$$

$$\Rightarrow 8.89 \times 10^{-5} = -\nu \frac{\sigma_y}{E}$$

$$\sigma_y = \frac{8.89 \times 10^{-5} \times 2 \times 10^5}{0.25} = 71.2 \text{ N/mm}^2$$

$$P_{a-a} = \sigma_y A = 71.2 \times 180 \times 10 = 12.8 \times 10^4 \text{ N}$$

$$2g \times 400 = P_{a-a}$$

$$g = \frac{12.8 \times 10^4}{800} = 160 \text{ N/mm}$$

$$P_x = 2gx = 2 \times 160x = 320x \text{ N}$$

$$\Delta = \int \frac{P_x dx}{AE} = \frac{320}{180 \times 10 \times 2 \times 10^5} \times \frac{x^2}{2} \Big|_0^{2000}$$

$$= 1.8 \text{ mm}$$

3-13

$$(a) F = \int_0^L f dy = \int_0^L k y^2 dy = \frac{k L^3}{3}$$

$$k = \frac{3F}{L^3}$$

$$dF = f dy = k y^2 dy$$

$$P = \int_0^y dP = \int_0^y k y^2 dy = \frac{k y^3}{3} = \frac{y^3}{L^3} F$$

$$\Delta = \int_0^L \frac{P}{AE} dy = \int_0^L \frac{F}{AF} \frac{y^3}{L^3} dy = \frac{FL}{4AE}$$

$$(b) \Delta = \frac{400 \times 10^3 (10 \times 10^3)}{4(64000)(10^6)}$$

$$= 1.56 \text{ mm}$$



3-14

$$(a) \Delta_{co} = \frac{PL}{AE} = \frac{\frac{2}{3} \times 2000 \times 60}{0.1 \times 20 \times 10^6} = 0.04"$$

$$\Delta_{AL} = \frac{PL}{AE} = \frac{\frac{1}{3} \times 2000 \times 100}{0.2 \times 10 \times 10^6} = 0.033"$$



$$\Delta_W = \Delta_{co} - \frac{1}{3} (\Delta_{co} - \Delta_{AL})$$

$$= 0.04 - \frac{1}{3} (0.04 - 0.033)$$

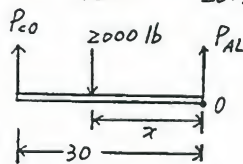
$$= 0.038"$$

$$(b) \Delta_{co} = \Delta_{AL}$$

$$\Rightarrow \left( \frac{PL}{AE} \right)_{AL} = \left( \frac{PL}{AE} \right)_{co}$$

$$P_{AL} = \frac{E_{AL} A_{AL}}{L_{AL}} \times \frac{L_{co}}{E_{co} A_{co}} P_{co}$$

$$= \frac{10 \times 10^6 \times 0.2}{100} \times \frac{60}{20 \times 10^6 \times 0.1} P_{co} = 0.6 P_{co}$$



$$P_{co} + P_{AL} = 2000$$

$$1.6 P_{co} = 2000$$

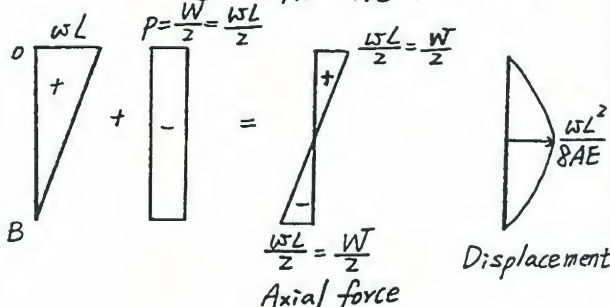
$$P_{co} = 1250 \text{ lb}$$

$$\sum M_o = 0 \Rightarrow 1250 \times 30 = 2000 \times x$$

$$x = 18.75 \text{ inch from the right end}$$

3-15

$$\Delta_B = 0 \quad \frac{WL}{2AE} = \frac{PL}{AE}, \quad P = \frac{WT}{2}$$



3-16

(a) subdivide L into 4 segments

$$\Delta_4 = \sum_{i=1}^4 \frac{P_i L_i}{EA}$$

$$= \frac{1}{EA} \times \frac{L}{4} (W + \frac{3}{4}W + \frac{2}{4}W + \frac{1}{4}W)$$

$$= \frac{1}{EA} \times \frac{L}{4} \times \frac{10}{4} W = \frac{5WL}{8EA}$$

(b) subdivide L into 10 segments

$$\Delta_{10} = \sum_{i=1}^{10} \frac{P_i L_i}{EA}$$

$$= \frac{1}{EA} \times \frac{L}{10} \times \frac{1}{10} (W + 2W + 3W + \dots + 10W)$$

$$= \frac{L}{EA} \times \frac{55}{100} W = \frac{11WL}{20EA}$$

(c) subdivide L into 20 segments

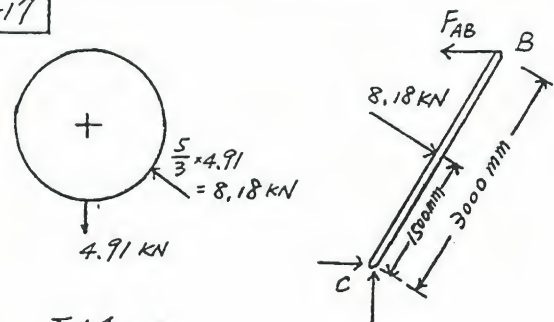
$$\Delta_{20} = \sum_{i=1}^{20} \frac{P_i L_i}{EA}$$

$$= \frac{1}{EA} \times \frac{L}{20} \times \frac{1}{20} (W + 2W + \dots + 20W)$$

$$= \frac{L}{EA} \times \frac{210W}{400} = \frac{21WL}{40EA}$$

$$\therefore \Delta_4 > \Delta_{10} > \Delta_{20} \approx \Delta = \frac{WL}{2EA}$$

3-17



$$\sum M_c = 0$$

$$8.18 \times 1500 = F_{AB} \times 3000$$

$$F_{AB} = 5.11 \text{ kN}$$

$$\Delta = \frac{F_{AB} L}{AE} = \frac{5.11 \times 10^3 (1800)}{5 (200 \times 10^3)}$$

$$= 9.20 \text{ mm}$$

3-18

vertical bar force from C:  $16 \times \frac{3}{4}$ vertical bar force from D:  $16 \times \frac{4}{3}$ 

$$\Delta = \frac{PL}{AE} = \frac{1 \times 10^3}{100 \times 200} \times 16 \times \left( \frac{3}{4} + \frac{4}{3} \right) = 1.67 \text{ mm}$$

3-19

$$\Delta = \frac{PL}{EA} = \frac{\sigma L}{E} = \frac{15 \times 20}{30 \times 10^3}$$

 $= 0.01''$  (elongation of each rod)

$$\Delta_B = \Delta = 0.01 \text{ in}$$

$$\Delta_D = 0$$

$$\Delta_E = \Delta = 0.01 \text{ in}$$

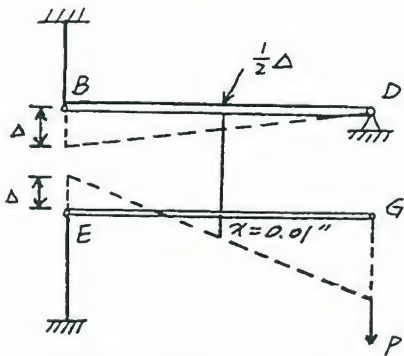
$$\Delta_{CF} = \frac{15 \times 10}{30 \times 10^3} = 0.005 \text{ in}$$

$$\chi - \frac{1}{2} \Delta = 0.005$$

$$\chi = 0.005 + 0.005 = 0.01$$

$$\frac{\Delta_G - \Delta}{2} = 0.001$$

$$\Delta_G = 0.02 + 0.01 = 0.03 \text{ in}$$



3-20

$$\Delta = \frac{WL^2}{2AE}$$

$$L^2 = \frac{2AE\Delta}{W} = \frac{2 \times 1 \times 10^6 \times 0.25}{1.17 \times 12}$$

$$= 356000 \text{ ft}^2$$

$$\therefore L = 596.76 \text{ ft}$$

3-21

$$\epsilon^n = E^{-1} \sigma, \quad \epsilon = \left( \frac{\sigma}{E} \right)^{\frac{1}{n}} = \left( \frac{Px}{Ax E} \right)^{\frac{1}{n}}$$

$$\Delta = \int_0^L \epsilon dx = \int_0^L \left( \frac{Wx}{AE} \right)^{\frac{1}{n}} dx$$

$$= \left( \frac{W}{AE} \right)^{\frac{1}{n}} \left( \frac{n}{n+1} \right) \left[ x^{\frac{n+1}{n}} \right]_0^L$$

$$\Delta = \left( \frac{W}{AE} \right)^{\frac{1}{n}} (L)^{\frac{1}{n}} \left( \frac{n}{n+1} \right) L$$

$$= \left( \frac{W}{AE} \right)^{\frac{1}{n}} \left( \frac{n}{n+1} \right) L$$

3-22

$$(a) \quad \sigma_1 = \frac{5}{1} = 5 \text{ ksi}$$

$$\sigma_2 = \frac{5}{0.5} = 10 \text{ ksi}$$

$$\Delta = \left[ \frac{5}{16000} + \left( \frac{5}{165} \right)^3 \right] \times 50 + \left[ \frac{10}{16000} + \left( \frac{10}{165} \right)^3 \right] \times 100$$

$$= 0.102''$$

$$(b) \quad d\epsilon = \frac{1}{16000} d\sigma + \frac{3}{165^2} \sigma^2 d\sigma$$

$$E = \frac{d\sigma}{d\epsilon} \Big|_{\sigma=0} = 16000 \text{ ksi}$$

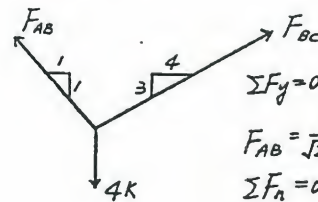
$$\Delta_E = \frac{5}{E} \times 50 + \frac{10}{E} \times 100$$

$$= \frac{5}{16000} \times 50 + \frac{10}{16000} \times 100 = 0.078''$$

$$\Delta_{\text{residual}} = \Delta - \Delta_E$$

$$= 0.102 - 0.078 = 0.024''$$

3-23



$$\sum F_y = 0$$

$$F_{AB} = \frac{1}{\sqrt{2}} + F_{BC} \frac{3}{5} = 20 \text{ k}$$

$$\sum F_x = 0$$

$$F_{AB} \left( \frac{1}{\sqrt{2}} \right) = F_{BC} \left( \frac{4}{5} \right)$$

$$\text{solve: } \Rightarrow F_{BC} = 14.3 \text{ k}$$

$$F_{AB} = 16.15 \text{ k}$$

Wire AB

$$\sigma_{0 \rightarrow 80} ; \Delta = \frac{PL}{AE} = \frac{80 \times 130 (900 \times \sqrt{2})}{130 (80 \times 10^3)} = 1.273 \text{ mm}$$

$$\sigma \rightarrow ; \Delta = \frac{PL}{AE} = \frac{(14300 - 10400)(700\sqrt{2})}{130(40 \times 10^3)} = 0.955 \text{ mm}$$

$$\Delta_{\text{Total}} = 1.27 + 0.96 = 2.23 \text{ mm}$$

Wire BC

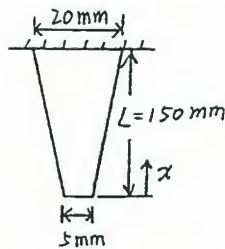
$$\sigma \rightarrow 80 \quad \Delta = \frac{PL}{AE} = \frac{80 \times 100(3.05 \times 10^3)}{100(80 \times 10^3)} = 3.05 \text{ mm}$$

$$80 \rightarrow \quad \Delta = \frac{PL}{AE} = \frac{(16150 - 8000)(3.05 \times 10^3)}{100(40 \times 10^3)}$$

$$= 6.21 \text{ mm}$$

$$\Delta_{\text{Total}} = 9.26 \text{ mm}$$

3-24



$$A_x = 4x[5 + \frac{(20-5)}{150}x] = 4(5 + \frac{1}{10}x)$$

$$P_x = r g \times \frac{1}{2} \times 4(10 + \frac{1}{10}x)x$$

$$= r g (20 + \frac{1}{5}x)x$$

$$\Delta = \int_0^{150} \frac{P_x}{AE} dx = \int_0^{150} \frac{r g (20 + \frac{1}{5}x)x}{4(5 + \frac{1}{10}x)E} dx$$

$$= \int_0^{150} \frac{r g (100 + x)x}{(100 + 2x)E} dx$$

$$= 7637.5 \frac{r g}{E}$$

3-25

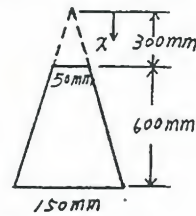
$$\Delta_A = \int_0^{L_A} \frac{P}{2E} dx = \frac{PL_A}{2E}$$

$$\Delta_B = \int_0^{L_B} \frac{P}{(1 + \frac{2x}{L_B})} dx = \frac{PL_B}{2E} \ln 3$$

$$\Delta_A = \Delta_B, \quad \frac{PL_A}{2E} = \frac{PL_B}{2E} \ln 3$$

$$\frac{L_A}{L_B} = \ln 3 = 1.10$$

3-26



$$d = \frac{x}{12}$$

$$A = \pi d^2 = \pi (\frac{x}{12})^2$$

$$P = [\frac{\pi}{3} (\frac{x}{12})^2 x - \frac{\pi}{3} \times 25^2 \times 300] r$$

$$= \frac{\pi r}{3} (\frac{x^3}{144} - 187500)$$

$$\Delta = \int_{300}^{900} \frac{P}{AE} dx = \int_{300}^{900} \frac{\pi r}{3} \frac{(\frac{x^3}{144} - 187500)}{\pi (\frac{x}{12})^2 E} dx$$

$$= \frac{r}{3E} \int_{300}^{900} \frac{x^3 - 27 \times 10^6}{x^2} dx$$

$$= \frac{r}{3E} [\frac{1}{2}x^2 + 27 \times 10^6 \frac{1}{x}] \Big|_{300}^{900}$$

$$= \frac{r}{3E} (36 \times 10^4 - 6 \times 10^4) = 10^5 \frac{r}{E}$$

3-27

$$P = \int_r^L (\frac{\partial A}{\partial r}) W^2 r = \frac{\partial A W^2}{2g} (L^2 - r^2)$$

$$\Delta = \int_0^L \frac{2P dr}{AE} = 2 \int_0^L \frac{\partial A W^2}{2g} (L^2 - r^2) \frac{dr}{AE}$$

$$= \frac{W^2}{gE} [L^2 r - \frac{r^3}{3}] \Big|_0^L = \frac{2W^2 L^3}{3gE}$$

3-28

$$F = \int_a^x -kx' dx' = -\frac{1}{2} kx'^2 \Big|_a^x$$

$$= -\frac{K}{2} (x^2 - a^2)$$

$$\Delta = \int_a^{2a} \frac{F dx}{AE} = \int_a^{2a} \frac{-K}{2EA} (x^2 - a^2) dx$$

$$= \frac{-K}{2AE} (\frac{x^3}{3} - a^2 x) \Big|_a^{2a}$$

$$= -\frac{K}{2AE} [\frac{1}{3}(8a^3 - a^3) - a^3] = \frac{-2Ka^3}{3EA}$$

$$P = \int_a^{2a} kx dx = \frac{kx^2}{2} \Big|_a^{2a} = \frac{k}{2} (4a^2 - a^2)$$

$$= \frac{3Ka^2}{2} \Rightarrow k = \frac{2P}{3a^2}$$

$$\therefore \Delta = -\frac{2a^3}{3EA} \times \frac{2P}{3a^2} = -\frac{4Pa}{9EA}$$



3-29

$$\begin{cases} F_{AB} \sin 26.6^\circ = F_{BC} \sin 45^\circ \\ F_{AB} \cos 26.6^\circ + F_{BC} \cos 45^\circ = 3 \end{cases} \Rightarrow F_{AB} = 2.24 \text{ kips} \\ F_{BC} = 1.42 \text{ kips}$$

$$\Delta_{AB} = \frac{2.24 \times 6.71}{0.25 \times 0.5 \times 10.6 \times 10^3} = 0.0113 \text{ in}$$

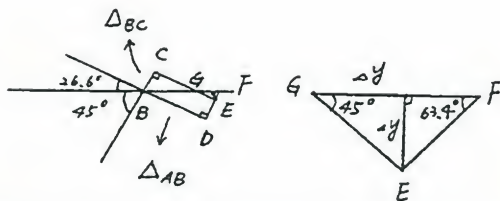
$$\Delta_{BC} = \frac{1.42 \times 8.49}{0.25 \times 0.875 \times 10.6 \times 10^3} = 5.20 \times 10^{-3} \text{ in}$$

$$GF = \frac{\Delta_{AB}}{\cos 26.6^\circ} - \frac{\Delta_{BC}}{\cos 45^\circ} = 5.28 \times 10^{-3} \text{ in}$$

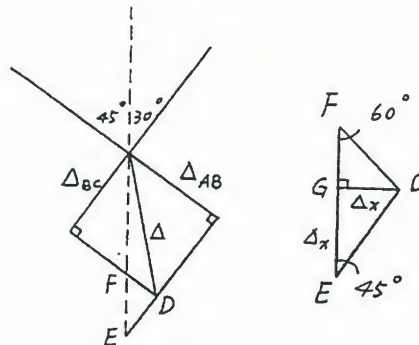
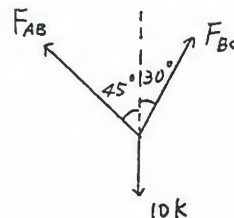
$$GF = \Delta y + \Delta y / \tan 63.4^\circ = 5.28 \times 10^{-3} \text{ in}$$

$$\therefore \Delta y = 3.52 \times 10^{-3} \text{ in}$$

$$\Delta x = \frac{\Delta_{BC}}{\cos 45^\circ} + \Delta y = 0.0109 \text{ in}$$



$$\begin{aligned} \Delta y &= \sqrt{2} \times \Delta_{AB} - \Delta x \\ &= \sqrt{2} \times 0.007321 - 0.00375 \\ &= 0.009978 \text{ in} \end{aligned}$$



3-30

$$\Sigma F = 0, \frac{1}{\sqrt{2}} F_{AB} + \frac{\sqrt{3}}{2} F_{BC} = 10 \quad F_{AB} = 5.177 \text{ k}$$

$$\frac{1}{\sqrt{2}} F_{AB} = \frac{1}{2} F_{BC} \Rightarrow F_{BC} = 7.321 \text{ k}$$

$$\Delta_{AB} = \left( \frac{PL}{AE} \right)_{AB} = \frac{5.177 \times 100 \sqrt{2}}{10^4} = 0.07321 \text{ in}$$

$$\Delta_{BC} = \left( \frac{PL}{AE} \right)_{BC} = \frac{7.321 \times 100 \times \frac{\sqrt{3}}{2}}{10^4} = 0.08454 \text{ in}$$

$$EF = \sqrt{2} \times \Delta_{AB} - \frac{2}{\sqrt{3}} \times \Delta_{BC}$$

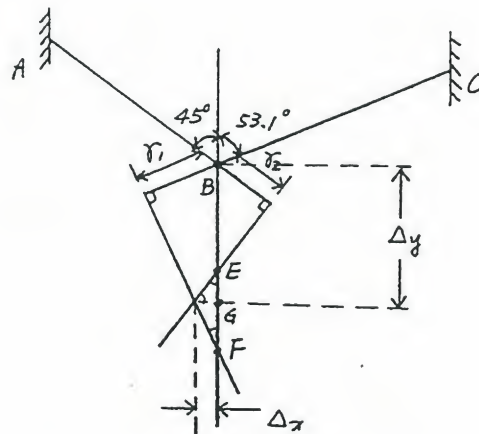
$$= \sqrt{2} \times 0.07321 - \frac{2}{\sqrt{3}} \times 0.08454$$

$$= 0.00592$$

$$EF = \frac{\Delta x}{\sqrt{3}} + \Delta x = 0.00592$$

$$\Delta x = 0.00375 \text{ in}$$

3-31



From Prob. 3-23

$$\gamma_1 = \frac{(14.3 \times 10^3) \times (3.05 \times 10^3)}{100 \times 80 \times 10^3} = 5.45 \text{ mm}$$

$$\gamma_2 = \frac{(16.15 \times 10^3) \times (0.9 \sqrt{2} \times 10^3)}{130 \times 80 \times 10^3} = 1.98 \text{ mm}$$

$$BF = BE + \Delta_x \tan 45^\circ + \frac{\Delta_x}{\tan 36.9^\circ}$$

$$\text{where } BF = \frac{r_1}{\cos 53.1^\circ}, \quad BE = \frac{r_2}{\cos 45^\circ}$$

$$\Delta_x = \frac{\frac{r_1}{\cos 53.1^\circ} - \frac{r_2}{\cos 45^\circ}}{\tan 45^\circ + \frac{1}{\tan 36.9^\circ}} = 2.69 \text{ mm}$$

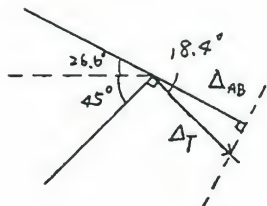
$$\Delta_y = \Delta_x + BE = 5.49 \text{ mm}$$

3-32

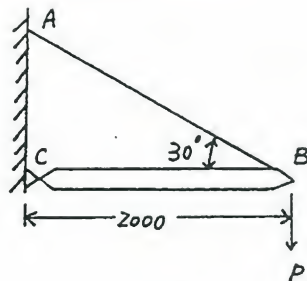
$$\Delta_{AB} = 8.656 \times 10^{-3} \text{ in}$$

$$\Delta_{BC} = 0$$

$$\begin{aligned} \Delta_T &= \frac{\Delta_{AB}}{\cos 18.4^\circ} \\ &= \frac{8.656 \times 10^{-3}}{\cos 18.4^\circ} \\ &= 9.122 \times 10^{-3} \text{ in} \end{aligned}$$



3-33



(a) crane stiffness

$P = 1$  (unit force)

$$P_{AB} \times \frac{1}{2} = 1$$

$$\Rightarrow P_{AB} = 2 \text{ (tensile)}$$

$$P_{BC} = P_{AB} \times \frac{\sqrt{3}}{2} = 1.732 \text{ (compressive)}$$

$$\begin{aligned} \Delta_{AB} &= \frac{PL}{AE} = \frac{2(2000 \times \frac{\sqrt{3}}{2})}{3 \times 10^{-4} (200 \times 10^9)} \\ &= 7.70 \times 10^{-5} \text{ mm} \end{aligned}$$

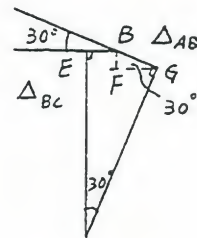
$$\begin{aligned} \Delta_{BC} &= \frac{1.732 \times 2000}{(3.2 \times 10^{-4})(200 \times 10^9)} \\ &= 5.41 \times 10^{-5} \text{ mm} \end{aligned}$$

$$EG = \Delta_{BC} + \frac{\sqrt{3}}{2} \Delta_{AB}$$

$$= 1.21 \times 10^{-4} \text{ mm}$$

$$X = EG \times \sqrt{3} + \frac{1}{2} \Delta_{AB} = 2.48 \times 10^{-4} \text{ mm}$$

$$K = \frac{P}{\Delta} = \frac{1}{2.48 \times 10^{-7}} = 4.0 \times 10^6 \text{ N/m}$$



(b) Deflection from 16 kN

$$\begin{aligned} \Delta &= \frac{P}{K} = \frac{16 \times 10^3}{4.0 \times 10^6} = 4.0 \times 10^{-3} \text{ m} \\ &= 4.0 \text{ mm} \end{aligned}$$

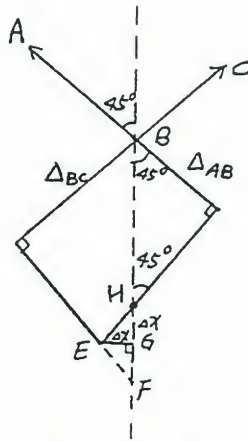
3-34

$$\Delta = \frac{mg}{K}$$

$$f = \frac{1}{2\pi} \sqrt{\frac{g}{\Delta}} = \frac{1}{2\pi} \sqrt{\frac{K}{m}}$$

$$= \frac{1}{2\pi} \sqrt{\frac{4.0 \times 10^6}{2000}} = 7.12 \text{ Hz}$$

3-35



$$\alpha = 13.0 \times 10^{-6} \text{ per } ^\circ\text{F}$$

$$\Delta_{AB} = 100 \times 13 \times 10^{-6} \times 900 \sqrt{2}$$

$$= 1.655 \text{ mm}$$

$$\Delta_{BC} = 100 \times 13 \times 10^{-6} \times 3050$$

$$= 3.965 \text{ mm}$$

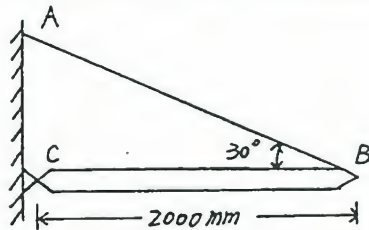
$$HF = \frac{5}{3} \Delta_{BC} - \sqrt{2} \Delta_{AB} = 4.268 \text{ mm}$$

$$HF = \Delta x + \Delta x \times \frac{4}{3} = 4.268 \text{ mm}$$

$$\Delta x = 1.829 \text{ mm}$$

$$\Delta y = \sqrt{2} \Delta_{AB} + \Delta x = 4.170 \text{ mm}$$

3-36



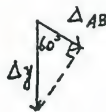
$$\Delta_{BC} = 0$$

$$\Delta_{AB} = 11.7 \times 10^{-6} (2000 \times \frac{2}{\sqrt{3}}) (80^\circ)$$

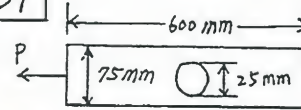
$$= 2.162 \text{ mm}$$

$$\Delta x = \Delta_{BC} = 0$$

$$\Delta y = \frac{\Delta_{AB}}{\cos 60^\circ} = 4.324 \text{ mm}$$



3-37



from graph  
on fig. 3-11

$$K = 2.18$$

$$P = \frac{\sigma A}{K} = \frac{0.22 (75-25) \times 6}{2.18} = 30.28 \text{ kN}$$

3-38

from graph on Fig. 3-11,

$$K = 1.75$$

$$\frac{\sigma}{\sigma_{\max}} = \frac{\frac{P}{A}}{K \frac{P}{A}} = \frac{1}{K} = \frac{1}{1.75} = 0.57$$

3-39

$$\frac{Y_1}{d} = \frac{8}{40} = 0.20, \quad K_1 = 1.63$$

$$\Rightarrow P_1 = \frac{40 \times 10 \times 60}{1.63} = 14.7 \text{ kN}$$

$$\frac{Y_2}{d} = \frac{12}{60} = 0.20, \quad K_2 = 2.30$$

$$\Rightarrow P_2 = \frac{(60-24) \times 10 \times 60}{2.30} = 9.4 \text{ kN}$$

3-40

small bar  $\frac{Y}{d} = \frac{6}{20} = 0.3, \quad K_1 = 1.53$

bend  $\frac{Y}{d} = \frac{20}{35} = 0.57, \quad K_2 = 1.37$

pin hole  $\frac{Y}{d} = \frac{10}{50} = 0.2, \quad K_3 = 2.3$

(a)  $P_1 = \frac{80}{1.53} \times 20 t_1 = 12000$

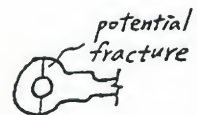
$$\Rightarrow t_1 = 11.5 \text{ mm}$$

$$P_2 = \frac{80}{1.37} \times 35 t_2 = 12000$$

$$\Rightarrow t_2 = 5.9 \text{ mm}$$

$$P_3 = \frac{80}{2.3} \times 30 t_3 = 12000$$

$$\Rightarrow t_3 = 11.5 \text{ mm}$$



(b) The potential fracture might occur in the pin hole section.



3-41

Number of cycles:  $10 \times 10^6$  $\Rightarrow$  stress amplitude 185 MPa

$$\frac{Y_1}{d} = \frac{2}{10} = 0.2 \Rightarrow K_1 = 1.63$$

$$\Rightarrow \sigma_{1\max} = 1.63 \times 185 = 301.55 \text{ MPa}$$

$$\text{or } 301.55 \times 10 \text{ mm}^2 = 3015.5 \text{ N} \\ = 3.0155 \text{ kN}$$

$$\frac{Y_2}{d} = \frac{4}{20} = 0.2 \Rightarrow K_2 = 2.30$$

$$\Rightarrow \sigma_{2\max} = 2.30 \times 185 = 425.5 \text{ MPa}$$

$$\text{or } 425.5 \times (20-8) \times 1 = 5106 \text{ N} \\ = 5.106 \text{ kN}$$

3-42

(a) from graph on Fig. 3-11,

$$\frac{Y}{D} = \frac{10}{60} = \frac{1}{6}, \quad K = 2.35$$

$$\sigma_{\max} = K \frac{P}{A} = 2.35 \times \frac{300}{60 \times 10} = 1.175 \text{ GPa}$$

$$(b) \Delta = \sum \frac{PL}{AE} = \frac{300 \times 120}{200 \times 60 \times 10} + \frac{300 \times 120}{200 \times 40 \times 10} \\ = 0.75 \text{ mm}$$

$$(c) \Delta = 0.02 \times 120 + \frac{350 \times 120}{200 \times 60 \times 10} = 2.75 \text{ mm}$$

$$(d) \Delta_{\text{res}} = \Delta - \Delta_E \\ = 2.75 - \left( \frac{350 \times 120}{200 \times 60 \times 10} + \frac{350 \times 120}{200 \times 40 \times 10} \right) \\ = 2.75 - 0.875 = 1.875 \text{ mm}$$

3-43

for aluminum,

$$U_{r, \text{elastic}} = \frac{44^2}{2 \times 10.6} = 91.32 \text{ psi}$$

$$U_{r, \text{hyper}} = \frac{60^2}{2 \times 10.6} = 169.81 \text{ psi}$$

$$\text{for magnesium, } U_{r, \text{elastic}} = \frac{22^2}{2 \times 6.5} = 37.23 \text{ psi}$$

$$U_{r, \text{hyper}} = \frac{40^2}{2 \times 6.5} = 123.68 \text{ psi}$$

$$\text{for steel, } U_{r, \text{elastic}} = \frac{36^2}{2 \times 30} = 21.6 \text{ psi}$$

$$U_{r, \text{hyper}} = \frac{65^2}{2 \times 30} = 325 \text{ psi}$$

3-44

for bar 12,

$$U \approx \frac{18.33 \times 0.5}{\frac{\pi}{4} \times 0.505^2 \times 0.5} = 91.51 \text{ ksi}$$

for bar 15,

$$U \approx \frac{18.33 \times 1}{\frac{\pi}{4} \times 0.505 \times 3.5} = 26.15 \text{ ksi}$$

3-45

from example 3-4

$$\sigma_{AB} = 17.8 \text{ ksi}, \quad P_{AB} = 2.23 \text{ kips}$$

$$L_{AB} = 6.71''$$

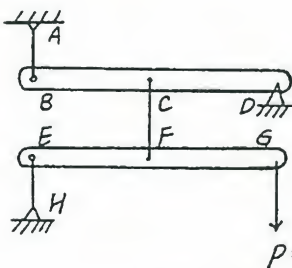
$$\sigma_{BC} = 12.9 \text{ ksi}, \quad P_{BC} = 2.83 \text{ kips}$$

$$L_{BC} = 8.49''$$

$$\frac{P\Delta}{2} = \sum \frac{\sigma^2}{2E} AL = \sum \frac{\sigma PL}{2E}$$

$$\Delta = \frac{1}{3 \times 10.6 \times 10^3} [17.8 \times 2.23 \times 6.71 + \\ 12.9 \times 2.83 \times 8.49] \\ = 0.018''$$

3-46



$$P = 300 \text{ lbs}$$

$$\text{Forces: } \sum M_F = 0 \Rightarrow F_{EH} = 300 \text{ lbs}$$

$$\sum M_E = 0 \Rightarrow F_{CF} = 600 \text{ lbs}$$

$$A_{CF} = \frac{F_{CF}}{\sigma} = \frac{600}{15 \times 10^3} = 0.04 \text{ in}^2$$

$$\sum M_D = 0 \Rightarrow F_{AB} = 300 \text{ lbs}$$

$$A_{AB} = \frac{F_{AB}}{\sigma} = \frac{300}{15 \times 10^3} = 0.02 \text{ in}^2$$

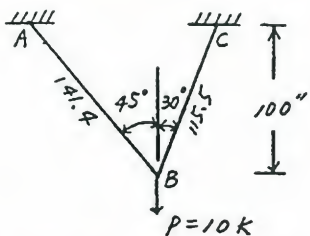
$$U = \frac{1}{2} \left[ \left( \frac{P^2 L}{AE} \right)_{AB} + \left( \frac{P^2 L}{AE} \right)_{CF} + \left( \frac{P^2 L}{AE} \right)_{EH} \right]$$

$$= \frac{1}{2} P \Delta$$

$$P \Delta = \left[ \frac{0.3^2 \times 20}{30 \times 10^3 \times 0.02} + \frac{0.3^2 \times 10}{30 \times 10^3 \times 0.02} + \frac{0.6^2 \times 20}{30 \times 10^3 \times 0.04} \right]$$

$$\therefore \Delta = 0.035 \text{ in}$$

3-47



$$\sum F_y = 0 \Rightarrow F_{AB} \cos 45^\circ + F_{CB} \cos 30^\circ = 10 \text{ k}$$

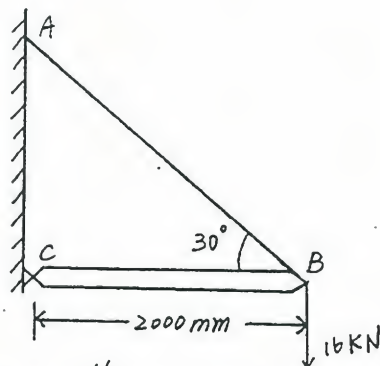
$$\sum F_x = 0 \Rightarrow F_{AB} \sin 45^\circ + F_{CB} \sin 30^\circ = 0$$

$$\text{solve, } F_{AB} = 5.177, F_{CB} = 7.231$$

$$U = \frac{1}{2} \frac{(5.177)^2 \times 141.4}{10^4} + \frac{1}{2} \frac{(7.231)^2 \times 115.5}{10^4} = \frac{10}{2} \Delta$$

$$\Rightarrow \Delta = 0.0998 \text{ in}$$

3-48



$$P_{AB} = \frac{16}{\sin 30^\circ} = 32 \text{ kN}$$

$$P_{BC} = 27.712 \text{ kN}$$

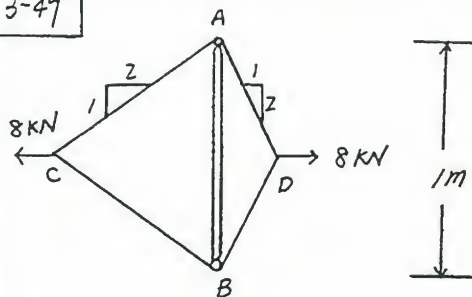
$$\frac{1}{2} P \Delta = \frac{1}{2} \frac{P_{AB}^2 L_{AB}}{A_{AB} E} + \frac{1}{2} \frac{P_{BC}^2 L_{BC}}{A_{BC} E}$$

$$\Rightarrow \frac{(32 \times 10^3)^2 (2.31)}{(300 \times 10^6) (200 \times 10^9)} + \frac{(27.712 \times 10^3)^2 (2.0)}{(320 \times 10^6) (200 \times 10^9)}$$

$$= 16 \times 10^3 \Delta$$

$$\Rightarrow \Delta = 0.004 \text{ m} = 4 \text{ mm}$$

3-49



$$T_{AD} = T_{DB} = 8.94 \text{ kN}$$

$$T_{CB} = T_{CA} = 4.47 \text{ kN}$$

$$T_{AB} = 8 \text{ kN} + 2 \text{ kN} = 10 \text{ kN}$$

$$\frac{1}{2} \times 2 \times \frac{(4.47 \times 1000)^2 \times 1.118}{20 \times 200 \times 1000}$$

$$+ \frac{1}{2} \times 2 \times \frac{(8.94 \times 1000)^2 \times 0.559}{40 \times 200 \times 1000}$$

$$+ \frac{1}{2} \frac{(10 \times 1000)^2 \times 1}{10 \times 200 \times 1000} = \frac{1}{2} \times 8.0 \Delta$$

$$\Delta = 3.42 \text{ mm}$$

3-50

$$\text{static force} = mg = 1.5 \times 9.81$$

$$= 14.715$$

$$\text{Bar 'A'} \Rightarrow \Delta_{ST} = \frac{PL}{AE} = \frac{14.7(2000)}{5^2 \pi (200 \times 10^3)}$$

$$= 0.0019 \text{ mm}$$

$$\sigma_{\max} = \frac{14.7}{5^2 \pi} \left( 1 + \sqrt{1 + \frac{2 \times 1000}{0.0019}} \right) = 192.3 \text{ MPa}$$

$$\text{Bar 'B'} \Rightarrow \Delta_{ST} = \frac{PL}{AE} = \frac{14.7 \times 2000}{\left(\frac{15}{4}\right)^2 \pi (200 \times 10^3)}$$

$$= 0.0008 \text{ mm}$$

$$\sigma_{\max} = \frac{14.7}{\frac{15^2}{4} \pi} \left( 1 + \sqrt{1 + \frac{2000}{0.0008}} \right) = 131.6 \text{ MPa}$$

$$\text{Bar 'C'} \Rightarrow \Delta_{ST} = \frac{14.7(1000)}{78.5(200 \times 10^3)} + \frac{14.7(1000)}{\frac{15^2}{4} \pi (200 \times 10^3)}$$

$$= 1.35 \times 10^{-3} \text{ mm}$$

for lower rod

$$\sigma_{\max} = \frac{14.7}{78.5} \left( 1 + \sqrt{1 + \frac{2000}{1.35 \times 10^{-3}}} \right) = 228.1 \text{ MPa}$$

for upper rod

$$\sigma_{\max} = \frac{78.5}{176.7} \times 228.1 = 101.3 \text{ MPa}$$

3-51

$$\Delta = 200 \text{ mm}$$

$$u = \frac{1}{2} k \Delta^2 = \frac{1}{2} m v^2$$

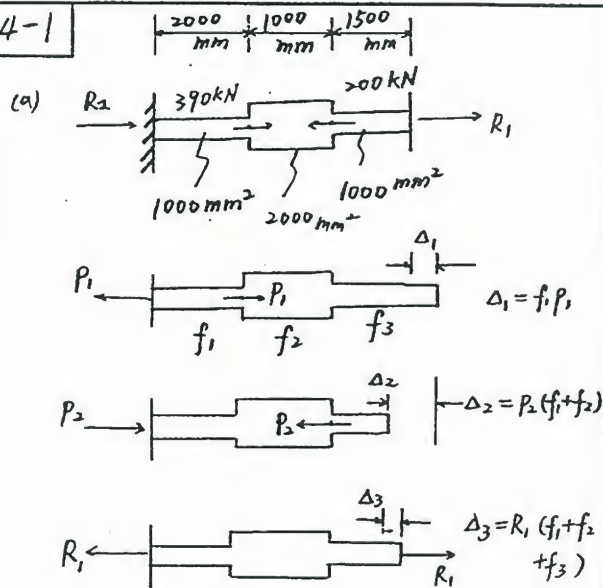
$$\Rightarrow k = \frac{m v^2}{\Delta^2} = \frac{(1 \text{ kg}) 3^2}{(0.02 \text{ m})^2}$$

$$= 45000 \text{ N/m}$$

$$= 45 \text{ kN/m}$$



4-1



$$f = \frac{L}{AE} \Rightarrow f_1 = 1 \times 10^{-5}$$

$$f_2 = 2.5 \times 10^{-6}$$

$$f_3 = 7.5 \times 10^{-6}$$

$$\text{also } \Delta_1 + \Delta_2 + \Delta_3 = 0$$

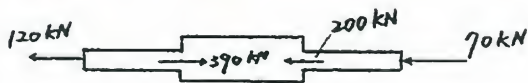
$$\Rightarrow f_1 P_1 + (-P_2)(f_1 + f_2) + R_1(f_1 + f_2 + f_3) = 0$$

$$\Rightarrow R_1 = \frac{f_1 P_1 + (-P_2)(f_1 + f_2)}{(f_1 + f_2 + f_3)}$$

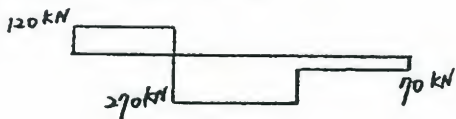
$$= \frac{1 \times 10^{-5}(390) + (-200)(1 \times 10^{-5} + 2.5 \times 10^{-6})}{(1 \times 10^{-5}) + (2.5 \times 10^{-6}) + (7.5 \times 10^{-6})}$$

$$= -70 \text{ kN}$$

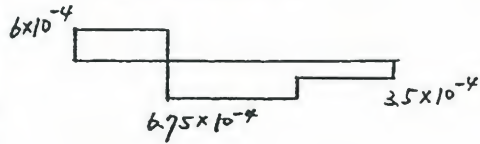
$$\Sigma F = 0, R_2 = -120 \text{ kN}$$



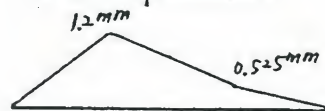
Axial force



Axial strain



Axial displacement



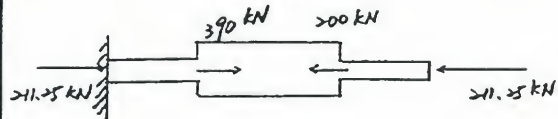
4-2

$$\Delta_1 + \Delta_2 + \Delta_3 + \Delta_4 = 0$$

$$R_1 = \frac{f_1 P_1 + (f_1 + f_2)(-P_2) + \alpha L \Delta t}{f_1 + f_2 + f_3}$$

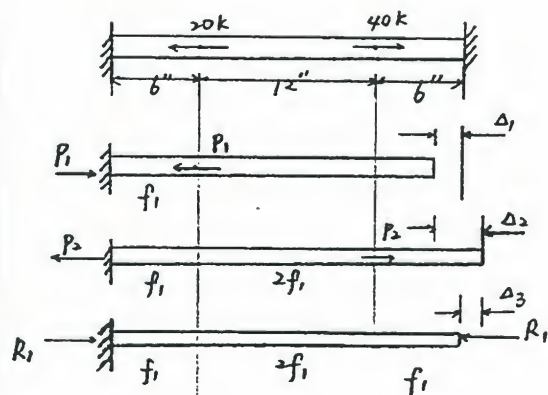
$$= \frac{(1 \times 10^{-5})(390) + (1 \times 10^{-5} + 2.5 \times 10^{-6})(-200) + 25 \times 10^{-6} \times 4.5 \times (-50)}{1 \times 10^{-5} + 2.5 \times 10^{-6} + 7.5 \times 10^{-6}}$$

$$= -211.25 \text{ kN}$$



4-3

$$f = \frac{L}{AE} \therefore f \propto L$$

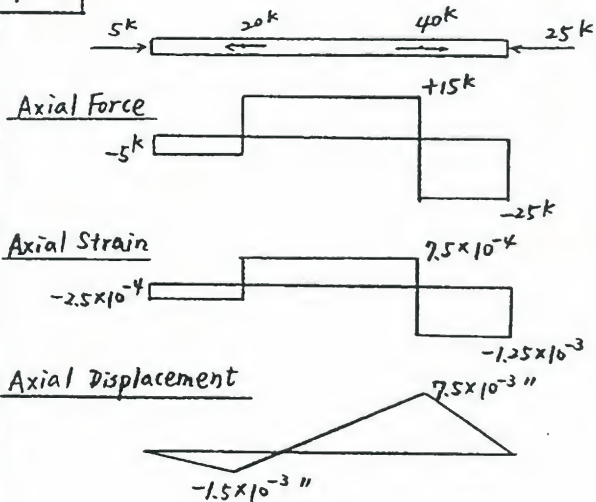


$$\Delta_1 + \Delta_2 + \Delta_3 = 0$$

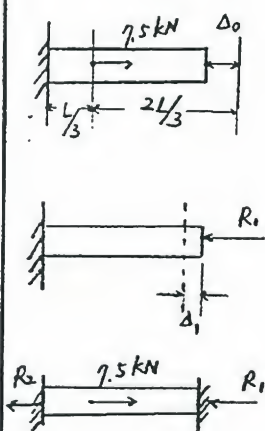
$$-P f_1 + P_2 2f_1 + (-R_1)(f_1) = 0$$

$$R_1 = \frac{3P_2 - P_1}{4} = \frac{3 \times 40 - 20}{4} = 25 \text{ kN}$$

4-3



4-4



From Fig. 2-25,  
for \$5 \times 10^8\$ cycles,  
\$S\_{e2} = 127\$ MPa

$$\Delta_0 = \frac{7.5 \times \frac{L}{3}}{EA} = \frac{2.5L}{EA}$$

$$\Delta_1 = \frac{-R_1 L}{EA}$$

$$\Delta_0 + \Delta_1 = 0$$

$$\frac{2.5L}{EA} = \frac{R_1 L}{EA}$$

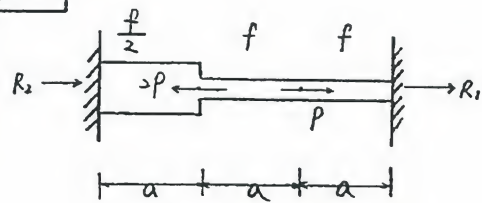
$$\therefore R_1 = 25 \text{ kN}$$

$$R_2 = 7.5 - 2.5 = 5 \text{ kN} \quad (\text{governs})$$

$$\frac{R_2}{A} \times \frac{1}{F.S} = S_{e2}$$

$$\frac{5 \times 10^3}{A} \times \frac{1}{1.8} = 127 \quad \therefore A = 21.87 \text{ mm}^2$$

4-5

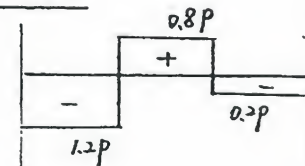


$$\Delta = \Delta_0 \quad 2p(2f) - pf = \frac{5}{2} R_2 f$$

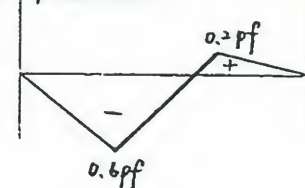
$$R_2 = 1.2p$$

$$R_1 = -0.2p$$

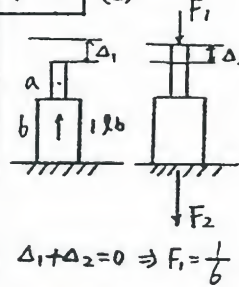
Axial Force



Axial Displacement



4-6 (I)



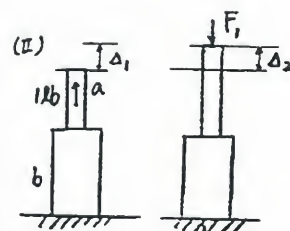
$$\Delta_1 = \frac{1 \times \frac{L}{4}}{2A_2 E} = \frac{L}{8A_2 E}$$

$$\Delta_2 = -\frac{F_1 \frac{L}{2}}{A_2 E} - \frac{F_1 \frac{L}{2}}{2A_2 E}$$

$$= -\frac{3F_1 L}{4A_2 E}$$

$$\Delta_1 + \Delta_2 = 0 \Rightarrow F_1 = \frac{1}{6}, F_2 = \frac{5}{6}$$

$$\Delta_{ab} = \frac{1}{6} \times \frac{L}{4} = \frac{L}{24A_2 E}$$



$$\Delta_1 = \frac{1 \times \frac{L}{4}}{A_2 E} + \frac{1 \times \frac{L}{2}}{2A_2 E} = \frac{L}{2A_2 E}$$

$$\Delta_2 = -\frac{3F_1 L}{4A_2 E}$$

$$\Delta_1 + \Delta_2 = 0 \Rightarrow F_1 = \frac{2}{3}, F_2 = \frac{1}{3}$$

$$\Delta_{ba} = \frac{1}{3} \times \frac{L}{4} = \frac{L}{24A_2 E}$$

$$\therefore \Delta_{ab} = \Delta_{ba}$$



4-7

statics:  $2F_{AL} + F_{st} = 1 \dots (1)$

geometry:  $\Delta_{AL} = \Delta_{st}$

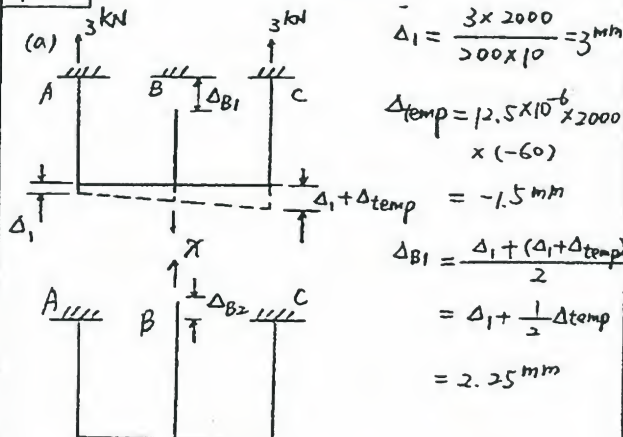
$$\frac{F_{AL} (25 \times 12)}{0.2 (10 \times 10^6)} = \frac{F_{st} (50 \times 12)}{0.3 (30 \times 10^6)}$$

$$F_{AL} = \frac{4}{9} F_{st} \dots (2)$$

Solving (1) & (2),

$$F_{st} = \frac{9}{17} \text{ k}, F_{AL} = \frac{4}{17} \text{ k}$$

4-8



$$\Delta_1 = \frac{3 \times 2000}{200 \times 10} = 3 \text{ mm}$$

$$\Delta_{temp} = 12.5 \times 10^{-6} \times 2000 \times (-60) = -1.5 \text{ mm}$$

$$\Delta_1 + \Delta_{temp} = -1.5 \text{ mm}$$

$$\Delta_{B1} = \frac{\Delta_1 + (\Delta_1 + \Delta_{temp})}{2}$$

$$= \Delta_1 + \frac{1}{2} \Delta_{temp}$$

$$= 2.25 \text{ mm}$$

$$\Delta_{B2} = \frac{\chi L}{AE} + \frac{\frac{1}{2} \chi L}{AE} = \frac{3 \chi L}{2AE}$$

$$\therefore \Delta_{B1} = \Delta_{B2}$$

$$2.25 = \frac{3 \chi L}{2AE}$$

$$\chi = 1.5 \text{ kN (central wire)}$$

$$F = 3 - \frac{1.5}{2} = 2.25 \text{ kN (side wires)}$$

(b) For wire slack,  $F_B = 0$

$$\Delta_A \leq \Delta_B$$

$$\frac{PL}{AE} \leq \alpha L \Delta T$$

$$\Rightarrow \Delta T \geq \frac{PL}{AE} \cdot \frac{1}{\alpha L} = \frac{3}{10 \times 200 \times 12.5 \times 10^{-6}} = 120^\circ \text{C}$$

4-9

$$F_{st} = \frac{9}{17} \times 500 = 264.71 \text{ lb}$$

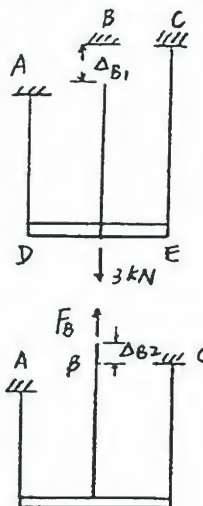
$$\Delta = \frac{264.71 \times 50}{(30 \times 10^6) \times 0.3} = 1.4706 \times 10^{-3} \text{ in}$$

$$f = \frac{\sqrt{g/\Delta}}{2} = \frac{\sqrt{32.2 \times 12 / (1.4706 \times 10^{-3})}}{2}$$

$$= 256.30 \text{ Hz}$$

4-10

Without slipping from left support ~



$$\Delta_{AD} = \frac{PL}{AE} = \frac{1.5 \times 1500}{10 \times 200} = 1.125 \text{ mm}$$

$$\Delta_{CE} = \frac{1.5 \times 2000}{10 \times 200} = 1.5 \text{ mm}$$

$$\therefore \Delta_{B1} = \frac{1}{2} (\Delta_{AD} + \Delta_{CE}) = 1.375 \text{ mm}$$

$$\Delta_{AD} = \frac{\frac{1}{2} F_B \times 1500}{10 \times 200} = 0.375 F_B$$

$$\Delta_{CE} = \frac{\frac{1}{2} F_B \times 2000}{10 \times 200} = 0.5 F_B$$

$$\therefore \Delta_{B2} = \frac{1}{2} (\Delta_{AD} + \Delta_{CE}) + \frac{F_B \times 2000}{10 \times 200} = 1.4375 F_B$$

$$\Delta_{B1} = \Delta_{B2} \Rightarrow F_B = 0.956 \text{ kN}$$

$$F_A = F_C = 1.022 \text{ kN}$$

With slipping from left support ~

$$\Delta_{AD} = 1.125 + 3 = 4.125 \text{ mm}$$

$$\Delta_{CE} = 1.5 \text{ mm}$$

$$\Delta_{B1} = \frac{1}{2} (\Delta_{AD} + \Delta_{CE}) = 2.8125 \text{ mm}$$

$$\Delta_{B2} = 1.4375 F_B$$

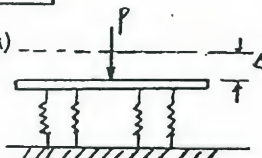
$$\Delta_{B1} = \Delta_{B2} \Rightarrow F_B = 1.957 \text{ kN}$$

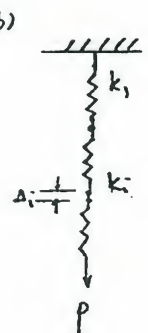
$$F_A = F_C = 0.522 \text{ kN}$$

additional force:  $\Delta F_A = \Delta F_C = -0.5 \text{ kN}, \Delta F_B = 1 \text{ kN}$



4-14

(a)   $P = \Delta K$   
 also,  $P = \Delta k_1 + \Delta k_2 + \dots + \Delta k_n$   
 $\Rightarrow K = \frac{P}{\Delta} = k_1 + k_2 + \dots + k_n$   
 $\therefore K = \sum_n k_i$

(b)   $P = \Delta K$   
 $\Delta = \Delta_1 + \Delta_2 + \dots + \Delta_n$   
 $\frac{P}{K} = \frac{P}{k_1} + \frac{P}{k_2} + \frac{P}{k_3} + \dots + \frac{P}{k_n}$   
 $\therefore \frac{1}{K} = \sum_n \frac{1}{k_i}$

4-15

$$\frac{1}{K} = \frac{1}{300} + \frac{1}{200} = \frac{5}{600} \Rightarrow K = 120 \text{ N/mm}$$

$$K_{\text{sys}} = 120 + 2 \times 250 = 620 \text{ N/mm}$$

$$\Delta = \frac{P}{K_{\text{sys}}} = \frac{6200}{620} = 10 \text{ mm}$$

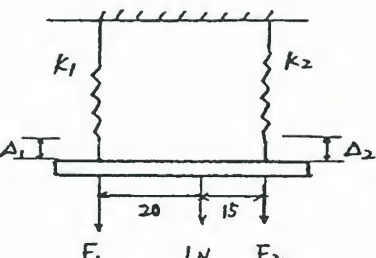
(a) Reactions:  $R = K \Delta$   
 $\left. \begin{array}{l} R = 120 \times 10 = 1200 \text{ N} \\ \text{or } R = 250 \times 10 = 2500 \text{ N} \end{array} \right\} \begin{array}{l} \text{(Top Reaction)} \\ \text{(Bottom Reaction)} \end{array}$

(b) Deflection between upper springs

$$\Delta_1 = \frac{1200}{300} = 4 \text{ mm}$$

$$\Delta_2 = \frac{1200}{200} = 6 \text{ mm}$$

4-16

  $F_1 = \frac{3}{7} N$   
 $F_2 = \frac{4}{7} N$   
 $\Delta_1 = \frac{3}{7 k_1}$   
 $\Delta_2 = \frac{4}{7 k_2}$



$$\begin{aligned} \Delta &= \Delta_1 + \frac{4}{7} (\Delta_2 - \Delta_1) \\ &= \frac{3}{7} \Delta_1 + \frac{4}{7} \Delta_2 \\ &= \frac{9}{49 k_1} + \frac{16}{49 k_2} \\ &= \frac{9 k_2 + 16 k_1}{49 k_1 k_2} \end{aligned}$$

$$K = \frac{1}{\Delta} = \frac{49 k_1 k_2}{9 k_2 + 16 k_1}$$

4-17

$$P = (2 K_{AR} + K_{st}) \Delta$$

$$\Rightarrow \Delta = \frac{P}{2 K_{AR} + K_{st}}$$

$$R_{AR} = K_{AR} \Delta = \frac{P K_{AR}}{2 K_{AR} + K_{st}}$$

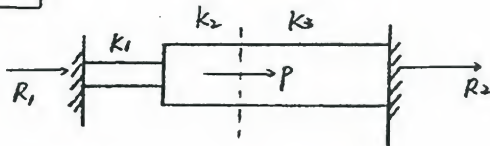
$$R_{st} = K_{st} \Delta = \frac{P K_{st}}{2 K_{AR} + K_{st}}$$

and  $K = \frac{EA}{L} \therefore K_{AR} : K_{st} = 4 : 9$

$$P = 1 \text{ k} \Rightarrow R_{AR} = \frac{4}{17} \text{ k}, R_{st} = \frac{9}{17} \text{ k}$$



4-18



$$K = \frac{EA}{L} \quad \therefore K_1 : K_2 : K_3 = 1 : 2 : 1$$

$$\begin{aligned} R_1 &= \Delta K_0 \\ R_2 &= \Delta K_3 \end{aligned} \quad \begin{aligned} P + \Delta K_0 + \Delta K_3 &= 0 \\ \Delta &= \frac{-P}{K_0 + K_3} \end{aligned}$$

$$\text{where } \frac{1}{K_0} = \frac{1}{K_1} + \frac{1}{K_2}$$

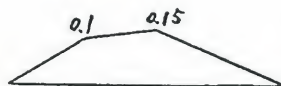
Solving:

$$R_1 = \frac{-PK_0}{K_0 + K_3} = -\frac{2}{5}P \quad (\leftarrow)$$

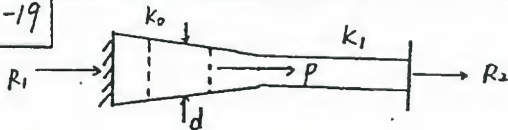
$$R_2 = \frac{-PK_3}{K_0 + K_3} = -\frac{3}{5}P \quad (\leftarrow)$$

Axial Displacement

$$\text{unit: } \frac{PL}{AE}$$



4-19



$$R_1 = +\Delta K_0, \quad R_2 = +\Delta K_1$$

$$P + \Delta K_0 + \Delta K_1 = 0 \quad \Delta = -\frac{P}{K_0 + K_1}$$

$$\text{where } \frac{1}{K_0} = \int \frac{1}{K_x} dx$$

$$K_x = \frac{AE}{L} = \frac{(30 - \frac{40x}{L})tE}{L}$$

$$\frac{1}{K_0} = \frac{L}{Et} \int_0^{L/2} \frac{dx}{(30L - 40x)}$$

$$= \frac{L}{Et} \times \frac{1}{40} \times \ln 3 \quad K_0 = 36.41 \frac{Et}{L}$$

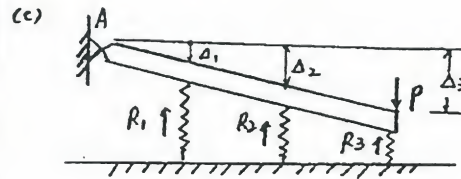
$$\text{and } K_1 = \frac{E 10t}{L/2} = 20 \frac{Et}{L}$$

$$\therefore K_0 : K_1 = 1.82 : 1$$

$$R_1 = \frac{-PK_0}{K_0 + K_1} = \frac{-P 1.82}{1.82 + 1} = -0.65P \quad (\leftarrow)$$

$$R_2 = -0.35P \quad (\leftarrow)$$

4-20 (a) &gt; (b)



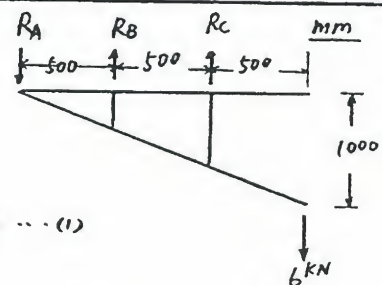
$$\Sigma M_A = 0$$

$$\Rightarrow (\Delta_1 k) a + (\Delta_2 k) 2a + (\Delta_3 k) 3a - P 3a = 0$$

$$\Delta_1 = \frac{1}{2} \Delta_2 = \frac{1}{3} \Delta_3 \quad \Rightarrow \Delta_1 = \frac{3P}{14k}$$

$$R_N = \Delta_N k \Rightarrow R_1 = \frac{3P}{14}, \quad R_2 = \frac{3P}{7}, \quad R_3 = \frac{9P}{14}$$

4-21



statics:

$$\Sigma M_A = 0$$

$$R_B + 2R_C = 18 \quad \dots (1)$$

Geometry:

$$\Delta_C = 2\Delta_B \quad \frac{R_C \times 2}{100} = 2 \times \frac{R_B \times 1}{80}$$

$$R_B = \frac{4}{5} R_C \quad \dots (2)$$

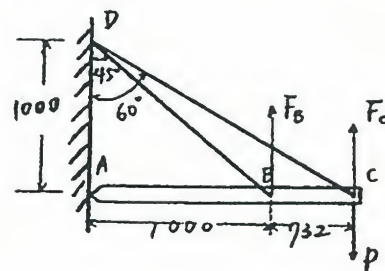
Solving (1) &amp; (2):

$$R_B = 5.14 \text{ kN } (\uparrow)$$

$$R_C = 6.43 \text{ kN } (\uparrow)$$

$$\Sigma F_y = 0 : \quad R_A = 6.43 + 5.14 - 6 = 5.57 \text{ kN } (\downarrow)$$

4-22



4-22

$$\sum M_A = 0$$

$$1.732P = 1.732F_C + F_B \quad \dots (1)$$

$$\Delta_C = 1.732 \Delta_B$$

$$\Delta_C = \frac{F_C L}{EA \cos^3 60^\circ}$$

$$\Delta_B = \frac{F_B L}{EA \cos^3 45^\circ}$$

$$\rightarrow \frac{F_C}{\cos^3 60^\circ} = \frac{F_B}{\cos^3 45^\circ} = 1.732$$

$$\rightarrow F_C = 0.612 F_B \quad \dots (2)$$

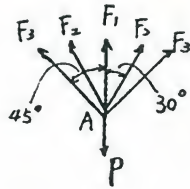
$$\text{Solving: } F_B = 6.73 \text{ kN}$$

$$F_C = 4.11 \text{ kN}$$

$$\text{the force in wire B: } \frac{6.73}{\cos 45^\circ} = 9.52 \text{ kN}$$

$$\text{in wire C: } \frac{4.11}{\cos 60^\circ} = 8.22 \text{ kN}$$

4-23



statics:

$$F_1 + 2F_2 \cos 30^\circ + 2F_3 \cos 45^\circ = P$$

$$\text{Geometry: } F_2 = F_1 \cos 30^\circ = \frac{3}{4} F_1$$

$$F_3 = F_1 \cos 45^\circ = \frac{1}{2} F_1$$

$$\text{Solving: } F_1 \approx \frac{P}{3}, F_2 = \frac{P}{4}, F_3 = \frac{P}{6}$$

Deflection at joint A due to load  $P=2$ 

MN:

$$\Delta = \frac{PL}{AE} = \frac{F_1 L}{AE} \quad (\text{using center bar})$$

$$= \frac{2L}{3AE}$$

$$= \frac{2 \times 2000}{3 \times 0.2 \times 500}$$

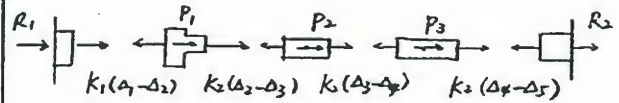
$$= 13.33 \text{ mm}$$

4-24

(a) 5 degrees of kinematic indeterminacies

3 degrees of static indeterminacies

(b)



$$\text{where } k_1 = 2k_2$$

Eqs.

$$1) -2k_2 \Delta_2 = P_1$$

$$2) 2k_2 \Delta_2 + k_2 \Delta_3 - k_2 \Delta_4 = -P_1 = -3P$$

$$3) -k_2 \Delta_2 + 2k_2 \Delta_3 - k_2 \Delta_4 = -P_2 = -2P$$

$$4) -k_2 \Delta_3 + 2k_2 \Delta_4 = -P_3 = -P$$

$$5) k_2 \Delta_4 = R_2$$

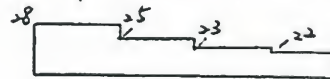
$$\text{Solving: } \Delta_2 = \frac{-R_1}{2k_2}, \Delta_4 = \frac{+R_2}{k_2}$$

$$\Delta_3 = \frac{3P - \frac{3}{2}R_1}{k_2} \quad \text{by } 2)$$

$$\Delta_3 = \frac{P + 2R_2}{k_2} \quad \text{by } 4)$$

$$\Rightarrow R_1 = (P - R_2) \frac{4}{3}$$

$$R_1 = -28P, R_2 = 22P$$

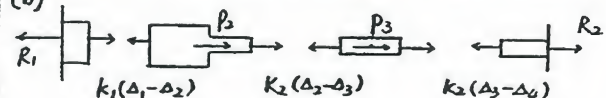


4-25

(a) 3 degrees of static indeterminacies

4 degrees of kinematic indeterminacies

(b)



$$\Delta_1 = \Delta_4 = 0$$

$$R_1 = -k_1 \Delta_2$$

$$k_1 \Delta_2 + k_2 (\Delta_2 - \Delta_3) = -P_2 = -2P$$

$$-k_2 (\Delta_2 - \Delta_3) + k_2 \Delta_3 = -P_3 = -P$$

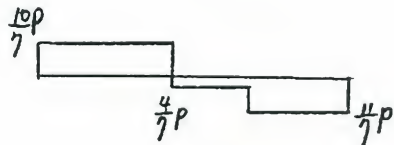
$$k_2 \Delta_3 = R_2$$

4-25

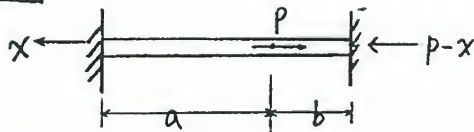
solving where  $\frac{1}{K_1} = \frac{1}{\frac{2AE}{L}} + \frac{1}{\frac{1AE}{L}}$

$$= \frac{3L}{2AE}$$

$$R_1 = \frac{10}{7}P, \quad R_2 = \frac{-11}{7}P$$



4-26



$$\sigma = k \epsilon^n$$

$$\epsilon = \sigma^{\frac{1}{n}} k^{-\frac{1}{n}}$$

$$\epsilon_a(a) = \epsilon_b(b)$$

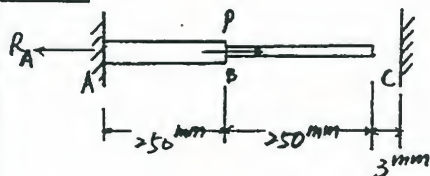
$$\sigma_a^{\frac{1}{n}} k^{-\frac{1}{n}} a = \sigma_b^{\frac{1}{n}} k^{-\frac{1}{n}} b$$

$$\sigma_a a^n = \sigma_b b^n$$

$$\left(\frac{X}{A}\right) a^n = \left(\frac{P-X}{A}\right) b^n$$

$$\therefore X = \frac{P}{\left(\frac{a}{b}\right)^n + 1}$$

4-27 (a)



Rod AB:

$$P_{yp} = \sigma_{yp} A = 200 \times 200 = 40000 \text{ N} = 40 \text{ kN}$$

$$\mu_{yp} = \frac{\sigma_{yp} L}{E} = \frac{200 \times 250}{2 \times 10^5} = 0.25 \text{ mm}$$

Rod BC:

$$P_{yp} = \sigma_{yp} A = 200 \times 100/1000 = 20 \text{ kN}$$

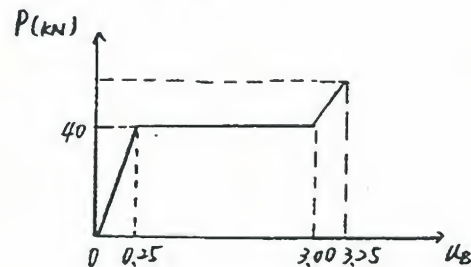
$$\mu_{yp} = \frac{\sigma_{yp} L}{E} = \frac{200 \times 250}{2 \times 10^5} = 0.25 \text{ mm}$$

$$P_{ult} = 40 + 20 = 60 \text{ kN}$$

$$\mu_B \text{ at ult} = (\mu_{yp})_{BC} + \text{gap}$$

$$= 0.25 + 3$$

$$= 3.25 \text{ mm}$$



$$(b) \quad 3 - 0.25 = 2.75 \text{ mm}$$

4-28

$$A_s = 4 \times (0.5)^2 \times \pi = 3.14 \text{ in}^2$$

$$A_c = 14^2 - 3.14 = 192.86 \text{ in}^2$$

$$(a) \quad \epsilon_{allow}^c = \frac{\sigma_{allow}^c}{E_c} = \frac{2000}{2 \times 10^6} = 1 \times 10^{-3}$$

$$\epsilon_{allow}^s = \frac{\sigma_{allow}^s}{E_s} = \frac{24 \times 10^3}{30 \times 10^6} = 0.8 \times 10^{-3} \text{ (governs)}$$

$$\therefore \epsilon_{allow} = 0.8 \times 10^{-3}$$

$$P = P_s + P_c = \epsilon_{allow} (A_s E_s + A_c E_c)$$

$$= 0.8 \times 10^{-3} (3.14 \times 30 \times 10^3 + 192.86 \times 2 \times 10^3)$$

$$= 384 \text{ kips}$$



4-28

$$\begin{aligned}
 (b) P_{ult} &= P_{ult}^c + P_{ult}^s \\
 &= 4.6 \times 192.86 + 60 \times 3.14 \\
 &= 1076 \text{ kips}
 \end{aligned}$$

4-29

$$\text{statics: } 2F_{AL} + F_{ST} = 100 \dots (1)$$

$$\text{geometry: } \Delta_{AL} = \text{gap} + \Delta_{ST}$$

$$\frac{F_{AL}(10)}{2(10 \times 10^3)} = 0.005 + \frac{F_{ST}(9.995)}{4(30 \times 10^3)}$$

$$6F_{AL} - 0.9995F_{ST} = 60 \dots (2)$$

Solving (1) &amp; (2):

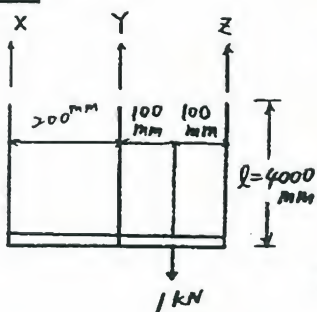
$$F_{ST} = 60 \text{ k}, F_{AL} = 20 \text{ k}$$

$$(a) \sigma_{ST} = \frac{F_{ST}}{A} = \frac{60}{4} = 15 \text{ ksi}$$

$$(b) \Delta_{AL} = \frac{F_{AL}L}{EA} = \frac{20 \times 10}{10 \times 10^3 \times 2} = 0.010 \text{ in}$$

$$(c) P_n = 40 \times 4 + 60 \times 4 = 400 \text{ kips}$$

4-30



statics:

$$\sum F_y = 0$$

$$X + Y + Z = 1000 \dots (1)$$

 $\sum M_A = 0$ 

$$3X + Y - Z = 0 \dots (2)$$

geometry:

$$\Delta Y = \frac{1}{2}(\Delta X + \Delta Z)$$

$$\frac{Y L}{EA} = \frac{1}{2} \left( \frac{X L}{EA} + \frac{Z L}{EA} \right) \Rightarrow X - 2Y + Z = 0 \dots (3)$$

Solving (1), (2) &amp; (3):

$$X = 83.3 \text{ N } (\uparrow)$$

$$Y = 333.3 \text{ N } (\uparrow)$$

$$Z = 583.4 \text{ N } (\uparrow)$$

4-31

$$\text{statics: } P_{AL} = P_{ST} = P \dots (1)$$

$$\text{geometry: } \Delta_{AL} = \Delta_{ST}$$

$$\alpha_{AL}(\Delta T)L - \sum_{AL} \frac{P_{AL}L}{AE}$$

$$= \alpha_{ST}(\Delta T)L + \frac{P_{ST}L}{AE}$$

$$12 \times 10^{-6}(110)(7) - \frac{P_{AL}(3)}{0.3(10^9)} - \frac{P_{AL}(4)}{0.4(10^9)}$$

$$= 6.5 \times 10^{-6}(110)(7) + \frac{P_{ST}(7)}{0.7 \times (3 \times 10^9)} \dots (2)$$

$$\text{Solving (1) & (2): } P = 1815 \text{ lb}$$

4-32

$$\Delta_1 = \frac{-R_1 L}{A_1 E_1} - \alpha_1 L(\Delta T) \quad \left. \begin{array}{l} \Delta_1 = \Delta_2 \end{array} \right\}$$

$$\Delta_2 = \frac{R_2 L}{A_2 E_2} - \alpha_2 L(\Delta T)$$

$$R_1 = R_2 = \sigma A = (20 \text{ MPa})(1000) = 20 \text{ kN}$$

$$\frac{-R_1 L}{A_1 E_1} - \alpha_1 L(\Delta T) = \frac{R_2 L}{A_2 E_2} - \alpha_2 L(\Delta T)$$

$$\Delta T = \left( \frac{R_1}{A_1 E_1} + \frac{R_2}{A_2 E_2} \right) \left( \frac{-1}{\alpha_1 - \alpha_2} \right) = -42.2^\circ$$

$$\text{TEMP} = 60^\circ - 42.2^\circ = 17.8^\circ \text{C}$$

4-33

$$\frac{F_2(L/\cos \alpha)}{AE} = \frac{F_1 L}{\frac{A}{3} E} \cos \alpha$$

$$\therefore F_2 = 3F_1 \cos^2 \alpha$$

$$F_1 + 2F_2 \cos \alpha = P$$

$$F_1 + 6F_1 \cos^3 \alpha = P$$

$$\Rightarrow F_1 = \frac{P}{1 + 6 \cos^3 \alpha} \dots (1)$$

$$F_2 = \frac{3P \cos^2 \alpha}{1 + 6 \cos^3 \alpha} \dots (2)$$

4-35

$$\Delta = \frac{(\delta P)}{k'} = \frac{212.52}{6887} = 0.0309 \text{ in}$$

$$\Delta = 2 \frac{1}{9E} \left( \frac{L^3}{2} - \frac{L^3}{6} \right)$$

$$= \frac{2}{3} \frac{1}{9E} L^3$$

4-36

From Prob. 3-28,  $P_x = \frac{2P}{3a^2} x$

$$\therefore EA \frac{d^2 u}{dx^2} = -\frac{2P}{3a^2} x$$

B.C. at  $x=2a$ ,  $EA \frac{du}{dx} = -P$

$$\Rightarrow EA \frac{du}{dx} = -\frac{P}{3a^2} x^2 + c_1$$

$$-P = -\frac{P}{3a^2} \times 4a^2 + c_1$$

$$c_1 = \frac{1}{3} P$$

$$\Rightarrow EA \frac{du}{dx} = -\frac{P}{3a^2} x^2 + \frac{1}{3} P$$

$$\Delta = \frac{1}{EA} \int_a^{2a} \left( -\frac{P}{3a^2} x^2 + \frac{1}{3} P \right) dx$$

$$= \frac{1}{EA} \left( -\frac{P}{9a^2} \times 7a^3 + \frac{1}{3} P \times a \right)$$

$$= -\frac{4Pa}{9EA}$$

4-37

$$AE \frac{d^2 u}{dr^2} = -\frac{1}{9} \frac{AW^2}{r}$$

$$AE \frac{du}{dr} = -\frac{1}{9} \frac{AW^2}{r} \left( \frac{r^2}{2} + c_1 \right)$$

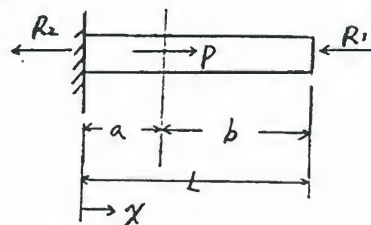
$$\frac{du}{dr} = 0, \text{ at } r=L, \therefore c_1 = -\frac{L^2}{2}$$

$$AE u = -\frac{1}{9} \frac{AW^2}{r} \left( \frac{r^3}{6} - \frac{L^2}{2} r + c_2 \right)$$

$$u=0, \text{ at } r=0, \therefore c_2=0$$

$$\Rightarrow u = -\frac{1}{9E} \frac{W^2}{r} \left( \frac{r^3}{6} - \frac{L^2}{2} r \right)$$

4-38



$$(a) EA \frac{d^2 u}{dx^2} = -P \delta(x-a)$$

B.C.  $\begin{cases} \text{at } x=0, & u=0 \\ \text{at } x=L, & u=0, EA \frac{du}{dx} = -R_1 \end{cases}$

$$EA \frac{du}{dx} = -P \langle x-a \rangle^0 + c_1$$

$$\text{at } x=L, EA \frac{du}{dx} = -R_1$$

$$-R_1 = -P + c_1$$

$$\therefore c_1 = P - R_1$$

$$EA u = -P \langle x-a \rangle^1 + (P - R_1)x + c_2$$

$$\text{at } x=0, u=0 \therefore c_2=0$$

$$\text{at } x=L, u=0$$

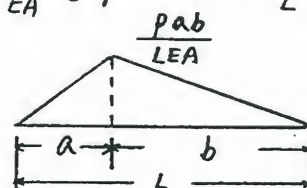
$$0 = -P(L-a) + (P - R_1)L$$

$$\Rightarrow R_1 = \frac{a}{L} P$$

$$R_2 = \frac{b}{L} P$$

$$(b) u = \frac{1}{EA} [-P \langle x-a \rangle^1 + (P - \frac{a}{L} P)x]$$

$$= \frac{1}{EA} [-P \langle x-a \rangle^1 + \frac{b}{L} P x]$$



5-1

$$K_s = \frac{G a b}{l_0} = \frac{0.64 \times 20 \times 40}{10}$$

$$= 51.2 \text{ N/mm}$$

5-3

for Aluminum,

$$U_{\text{shear}} = \frac{\tau^2}{2G} = \frac{25^2}{2 \times 4} = 78.125 \text{ psi}$$

$$U_r = \frac{\sigma_p^2}{2E} = \frac{60^2}{2 \times 10.6} = 169.81 \text{ psi}$$

for steel,

$$U_{\text{shear}} = \frac{\tau^2}{2G} = \frac{24^2}{2 \times 12} = 24 \text{ psi}$$

$$U_r = \frac{\sigma_p^2}{2E} = \frac{65^2}{2 \times 30} = 70.42 \text{ psi}$$

5-3

from eq. (5-14)

(a) state of plane stress

$$\sigma_x = 20, \quad \sigma_y = 10, \quad \sigma_z = 0$$

$$\epsilon_z = \frac{1}{E} [\sigma_z - \nu(\sigma_x + \sigma_y)]$$

$$= \frac{1}{30 \times 10^3} [0 - 0.25(20 + 10)]$$

$$= -2.5 \times 10^{-4}$$

$$e_x = \epsilon_x \cdot l = \frac{1}{30 \times 10^3} (20 - 0.25 \times 10) \times 4$$

$$= 2.3 \times 10^{-3} \text{ in}$$

$$e_y = \frac{1}{30 \times 10^3} (10 - 0.25 \times 20) \times 4$$

$$= 6.67 \times 10^{-4} \text{ in}$$

(b) state of plain strain

$$\sigma_z = \nu(\sigma_x + \sigma_y)$$

$$= 0.25(20 + 10)$$

$$= 7.5 \text{ ksi}$$

$$e_x = \frac{1}{30 \times 10^3} (20 - 0.25 \times 17.5) \times 4$$

$$= 2.08 \times 10^{-3} \text{ in}$$

$$e_y = \frac{1}{30 \times 10^3} (10 - 0.25 \times 27.5) \times 4$$

$$= 4.17 \times 10^{-4} \text{ in}$$

5-4

$$\Delta_x = \alpha \times \frac{1}{E} [\sigma_x - \nu(\sigma_y + \sigma_z)]$$

$$= 200 \times \frac{1}{140 \times 10^3} [-4.8 - 0.35(2.40 + 2)]$$

$$= -9.06 \times 10^{-3} \text{ mm}$$

$$\Delta_y = \alpha \times \frac{1}{E} [\sigma_y - \nu(\sigma_x + \sigma_z)]$$

$$= 100 \times \frac{1}{140 \times 10^3} [2.40 - 0.35(-4.80 + 2)]$$

$$= 2.41 \times 10^{-3} \text{ mm}$$

$$\Delta_z = \alpha \times \frac{1}{E} [\sigma_z - \nu(\sigma_x + \sigma_y)]$$

$$= 120 \times \frac{1}{140 \times 10^3} [2 - 0.35(-4.8 + 2.4)]$$

$$= 2.43 \times 10^{-3} \text{ mm}$$

5-5

$$\sigma_x = 2.4 \text{ MPa}$$

$$\sigma_y = -1.2 \text{ MPa}$$

$$\sigma_z = -2.0 \text{ MPa}$$

$$e_x = \frac{1}{140 \times 10^3} [2.4 - 0.35(-1.2 - 2.0)] \times 200$$

$$= 5.029 \times 10^{-3} \text{ mm}$$

$$e_y = \frac{1}{140 \times 10^3} [-1.2 - 0.35(2.4 - 2.0)] \times 100$$

$$= -9.575 \times 10^{-4} \text{ mm}$$

$$e_z = \frac{1}{140 \times 10^3} [-2.0 - 0.35(2.4 - 1.2)] \times 120$$

$$= -2.075 \times 10^{-3} \text{ mm}$$

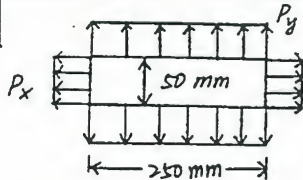


$$\sigma_x \neq 0 \quad \sigma_y = \sigma_z = 0$$

$$\epsilon'_x = \frac{1}{140 \times 10^3} \sigma_x \times 2000 = 5.029 \times 10^{-3} \text{ mm}$$

$$\sigma_x = 3.52 \text{ MPa}$$

5-6



$$(a) \quad \sigma_x = \frac{100}{50 \times 10} = 0.2 \text{ kN/mm}^2$$

$$\sigma_y = \frac{200}{250 \times 10} = 0.08 \text{ kN/mm}^2$$

$$\sigma_z = 0$$

$$\begin{aligned} \epsilon_z &= \frac{1}{E} [\sigma_z - \nu (\sigma_x + \sigma_y)] \\ &= \frac{1}{200} [-0.25 (0.2 + 0.08)] \\ &= -3.50 \times 10^{-4} \end{aligned}$$

$$\Delta L_z = -3.50 \times 10^{-4} (10) = -3.5 \times 10^{-3} \text{ mm}$$

$$(b) \quad -3.50 \times 10^{-4} = \frac{1}{E} (-0.25 \sigma_x)$$

$$\begin{aligned} \sigma_x &= 3.50 \times 10^{-4} \times \frac{200}{0.25} \\ &= 0.28 \text{ kN/mm}^2 \end{aligned}$$

$$P_x = 0.28 \times 50 \times 10 = 140 \text{ kN}$$

5-7

$$\sigma_x = \frac{7.5 \times 10^3}{15 \times 5} = 100 \text{ MPa}$$

$$\sigma_y = 0$$

$$\sigma_z = -\frac{5 \times 10^3}{20 \times 5} = -50 \text{ MPa}$$

$$\epsilon_x = \frac{1}{110 \times 10^3} [100 - 0.33 (0 - 50)] = 1.059 \times 10^{-3}$$

$$\epsilon_y = \frac{1}{110 \times 10^3} [0 - 0.33 (100 - 50)] = -1.5 \times 10^{-4}$$

$$\epsilon_z = \frac{1}{110 \times 10^3} [-50 - 0.33 (100 + 0)] = -7.545 \times 10^{-4}$$

5-8

$$\lambda = \frac{\nu E}{(1+\nu)(1-2\nu)} = \frac{0.25 \times 200}{1.25 \times 0.5} = 80 \text{ GPa}$$

$$\mu = \frac{E}{1+\nu} = \frac{200}{1+0.25} = 160 \text{ GPa}$$

5-9

$$G = \frac{E}{2(1+\nu)} = \frac{180}{2(1+0.25)} = 72 \text{ MPa}$$

5-10

$$G = \frac{E}{2(1+\nu)} = \frac{16}{2(1+0.45)} = 5.52 \text{ MPa}$$

$$K = \frac{E}{3(1-2\nu)} = \frac{16}{3(1-2 \times 0.45)} = 53.33 \text{ MPa}$$

5-11

$$\text{from eq. (5-21), } G = \frac{E}{2(1+\nu)}$$

$$\nu = \frac{E - 2G}{2G}$$

$$\nu_A = \frac{10.6 - 2 \times 4.0}{2 \times 4.0} = 0.325$$

$$\nu_s = \frac{30 - 2 \times 12}{2 \times 12} = 0.25$$

5-12

for aluminum,

$$\nu = \frac{E - 2G}{2G} = \frac{10 - 2 \times 3.75}{2 \times 3.75} = 0.33$$

$$\begin{aligned} K &= \frac{E}{3(1-2\nu)} = \frac{10 \times 10^6}{3(1-2 \times 0.33)} = 9.8 \times 10^6 \text{ psi} \\ &= 67.55 \text{ GPa} \end{aligned}$$

for steel,

$$\nu = \frac{30 - 2 \times 12}{2 \times 12} = 0.25$$

$$\begin{aligned} K &= \frac{30 \times 10^6}{3(1-2 \times 0.25)} = 20 \times 10^6 \text{ psi} \\ &= 137.8 \text{ GPa} \end{aligned}$$

5-13

for glass

$$G_g = \frac{65}{2(1+0.22)} = 26.64 \text{ MPa}$$

$$K_g = \frac{65}{3(1-2 \times 0.22)} = 38.69 \text{ MPa}$$

for steel

$$G_s = \frac{200}{2(1+0.25)} = 80 \text{ GPa}$$

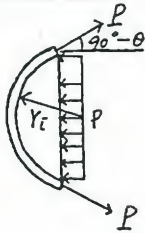
$$K_s = \frac{200}{3(1-2 \times 0.25)} = 133.33 \text{ GPa}$$

for lead

$$G_l = \frac{15}{2(1+0.45)} = 5.17 \text{ GPa}$$

$$K_l = \frac{15}{3(1-2 \times 0.45)} = 50 \text{ GPa}$$

5-14



$$\sum F_x = 0$$

$$2P \cos(90^\circ - \theta) = 2PL Y_i \sin \theta$$

$$P = PL Y_i$$

$$\sigma_i = \frac{P}{Lt} = \frac{P Y_i}{t}$$

5-15

$$(a) p = \frac{0.5}{15} \times \frac{80}{5} = 0.533 \text{ ksi}$$

$$(b) p = \frac{2 \times 0.5}{15} \times 80 = 5.33 \text{ ksi}$$

5-16

$$60 \times 10^6 = \frac{0.75 \times 80 \times 10^3 \times 9.8}{t}$$

$$t = 0.0098 \text{ m}$$

$$= 9.8 \text{ mm}$$

5-17

$$(a) 0.8 \times 160 = \frac{(9800 \times 0.721 \times 5) \times 6 \times 10^{-3}}{(t-3)}$$

$$t = 4.66 \text{ mm, use 5 mm plate}$$

$$(b) \epsilon = \frac{\Delta Y}{Y} = \frac{PY}{2t} \cdot \frac{1}{E} (2-\nu)$$

$$= \frac{0.8 \times 160}{2} \times \frac{1}{200 \times 10^3} (2-0.25)$$

$$= 5.6 \times 10^{-4}$$

$$\Delta d = d \epsilon = 12000 \times 5.6 \times 10^{-4}$$

$$= 6.72 \text{ mm}$$

5-18

$$(a) \sigma_i = \frac{PY}{t}$$

$$\frac{400 \times 10^3}{5} = \frac{20 \times 101 \times Y}{20 \times 10^{-3}}$$

$$Y = 0.79 \text{ m, } d = 1.58 \text{ m}$$

(b) from table 1 of Appendix, the material may be Aluminum

$$2024-T4, E = 73 \text{ GPa, } \nu = 0.335$$

$$\epsilon = \frac{1}{73 \times 10^3} (80 - 0.335 \times 40) = 9.18 \times 10^{-4}$$

$$\Delta d = 1.58 \times 9.18 \times 10^{-4} = 1.45 \times 10^{-3} \text{ m}$$

5-19

$$n = \frac{F}{A\sigma} = \frac{\pi \left(\frac{300}{2}\right)^2 \times 1.5}{195 \times 50} = 14.8$$

use 15 x @ 19 mm bolts

5-20

$$P_{cr} = P_{bottom} = wh = 9.802 \times 4$$

$$(dxs) P_{cr} = (2A) \sigma_{all}$$

$$S = \frac{2(0.03 \times 0.006) \times 90000}{6 \times (9.802 \times 4)} = 0.138 \text{ m}$$

Similarly, use 0.22 m, 0.33 m & 0.66 m spacings respectively for the upper three meters.

5-21

$$\sigma_1 = \frac{P_i Y_i}{t} = \frac{120 \times 59}{1} = 7080 \text{ psi}$$

$$\sigma_2 (\text{accurate}) = \frac{P_i Y_i^2}{Y_o^2 - Y_i^2} = \frac{120 \times 59^2}{60^2 - 59^2} = 3510 \text{ psi}$$

$$\epsilon = \frac{1}{E} (\sigma_1 - \nu \sigma_2) = \frac{1}{29 \times 10^6} [7080 - 0.25 \times 3510] = 2.13 \times 10^{-4} \text{ in/in}$$

$$\Delta_c = \pi d \epsilon = \pi \times 119 \times 2.13 \times 10^{-4} = 0.080''$$

$$\Delta_d = d \epsilon = 119 \times 2.13 \times 10^{-4} = 0.025''$$

5-22

$$\alpha (\Delta T) L = \frac{PL}{EA}$$

$$(a) 20 \times 10^{-6} (150) = \frac{P}{70 (5 \times 50)}, P = 52.5 \text{ kN}$$

$$\sigma_{\text{hoop}} = \frac{52.5 \times 10^3}{5 \times 50} = 210 \text{ MPa}$$

$$(b) \sigma_{\text{bearing}} = \frac{F}{A} = \frac{52.5 \times 10^3 \times 2}{200 \times 50} = 10.5 \text{ MPa}$$

5-23

$$\sigma_1 = \frac{1 \times 1000}{10} = 100 \text{ MPa}$$

$$\Delta d_1 = \frac{2000}{200 \times 10^3} (100 - 0.3 \times 50) = 0.85 \text{ mm}$$

$$\Delta d_2 = 2000 \times 12 \times 10^{-6} (-50) = -1.2 \text{ mm}$$

$$\Delta d_3 = -2000 \times 18 \times 10^{-6} (-50) = -1.8 \text{ mm}$$

$$\sigma = E (\Delta d_1 + \Delta d_2 - \Delta d_3) / d = 140 \times 10^3 \times \frac{1.45}{2000} = 101.5 \text{ MPa}$$

5-24

$$\text{static: } P_s = P_B = P$$

$$\text{geometry: } \Delta_s = \Delta_B$$

$$\frac{P_s (\pi D)}{E_s A_s} = \alpha_B (\Delta T) (\pi D) - \frac{P_B (\pi D)}{E_B A_B}$$

$$\frac{P}{30 \times 10^6 \times (1 \times 0.75)} = 10.7 \times 10^{-6} (100) - \frac{P}{16 \times 10^6 \times (1 \times 0.75)}$$

$$P = 2791.30 \text{ lb}$$

$$\sigma = \frac{P}{A} = \frac{2.79}{1 \times 0.75} = 11.17 \text{ ksi}$$

5-25

$$\sigma_{1s} = \frac{(2 - \bar{P}) \times 200}{4} = 100 - 50 \bar{P}$$

$$\sigma_{1a} = \frac{\bar{P} \times 200}{4} = 50 \bar{P}$$

$$\Delta d_s = \Delta d_a$$

$$\frac{100 - 50 \bar{P}}{200} = \frac{50 \bar{P}}{70}$$

$$\bar{P} = 0.518 \text{ MPa}$$

$$\sigma_{1a} = 50 \times 0.518 = 25.93 \approx 26 \text{ MPa}$$

5-26

$$\text{statics: } P_s = P_a = P$$

$$\text{geometry: } \Delta_s = \Delta_a$$

$$\alpha_s \Delta T (\pi D_s) + \frac{P_s (\pi D_s)}{E_s A_s} = \alpha_a \Delta T (\pi D_a) - \frac{P_a (\pi D_a)}{E_a A_a}$$

$$1.2 \times 10^{-6} \times 40 \times 280 + \frac{P_s \times 280}{200 \times 280 \times 0.8} =$$

$$23 \times 10^{-6} \times 40 \times 300 - \frac{P_a \times 300}{70 \times \frac{\pi}{4} \times 20^2}$$

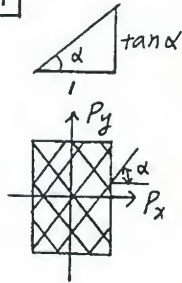
$$0.019891852 \times P = 0.26556$$

$$P = 13.2 \text{ kN}$$

$$\sigma_{s1} = \frac{13.2 \times 10^3}{280 \times 0.8} = 58.93 \text{ MPa}$$



5-27



$$P_y = F_n \sin \alpha$$

$$P_x = F_n \cos \alpha$$

$$\frac{P_y}{1} = z \frac{P_x}{\tan \alpha}$$

$$F_n \sin \alpha = z F_n \frac{\cos \alpha}{\sin \alpha}$$

$$\tan^2 \alpha = z; \quad \alpha = 55^\circ$$

$$= \frac{P_i r_i^2}{r_o^2 - r_i^2} (1 + \beta)$$

$$\therefore \frac{\sigma_{t \max}}{\sigma_{t \text{ ave}}} = \frac{1 + \beta^2}{1 + \beta}$$

5-30

$$\sigma_{t \max} = P_i r_i^2 \left[ \frac{1}{r_o^2 - r_i^2} + \frac{r_o^2}{(r_o^2 - r_i^2) r_i^2} \right]$$

$$\sigma_{t \max} = P_i \left[ \frac{r_i^2}{r_o^2 - r_i^2} + \frac{1}{1 - (\frac{r_i}{r_o})^2} \right]$$

$$\lim_{r_o \rightarrow \infty} \sigma_{t \max} = \lim_{r_o \rightarrow \infty} P_i \left( \frac{r_i}{r_o^2 - r_i^2} + \frac{1}{1 - (\frac{r_i}{r_o})^2} \right)$$

$$= P_i (0 + 1) = P_i$$

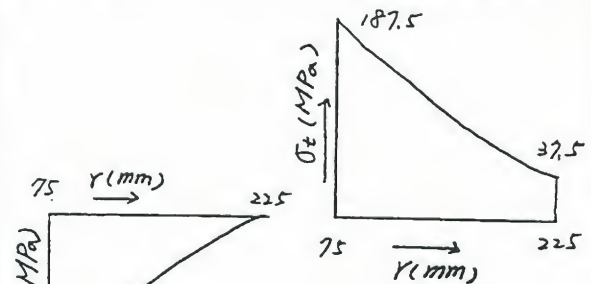
5-31

$$r_i = 75 \text{ mm}, \quad r_o = 225 \text{ mm}, \quad P_i = 150 \text{ MPa}$$

$$(a) \quad \sigma_r = \frac{P_i r_i^2}{r_o^2 - r_i^2} \left( 1 - \frac{r_o^2}{r^2} \right) = 18.75 \left( 1 - \frac{50625}{r^2} \right) \text{ MPa}$$

$$\sigma_t = \frac{P_i r_i^2}{r_o^2 - r_i^2} \left( 1 + \frac{r_o^2}{r^2} \right) = 18.75 \left( 1 + \frac{50625}{r^2} \right) \text{ MPa}$$

$r \text{ (mm)}$	75	100	125	150	175	200	225
$\sigma_r \text{ (MPa)}$	150	76.2	42.0	23.4	12.2	5.0	0
$\sigma_t \text{ (MPa)}$	187.5	113.7	79.5	60.9	49.7	42.5	37.5



5-29

$$\text{Let } \beta = \frac{r_o}{r_i}$$

from eq. 5-49,

$$\sigma_{t \max} = \frac{P_i r_i^2}{r_o^2 - r_i^2} (1 + \beta^2)$$

$$\sigma_{t \text{ ave}} = \frac{F_t}{A} = \frac{1}{(r_o - r_i) \pi} \int_{r_i}^{r_o} \sigma_t (1 \times dr)$$

$$= \frac{P_i r_i^2}{r_o^2 - r_i^2} \left( \frac{1}{r_o - r_i} \right) \left[ \int_{r_i}^{r_o} dr + \int_{r_i}^{r_o} \frac{r_o^2}{r^2} dr \right]$$

$$= \frac{P_i r_i^2}{r_o^2 - r_i^2} \left( \frac{1}{r_o - r_i} \right) (r_o - r_i) \left[ 1 + \frac{r_o}{r_i} \right]$$

$$(b) \quad \sigma_{t \max} = \frac{P_i r_o^2}{r_o^2 - r_i^2} = \frac{150 (225^2)}{225^2 - 75^2} = 168.8 \text{ MPa}$$

$$(c) \quad A_1 = \frac{(1+\nu)(1-2\nu)}{E} \left( \frac{P_i r_i^2 - P_o r_o^2}{r_o^2 - r_i^2} \right)$$

$$= \frac{1.25 \times 0.5}{200 \times 10^3} \left( \frac{150 \times 75^2}{225^2 - 75^2} \right) = 5.86 \times 10^{-5}$$

$$A_2 = \frac{1+\nu}{E} \left( \frac{P_i r_i^2 r_o^2}{r_o^2 - r_i^2} \right) = \frac{1.25}{200 \times 10^3} \left( \frac{150 \times 75^2 \times 225^2}{225^2 - 75^2} \right) = 5.93$$

$$u = A_1 r + A_2 \left( \frac{1}{r} \right)$$

$$u_i = 5.86 \times 10^{-5} (75) + \frac{5.93}{75} = 8.35 \times 10^{-2} \text{ mm}$$

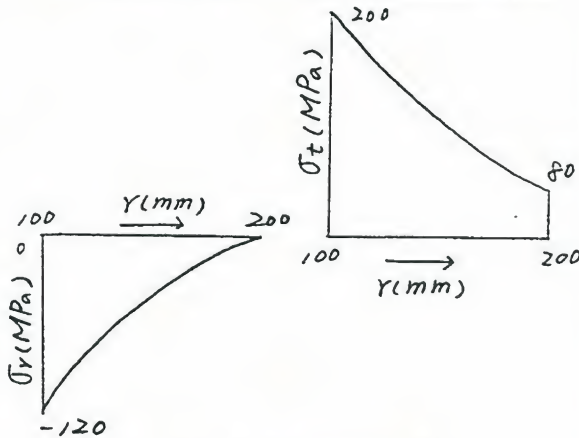
$$u_o = 5.86 \times 10^{-5} (225) + \frac{5.93}{225} = 3.96 \times 10^{-2} \text{ mm}$$

5-32  $r_i = 100 \text{ mm}, r_o = 200 \text{ mm}, P_i = 120 \text{ MPa}$

$$(a) \sigma_r = \frac{P_i r_i^2}{r_o^2 - r_i^2} \left( 1 - \frac{r_o^2}{r^2} \right) = 40 \left( 1 - \frac{40000}{r^2} \right) \text{ MPa}$$

$$\sigma_t = \frac{P_i r_i^2}{r_o^2 - r_i^2} \left( 1 + \frac{r_o^2}{r^2} \right) = 40 \left( 1 + \frac{40000}{r^2} \right) \text{ MPa}$$

$r \text{ (mm)}$	100	125	150	175	200
$-\sigma_r \text{ (MPa)}$	120	62.4	31.1	12.2	0
$\sigma_t \text{ (MPa)}$	200	142.4	111.1	92.2	80



$$(b) \tau_{\max} = \frac{P_i r_o^2}{r_o^2 - r_i^2} = \frac{120 \times 200^2}{200^2 - 100^2} = 160 \text{ MPa}$$

$$(c) A_1 = \frac{(1+\nu)(1-2\nu)}{E} \left( \frac{P_i r_i^2 - P_o r_o^2}{r_o^2 - r_i^2} \right) = \frac{1.25 \times 0.5}{200 \times 10^3} \left( \frac{120 \times 100^2}{200^2 - 100^2} \right) = 1.25 \times 10^{-4}$$

$$A_2 = \frac{1+\nu}{E} \left( \frac{P_i r_i^2 r_o^2}{r_o^2 - r_i^2} \right) = \frac{1.25}{200 \times 10^3} \left( \frac{120 \times 200^2 \times 100^2}{200^2 - 100^2} \right) = 10$$

$$u_i = 1.25 \times 10^{-4} (100) + \frac{10}{100} = 11.25 \times 10^{-2} \text{ mm}$$

$$u_o = 1.25 \times 10^{-4} (200) + \frac{10}{200} = 7.5 \times 10^{-2} \text{ mm}$$

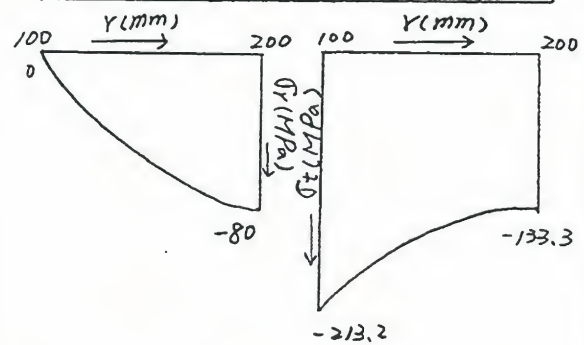
5-33

$$r_i = 100 \text{ mm}, r_o = 200 \text{ mm}, P_o = 80 \text{ MPa}$$

$$(a) \sigma_r = -\frac{P_o r_o^2}{r_o^2 - r_i^2} \left( 1 - \frac{r_i^2}{r^2} \right) = -106.6 \left( 1 - \frac{10000}{r^2} \right) \text{ MPa}$$

$$\sigma_t = -\frac{P_o r_o^2}{r_o^2 - r_i^2} \left( 1 + \frac{r_i^2}{r^2} \right) = -106.6 \left( 1 + \frac{10000}{r^2} \right) \text{ MPa}$$

$r \text{ (mm)}$	100	125	150	175	200
$-\sigma_r \text{ (MPa)}$	0	38.8	59.2	71.8	80.0
$-\sigma_t \text{ (MPa)}$	213.2	174.8	154.0	141.4	133.3



$$(b) \tau_{\max} = \frac{(\sigma_t)_{\max}}{2} = -\frac{213.2}{2} = -106.6 \text{ MPa}$$

$$(c) A_1 = \frac{(1+\nu)(1-2\nu)}{E} \left( \frac{-P_o r_o^2}{r_o^2 - r_i^2} \right) = \frac{1.25 \times 0.5}{200 \times 10^3} \times \frac{(-80)(200^2)}{200^2 - 100^2} = -3.33 \times 10^{-4}$$

$$A_2 = \frac{1+\nu}{E} \left( \frac{-P_o r_i^2 r_o^2}{r_o^2 - r_i^2} \right) = \frac{1.25}{200 \times 10^3} \left( \frac{-80 \times 100^2 \times 200^2}{200^2 - 100^2} \right) = -6.67$$

$$u = A_1 r + \frac{A_2}{r}$$

$$u_i = -3.33 \times 10^{-4} (100) - \frac{6.67}{100} = -0.1 \text{ mm}$$

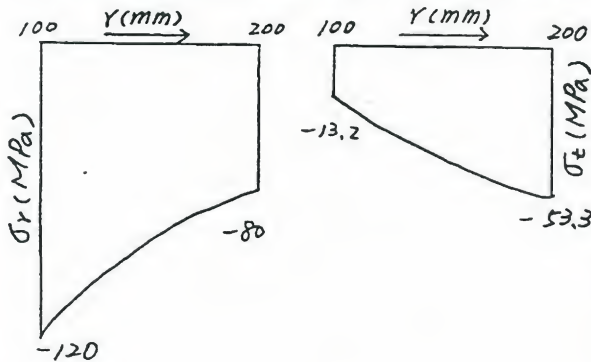
$$u_o = -3.33 \times 10^{-4} (200) - \frac{6.67}{200} = -0.1 \text{ mm}$$



5-34

$P_i = 120 \text{ MPa}$ ,  $P_o = 80 \text{ MPa}$   
superposing Prob. 5-32 and 5-33

$r \text{ (mm)}$	100	125	150	175	200
$-\sigma_r \text{ (MPa)}$	120	101.2	90.3	84.0	80.0
$-\sigma_t \text{ (MPa)}$	13.2	32.4	42.9	47.2	53.3



$$(b) \quad Z_{\max} = \frac{\sigma_{t\max} - \sigma_{r\max}}{2}$$

$$= \frac{-13.2 + 120}{2} = 53.4 \text{ MPa}$$

$$(c) \quad U_i = 11.25 \times 10^{-2} - 0.1 = 1.25 \times 10^{-2} \text{ mm}$$

$$U_o = 7.5 \times 10^{-2} - 0.1 = 2.5 \times 10^{-2} \text{ mm}$$

5-35

$$F_o = P_o (A_{\text{proj.}})_o = 80(400 \times 1) = 32 \text{ kN} (\rightarrow)$$

$$F_i = P_i (A_{\text{proj.}})_i = -160(200 \times 1) = -32 \text{ kN} (\leftarrow)$$

$$C_1 = \frac{P_i r_i^2 - P_o r_o^2}{r_o^2 - r_i^2} = \frac{160 \times 200^2 \times 100^2}{200^2 - 100^2} = 1.066 \times 10^6$$

$$F_t = 2 \int_{100}^{200} \sigma_t (1 \times dr) = 2 \int_{100}^{200} (C_1 + \frac{C_2}{r^2}) dr$$

$$= 2(100 C_1 + \frac{C_2}{200}) = 0$$

$$\Sigma F_x = F_o + F_i + F_t = 32 - 32 + 0 = 0$$

5-36

$$Z_{\max} = \frac{\sigma_{yp}}{2} = \frac{P_i r_o^2}{r_o^2 - r_i^2} = \frac{P_i}{1 - (\frac{r_i}{r_o})^2}$$

$$(a) \quad \frac{40}{2} = \frac{8 \times 2}{1 - (\frac{r_i}{r_o})^2}$$

$$1 - (\frac{r_i}{r_o})^2 = \frac{4}{5}, \quad r_o = 4.47''$$

$$d_o = 8.94''$$

$$(b) \quad P_{ult} = \sigma_{yp} \ln(\frac{a}{b})$$

$$3(-8) = 40 \ln(\frac{2}{b})$$

$$\frac{2}{b} = e^{-\frac{24}{40}} = 0.549$$

$$b = 3.64 \text{ in}$$

$$d_o = 2b = 7.29 \text{ in}$$

5-37

inner cylinder:  $r_i = 3''$ ,  $r_o = 5''$ ,  $P_i = 0$

$$(a) \quad A_1 = \frac{(1+0.25)(1-0.5)}{30 \times 10^3} \times \frac{(-P_o)(25)}{(25-9)}$$

$$= -3.26 \times 10^{-5} P_o$$

$$A_2 = \frac{(1+0.25)}{30 \times 10^3} \times \frac{(-P_o)(9)(25)}{(25-9)}$$

$$= -5.86 \times 10^{-4} P_o$$

$$U_i(5'') = [3.26 \times 10^{-5}(5) + \frac{5.86 \times 10^{-4}}{5}] (-P_o)$$

$$= -2.80 \times 10^{-4} P_o \text{ in}$$

outer cylinder:  $r_i = 5''$ ,  $r_o = 8''$ ,  $P_o = 0$

$$A_1 = \frac{(1+0.25)(1-0.5)}{30 \times 10^3} \times \frac{P_i(25)}{(64-25)} = 1.34 \times 10^{-5} P_i$$

$$A_2 = \frac{(1+0.25)}{30 \times 10^3} \times \frac{P_i(25)(64)}{64-25} = 1.71 \times 10^{-3} P_i$$

$$U_o(5'') = [1.34 \times 10^{-5}(5) + \frac{1.71 \times 10^{-3}}{5}] P_i$$

$$= 4.09 \times 10^{-4} P_i \text{ in}$$

$$|U_i(5'')| + |U_o(5'')| = \frac{0.01}{2}$$

$$(2.80 + 4.09) \times 10^{-4} P = 0.005$$

$$P = 7.26 \text{ ksi}$$

(b) inner cylinder:  $r_i = 3''$ ,  $r_o = 5''$ ,  $P_o = 7.26 \text{ ksi}$

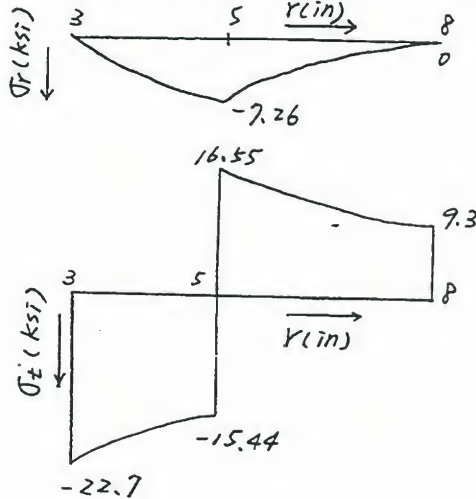
$$\sigma_r = -\frac{7.26(25)}{25-9} (1 - \frac{9}{r^2}) = -11.35 (1 - \frac{9}{r^2})$$



$$\sigma_t = \frac{7.26(25)}{64-25} \left(1 + \frac{64}{r^2}\right) = 4.65 \left(1 + \frac{64}{r^2}\right)$$

r (in)	3.0	3.5	4.0	4.5	5.0
$-\sigma_r$ (ksi)	0.0	3.01	4.97	6.31	7.26
$-\sigma_t$ (ksi)	22.7	19.69	17.73	16.39	15.44

r (in)	5.0	5.5	6.0	6.5	7.0	7.5	8.0
$-\sigma_r$ (ksi)	7.25	5.19	3.62	2.39	1.42	0.64	0.0
$\sigma_t$ (ksi)	16.55	14.49	12.92	11.69	10.72	9.94	9.3



(c)  $\sigma_t = 20 + 22.7 = 42.7 \text{ ksi}$

$$\sigma_t(r_i) = \frac{P_i (r_i^2 + r_o^2)}{r_o^2 - r_i^2}$$

$$42.7 = \frac{P_i (3^2 + 8^2)}{8^2 - 3^2}, \quad P_i = 32.2 \text{ ksi}$$

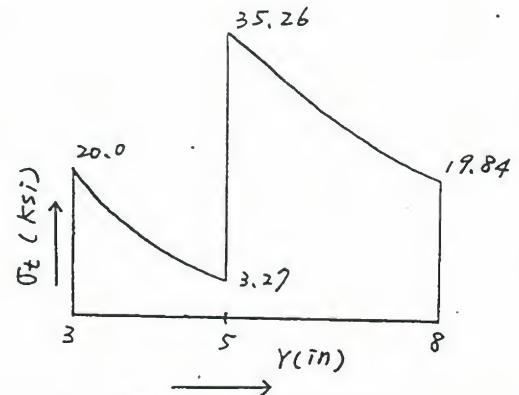
(d) due to  $P_i = 32.2 \text{ ksi}$

$$\sigma_t = \frac{P_i r_i^2}{r_o^2 - r_i^2} \left(1 + \frac{r_o^2}{r^2}\right) = 5.27 \left(1 + \frac{64}{r^2}\right)$$

r (in)	3.0	3.5	4.0	4.5	5.0	5.5	6.0	6.5	7.0	7.5	8.0
$\sigma_t$ (ksi)	22.7	19.69	17.73	16.39	15.44	14.49	12.92	11.69	10.72	9.94	9.3

superposing,

r (in)	3.0	3.5	4.0	4.5	5.0	5.5	6.0	6.5	7.0	7.5	8.0
$\sigma_t$ (ksi)	20.0	12.10	8.66	5.57	3.27						
						35.26	31.93	27.57	24.98	22.83	21.57



5-38 For a thin disk,  $\sigma_z = 0$

$$\frac{du}{dr} = \epsilon_r = \frac{1}{E} (\sigma_r - \nu \sigma_t)$$

or  $\sigma_r - \nu \sigma_t = E \frac{du}{dr} \dots (1)$

$$\frac{u}{r} = \epsilon_t = \frac{1}{E} (-\nu \sigma_r + \sigma_t)$$

or  $-\nu \sigma_r + \sigma_t = E \frac{u}{r} \dots (2)$

solving (1) & (2)

$$\sigma_r = \frac{E}{(1-\nu^2)} \left( \frac{u}{r} + \nu \frac{du}{dr} \right)$$

$$\sigma_t = \frac{E}{(1-\nu^2)} \left( \nu \frac{u}{r} + \frac{du}{dr} \right)$$

$$F_{inertia} = \frac{\int r d\phi dr}{g} \omega^2 r$$

$$\sigma_{inertia} = \frac{F_{inertia}}{d\phi dr} = \frac{\sigma \omega^2 r^2}{g}$$

$$\sigma_t - \sigma_r - r \frac{d\sigma_r}{dr} - \frac{\sigma \omega^2 r^2}{g} = 0$$

$$\frac{E}{(1-\nu^2)} \left[ \nu \frac{u}{r} + \frac{du}{dr} \right] - \frac{E}{(1-\nu^2)} \left[ \frac{u}{r} + \nu \frac{du}{dr} \right]$$

$$- \frac{rE}{(1-\nu^2)} \left[ \frac{1}{r} \frac{du}{dr} - \frac{u}{r^2} + \nu \frac{d^2u}{dr^2} \right] = 0$$

Simplifying,

$$\frac{d^2u}{dr^2} + \frac{1}{r} \frac{du}{dr} - \frac{u}{r^2} + \frac{(1-\nu^2)}{\nu E} \frac{\sigma \omega^2 r}{g} = 0$$

5-39

$$r_i = a, \quad r_o = 2a$$

$$(a) \quad \sigma_r = \sigma_{yp} \ln\left(\frac{r}{c}\right) - \frac{\sigma_{yp}}{2} \left( \frac{b^2 - c^2}{b^2} \right)$$

$$\text{for } c = 1.5a$$

$$\begin{aligned} P_i = \sigma_r(a) &= 350 \left[ \ln\left(\frac{a}{1.5a}\right) - \frac{1}{2} \frac{(2a)^2 - (1.5a)^2}{(2a)^2} \right] \\ &= 350 [-0.41 - 0.22] \\ &= -220.5 \text{ MPa} \end{aligned}$$

(b) plastic zone:  $0 < r < 1.5a$

$$\sigma_r = 350 \left[ \ln\left(\frac{r}{1.5a}\right) - 0.22 \right]$$

$$\sigma_t = 350 + \sigma_r$$

elastic zone:  $1.5a < r < 2a$

$$\sigma_r = \frac{76.6}{\left(\frac{2}{1.5}\right)^2 - 1} \left( 1 - \frac{4a^2}{r^2} \right) = 98.44 \left( 1 - \frac{4a^2}{r^2} \right)$$

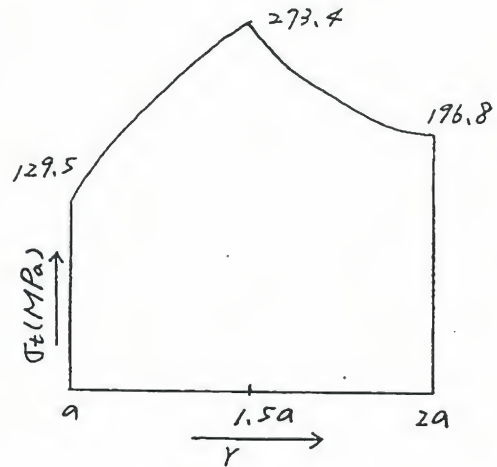
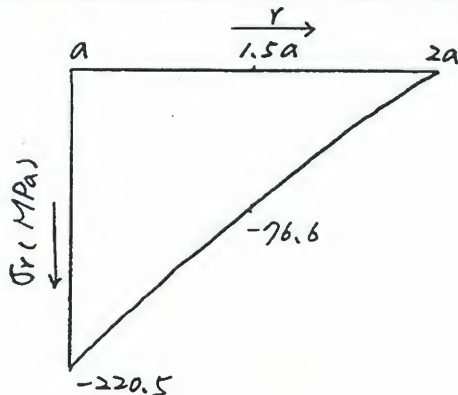
$$\sigma_t = \frac{26.6}{\left(\frac{2}{1.5}\right)^2 - 1} \left( 1 + \frac{4a^2}{r^2} \right) = 98.44 \left( 1 + \frac{4a^2}{r^2} \right)$$

plastic zone:

$r$	$a$	$1.1a$	$1.2a$	$1.3a$	$1.4a$	$1.5a$
$-\sigma_r (\text{MPa})$	220.5	185.1	154.7	126.6	100.7	76.6
$\sigma_t (\text{MPa})$	129.5	164.9	195.3	223.4	249.3	273.4

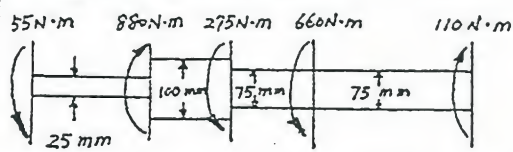
elastic zone:

$r$	$1.5a$	$1.6a$	$1.7a$	$1.8a$	$1.9a$	$2.0a$
$-\sigma_r (\text{MPa})$	76.6	55.4	37.8	23.1	10.6	0.0
$\sigma_t (\text{MPa})$	273.4	252.3	234.6	219.9	207.5	196.8



$$\begin{aligned} (c) \quad P_{ult} = \sigma_r(a) &= \sigma_{yp} \ln\left(\frac{a}{b}\right) \\ &= 350 \ln\left(\frac{a}{2a}\right) \\ &= 350 (-0.693) \\ &= -242.6 \text{ MPa} \end{aligned}$$

6-1



$$T_1 = 55 \text{ N}\cdot\text{m}, T_2 = 825 \text{ N}\cdot\text{m}, T_3 = 550 \text{ N}\cdot\text{m}, T_4 = 110 \text{ N}\cdot\text{m}$$

$$\tau_1 = \frac{T_1 C_1}{J_{P1}} = \frac{55 \left( \frac{25}{2} \times 10^{-3} \right)}{\frac{\pi}{2} \left( \frac{25}{2} \times 10^{-3} \right)^4} = 1.79 \times 10^7 \text{ N/m}^2$$

$$\tau_2 = \frac{T_2 C_2}{J_{P2}} = \frac{825 \left( 50 \times 10^{-3} \right)}{\frac{\pi}{2} \left( 50 \times 10^{-3} \right)^4} = 4.2 \times 10^6 \text{ N/m}^2$$

$$\tau_3 = \frac{T_3 C_3}{J_{P3}} = \frac{550 \left( \frac{75}{2} \times 10^{-3} \right)}{\frac{\pi}{2} \left( \frac{75}{2} \times 10^{-3} \right)^4} = 6.64 \times 10^6 \text{ N/m}^2$$

$$\therefore \tau_{\max} = \tau_1 = 1.79 \times 10^7 \text{ N/m}^2$$

$$6-2 \quad \frac{T_2}{T_1} = \frac{J_{P2}}{J_{P1}} = \frac{\frac{\pi}{2} (r^4 - b^4)}{\frac{\pi}{2} r^4}$$

$$T_2 = T_1 \cdot \frac{(100)^4 - (50)^4}{(100)^4} = 0.938 T_1$$

$$\text{Loss} = 100\% - 93.8\% = 6.2\%$$

$$6-3 \quad \tau_{\max} = \frac{T c}{J_P}$$

$$\tau_{\max} = \frac{T \cdot 20}{\frac{\pi}{32} (40)^4} = \frac{T \cdot 30}{\frac{\pi}{32} (60^4 - D_i^4)}$$

$$T \cdot (60 - D_i^4) = T \cdot \frac{30}{20} (40)^4$$

$$D_i = 54.95 \text{ mm}$$

$$t = \frac{60 - 54.95}{2} = 2.525 \text{ mm}$$

$$\frac{W_s}{W_t} = \frac{(40)^2}{(60)^2 - (54.95)^2} = \frac{1}{9.675}$$

6-4

$$\tau = \sigma_1 = -\sigma_2$$

$$\epsilon = 0.004, E = 180, G = 70$$

$$\sigma_1 = -\sigma_2 = 180(0.004) = 0.72 = \tau$$

$$\phi = \frac{T \cdot L}{G \cdot J_P}, \tau = \frac{T \cdot c}{J_P} \Rightarrow T = \frac{\tau J_P}{c}$$

$$\therefore \phi = \frac{\tau \cdot J_P \cdot L}{c \cdot G \cdot J_P} = \frac{(0.72)(500)}{(10)(70)}$$

$$\phi = 0.514 \text{ radians}$$

$$6-5 \quad (a) T = \frac{159 \times 400}{2} = 31,800 \text{ N}\cdot\text{m}$$

$$\tau_{\max} = \frac{T c}{J_P} = \frac{31800 (0.050)}{\frac{\pi}{2} (0.050)^4}$$

$$\tau_{\max} = 161.96 \text{ MPa}$$

$$(b) (100)^3 \cdot 2 = d^3 \cdot 4$$

$$d = 79.4 \text{ mm}$$

$$6-6 \quad T = \frac{63000 \text{ Hp}}{N} = \frac{63000 \times 90}{N}$$

$$= 9000 \text{ in}\cdot\text{lbs}$$

$$\frac{J_P}{c} = \frac{T}{\tau} = \frac{9000}{5750} = 1.565 = \frac{\pi c^3}{2}$$

$$c = \sqrt[3]{\frac{2}{\pi} \cdot 1.565} = 1.0 \text{ in}, d = 2.0 \text{ in}$$

6-7

$$T_L = T_s, f_L = \frac{28}{90} f_s$$

Same material; same allow.  $\tau_{\max}$

$$d_L^3 \cdot f_L = d_s^3 \cdot f_s$$

$$d_L^3 \cdot \frac{28}{90} f_s = (12)^3 f_s$$

$$d_L^3 = \frac{90}{28} (12)^3$$

$$d_L = 17.7 \text{ mm}$$



$$6-8 \quad T_i = \frac{159(500)}{30} = 2650 \text{ N}\cdot\text{m}$$

$$\tau_{\max} = 60 \times 10^6 = \frac{T_i c_i}{I_p} = \frac{2650 c_i}{\frac{\pi}{2} c_i^4}$$

$$c_i^3 = \frac{2(2650)}{\pi 60 \times 10^6} \quad c_i = 0.030 \text{ m} = 30 \text{ mm}$$

$d_i = 2 c_i = 60 \text{ mm}$  dia for the internal gear

Larger gear is of the same material

Hence,  $d_L^3 f_L = d_i^3 f_i$

$$d_L^3 = d_i^3 \frac{f_i}{f_L} = (60)^3 \frac{30}{10}$$

$$d_L = 86.5 \text{ mm}$$

6-9 From the S-N diagram in Fig 2-25, for  $N = 5 \times 10^8$  cycles,  $S = 131 \text{ MPa}$

$$\tau = \frac{S}{2} = \frac{131}{2} = 65.5 \text{ MPa}$$

$$\frac{T}{\tau} = \frac{I_p}{c}, \quad T = 65.5 \times 10^6 \frac{\pi (0.02)^3}{2} = 823.1$$

Using a safety factor of 1.8

$$T_{\text{allow}} = \frac{823.1}{1.8} = 457.28 \text{ N}\cdot\text{m}$$

$$6-10 \quad (a) \quad T = \frac{63000 \text{ Hp}}{N} = \frac{63000 (300)}{75}$$

$$T = 252,000 \text{ in}\cdot\text{lbs}$$

$$\frac{I_p}{c} = \frac{T}{\tau} = \frac{252,000}{8,050} = 31.5$$

$$\frac{I_p}{c} = \frac{\pi}{2} \left[ \frac{c^4 - (c/1.2)^4}{c} \right] = 0.813 c^3 = 31.5$$

$$c^3 = 38.732, \quad c = 3.38 \text{ in}$$

$$D_o = 2 c = 6.77 \text{ in}$$

$$D_i = \frac{D_o}{1.2} = 5.64 \text{ in}$$

$$(b) \quad \frac{I_p}{c} = \frac{T}{\tau} = 31.5 = \frac{\pi}{2} c^3$$

$$c^3 = 20.054, \quad c = 2.72 \text{ in}$$

$$D = 2 c = 5.44 \text{ in}$$

dia of equiv solid shaft

6-11

$$T = \frac{63000}{N} \text{ Hp}$$

$$T_{25} = \frac{63000 (25)}{26.3} = 6.00 \times 10^4 \text{ in}\cdot\text{lb}$$

$$T_{75} = \frac{63000 (75)}{26.3} = 1.80 \times 10^5 \text{ in}\cdot\text{lb}$$

$$\frac{I_p}{c} = \frac{T}{\tau}; \quad \left(\frac{I_p}{c}\right)_{\text{left}} = \frac{6 \times 10^4}{6000} = 10 \text{ in}^3$$

$$\frac{\pi}{2} c^3 = 10, \quad c = \left(\frac{20}{\pi}\right)^{\frac{1}{3}} = 1.85 \text{ in}$$

$$\left(\frac{J}{C}\right)_{\text{right}} = \frac{1.8 \times 10^5}{6000} = 30, \quad c = \left(\frac{60}{\pi}\right)^{\frac{1}{3}} = 2.67 \text{ in}$$

$$\therefore D_{\text{left}} = 3.71 \text{ in.} \quad D_{\text{right}} = 5.35 \text{ in}$$

6-12

see fig. 6-13

$$r = 12 \text{ mm}, \quad \frac{r}{\frac{1}{2}d} = \frac{12}{\frac{1}{2} \times 75} = 0.32$$

$$\frac{D}{d} = \frac{150}{75} = 2.00$$

by extrapolation,  $K = 1.3$

$$\tau = K \frac{Tc}{I_p} = 1.3 \frac{2.7 \times (37.5 \times 10^{-3})}{\frac{\pi}{2} (37.5 \times 10^{-3})^4} \times 10^{-3} = 42.37 \text{ MPa}$$

$$r = 3 \text{ mm}, \quad \frac{r}{\frac{1}{2}d} = \frac{3}{\frac{1}{2} \times 75} = 0.08$$

$$\frac{D}{d} = 2.00$$

$$K = 1.85$$

$$\tau = \frac{1.85}{1.3} (42.37) = 60.30 \text{ MPa}$$

6-13

$$T = \frac{63000 \times 110}{100} = 69300 \text{ in-lb}$$

$$K = \frac{\tau I_P}{TC} = \frac{8000 \left( \frac{\pi}{2} \times 2^4 \right)}{69300 (2)} = 1.45$$

$$\frac{D}{d} = \frac{6}{4} = 1.5$$

from Fig. 4-13,  $\frac{r}{\frac{1}{2}d} = 0.155$

$$r = (0.155) \frac{4}{2} = 0.310 \text{ in.}$$

6-14 (a)  $D = 40$ ,  $d = 20$ ,  $r = 1$

$$\frac{D}{d} = 2, \quad \frac{r}{d/2} = 0.1$$

From Fig 6-13 of stress concentration factors,

$$K \approx 1.75 \text{ for } \frac{D}{d} = 2 \text{ and } \frac{r}{d/2} = 0.1$$

$$\tau_{\max} = K \frac{TC}{I_P} \text{ (compute } \frac{C}{I_P} \text{ from } d)$$

$$= T \cdot (1.75) \frac{2}{\pi (0.010)^3}$$

$$\tau_{\max} = 1,114,085 \text{ T Pascals}$$

or  $1.114 \text{ T MPa}$

(b) Here  $\frac{D}{d} = \frac{30}{18} = 1.67$

$$\text{and } \frac{r}{d/2} = \frac{2}{9} = 0.222$$

$$K \approx 1.4 \text{ (from Fig 6-13)}$$

$$\tau_{\max} = K \frac{TC}{I_P} = T (1.4) \frac{2}{\pi (0.009)^3}$$

$$\tau_{\max} = 1,222,589 \text{ T Pascals}$$

or  $1.223 \text{ T MPa}$

Shaft assembly (a) causes lower stresses than the assembly (b), a more generous 2mm fillet radius in case (a) would further reduce the stress

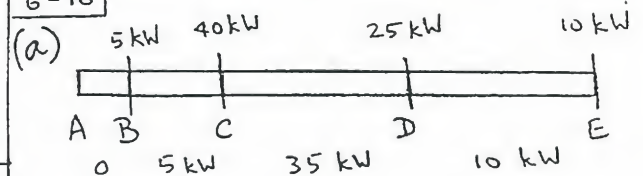
6-15

$$\phi = \frac{TL}{GI_P} = \frac{\tau I_P}{C} \frac{L}{GI_P} = \frac{\tau L}{CG}$$

$$L = \frac{\phi CG}{\tau} = \frac{2\pi (0.003) (27 \times 10^9)}{42 \times 10^6}$$

$$= 12.12 \text{ m}$$

6-16



$$T_{\max} = \frac{159 \text{ (kW)}_{\max}}{f} = \frac{159 \times 35}{3}$$

$$T_{\max} = 1855 \text{ N.m}$$

$$I_P = \frac{\pi r^4}{2} = \frac{\pi}{2} (0.025)^4 = 6.14 \times 10^{-7} \text{ m}^4$$

$$I_P/c = \frac{6.14 \times 10^{-7}}{0.025} = 2.4544 \times 10^{-5}$$

$$\tau_{AB} = 0$$

$$\tau_{CD} = T \frac{C}{I_P} = \frac{1855}{2.4544 \times 10^{-5}} = 75.58 \times 10^6 \text{ N/m}^2$$

or  $75.58 \text{ MPa}$

$$\tau_{BC} = \frac{5}{35} \tau_{CD} = \frac{5}{35} (75.58) = 10.8 \text{ MPa}$$

$$\tau_{DE} = \frac{10}{35} \tau_{CD} = \frac{10}{35} (75.58) = 21.6 \text{ MPa}$$

(b) from eqn (6-15)

$$\phi_{AE} = -\phi_{AB} - \phi_{BC} + \phi_{CD} + \phi_{DE}$$

$$\phi_{AB} = 0 \text{ for } \tau_{AB} = 0$$

$$\phi_{BC} = \frac{1855 \left( \frac{5}{35} \right) 3}{6.14 \times 10^{-7} \times 84 \times 10^9} = 0.0154$$

$$\phi_{CD} = \frac{1855 \times 6}{6.14 \times 10^{-7} \times 84 \times 10^9} = 0.2158$$

$$\phi_{DE} = 0.2158 \left( \frac{10}{35} \right) = 0.0616$$

$$\text{Then, } \phi_{AE} = 0.262 \text{ rad} = 15.01 \text{ deg.}$$

6-17  $L = 6$ ,  $D_i/D_o = 0.5$

(a)

$$I_p = \frac{T L}{\phi G} = \frac{1 \times 6}{0.1 \frac{2\pi}{360} (12 \times 10^6)}$$

$$I_p = 2.865 \times 10^{-4}$$

$$I_p = \frac{\pi}{2} \left[ C^4 - \left( \frac{C}{2} \right)^4 \right] = 2.865 \times 10^{-4}$$

$$C^4 = 1.945 \times 10^{-4}$$

$$C = 0.12 \text{ in}, D_o = 2C = 0.24 \text{ in}$$

(b) From eqn. (6-19)

$$K_t = \frac{\pi \times 6 \times 12 \times 10^6}{\frac{1}{0.12^2} - \frac{1}{0.24^2}} = 4.34 \times 10^6 \text{ in-lb/rad}$$

6-18 (a)  $L = 1000 \text{ mm}$ ,  $c = 30 \text{ mm}$

Criterion: strength

$$T_{\max, \text{steel}} = 2 T_{\max, \text{Al}}$$

$$\frac{T C}{I_{p\text{st}}} = 2 \frac{T C}{I_{p\text{Al}}}$$

$$\frac{\pi C^4}{2} = 2 \frac{\pi}{2} (C^4 - \gamma_i^4)$$

$$\gamma_i = 25.2 \text{ mm}$$

Criterion: stiffness

$$\phi_{\text{Al}} = \phi_{\text{steel}}$$

$$\frac{T L}{G_{\text{Al}} I_{p\text{Al}}} = \frac{T L}{G_{\text{st}} I_{p\text{st}}}$$

$$28 \frac{\pi C^4}{2} = 84 \frac{\pi}{2} (C^4 - \gamma_i^4)$$

$$\gamma_i = 27.1 \text{ mm}$$

(b) Strength controls -

$$\text{use } \gamma_i = 25 \text{ mm}$$

6-19  $\phi = \frac{T L}{I_p G}$

$$\phi_{A1} = \phi_A \text{ due to } 600 \text{ N.m torque}$$

$$\phi_{A2} = \phi_A \text{ due to rotation of gear C}$$

$$\phi_{A1} = \frac{600 \times 6}{\frac{\pi}{2} (0.03)^4 (84 \times 10^9)} = 0.0337 \text{ rad.}$$

$$\phi_C = \frac{(400/200) 600 \times 3}{\frac{\pi}{2} (0.03)^4 (84 \times 10^9)} = 0.0337 \text{ rad}$$

$$\phi_{A2} = 2 \phi_C = 0.0674 \text{ rad}$$

$$\phi_A = \phi_{A1} + \phi_{A2} = 0.1011 \text{ rad} = 5.79^\circ$$

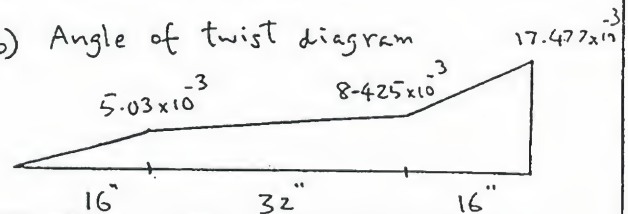
6-20  $I_{p3} = 7.952 \text{ in}^4$ ,  $\phi_1 = 5.030 \times 10^{-3}$

(a)  $I_{p2} = 1.571 \text{ in}^4$ ,  $\phi_2 = 3.395 \times 10^{-3}$

$$I_{p0} = 1.473 \text{ in}^4$$

$$\phi = \phi_1 + \phi_2 + \phi_3 = 17.477 \times 10^{-3} \text{ rad} = 1.0^\circ$$

(b) Angle of twist diagram



6-21  $T = \frac{119 \times \text{hp}}{f}$  and  $\phi = \frac{T L}{I_p G}$

$$\phi = 6^\circ = 6 \times \frac{\pi}{180} \text{ rad} = \frac{119 \text{ hp} (0.3)}{(20) \frac{\pi}{2} (0.006)^4 (84 \times 10^9)}$$

$$h_p = 10$$

Power input required: 10 hp

6-22 From eqn. (6-19)

$$K_t = \frac{\pi \times 1 \times 70 \times 10^6}{\frac{1}{0.020^2} - \frac{1}{0.030^2}}$$

$$K_t = 1.583 \times 10^5 \text{ N.m/rad}$$

Torsional flexibility,  $f_t = 1/K_t$

$$f_t = 6.32 \times 10^{-6} \frac{\text{rad}}{\text{N.m}}$$



6-23  $L = 0.5 \text{ m}$ ,  $d = 0.024 \text{ m}$ ,  $G = 83 \text{ MPa}$

$$k_t = \frac{I_p G}{L} = \frac{\pi}{2} \frac{(0.012)^4 (83 \times 10^6)}{0.5}$$

$$k_t = 5.407$$

$$I_{mz} = m R^2 / 2 = 2 R^2 / 2 = R^2$$

$$f_n = \frac{1}{2\pi} \sqrt{\frac{k_t}{I_{mz}}} = \frac{1}{2\pi} \sqrt{\frac{5.407}{R^2}}$$

$$f_n = 0.37/R$$

(b) Using Al alloy:

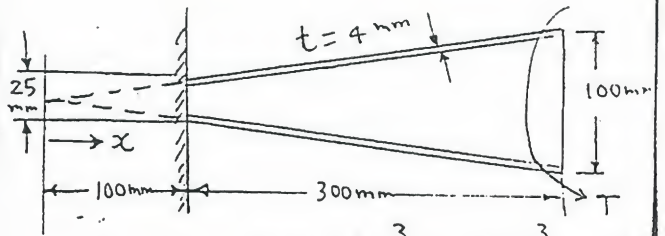
$$k_t = \frac{\pi}{32} \frac{d^4 (26 \times 10^9)}{0.5} = 5.105 \times 10^9 d^4$$

$$f_n = \frac{0.37}{R} = \frac{1}{2\pi} \sqrt{\frac{5.105 \times 10^9 d^4}{R^2}}$$

$$\frac{0.37}{R} = \frac{1}{2\pi R} (7.145 \times 10^4 d^2)$$

$$d = 0.0057 \text{ m} \text{ or } 5.7 \text{ mm}$$

6-25



$$C = \frac{x}{8}, \quad I_p = 2\pi C^3 t = 2\pi \left(\frac{x}{8}\right)^3 (0.004)$$

$$= 4.9 \times 10^{-5} x^3$$

$$\phi = \int \frac{T dx}{I_p G} = \frac{T}{G} \int_{0.1}^{0.4} \frac{dx}{4.9 \times 10^{-5} x^3}$$

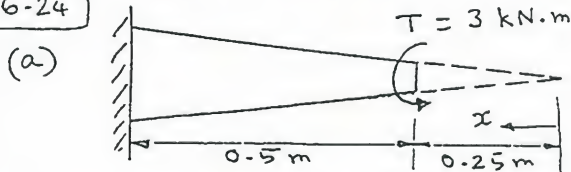
$$= \frac{T}{4.9 \times 10^{-5} G} \left| -\frac{x^{-2}}{2} \right|_{0.1}^{0.4}$$

$$\phi = \frac{T}{4.9 \times 10^{-5} G} \frac{1}{2} \left( \frac{1}{0.1^2} - \frac{1}{0.4^2} \right)$$

$$k_t = \frac{T}{\phi} = 1.045 \times 10^{-6} G \text{ (N.m/rad)}$$

(b)  $f_t = 1/k_t = 9.569 \times 10^5 \text{ G (rad/N.m)}$

6-24



$$I_p = \frac{\pi}{2} C^4 = \frac{\pi}{2} \left(\frac{x}{10}\right)^4 = \frac{\pi x^4}{2 \times 10^4}$$

$$\phi = \int \frac{T dx}{I_p G} = \frac{3000}{200 \times 10^9} \frac{2 \times 10^4}{\pi} \int_{0.25}^{0.75} \frac{dx}{x^4}$$

$$= \frac{3}{\pi} 10^{-4} \left| -\frac{x^{-3}}{3} \right|_{0.25}^{0.75}$$

$$= \frac{1}{\pi} 10^{-4} \left( \frac{1}{0.25^3} - \frac{1}{0.75^3} \right)$$

$$\phi = 1.962 \times 10^{-3} \text{ rad}$$

(b)  $f_t = \frac{\phi}{T} = \frac{1.962 \times 10^{-3}}{3000}$

$$f_t = 6.539 \times 10^{-7} \text{ (rad/N.m)}$$

6-26

$$I_p G \frac{d^2 \phi}{dx^2} = -t_x = -kx$$

$$T = I_p G \phi' = -\frac{1}{2} kx^2 + C_1$$

Since  $T(0) = 0$ ,  $C_1 = 0$

$$I_p G \phi = -\frac{1}{6} kx^3 + C_2$$

Since  $\phi(L) = 0$ ,  $C_2 = kL^3/6$

$$\phi(0) = \frac{C_2}{I_p G} = \frac{kL^3}{6I_p G}$$

6-27

$$T_A = \int_0^L \frac{t_0}{L} x dx = \frac{t_0 L}{2}$$

$$T(x) = T_A - \int_0^x \frac{t_0}{L} x dx = \frac{t_0 L}{2} - \frac{t_0 x^2}{2L}$$

$$\phi_B = \int \frac{T(x)}{G I_p} dx = \frac{t_0 L^2}{3 G I_p}$$

6-28

$$T_A = \int_0^2 (200 + 240 \frac{x}{2}) dx = 640$$

$$T(x) = 640 - \int_0^x (200 + 120x) dx = 640 - 200x - 60x^2$$

$$\phi(B) = \int_0^2 \frac{T(x)}{G I_p} dx = \frac{720}{G I_p}$$

6-29  $E_{st} = 210 \text{ GPa}$ ,  $E_{AL} = \frac{1}{3} E_{st} = 70 \text{ GPa}$

(a) Using Table 1 of Appendix,

$G_{st} = 83 \text{ GPa}$ ,  $G_{AL} = 26 \text{ GPa}$

$I_{pst} = \frac{\pi}{2} (0.025)^4 = 6.14 \times 10^{-7} \text{ m}^4$

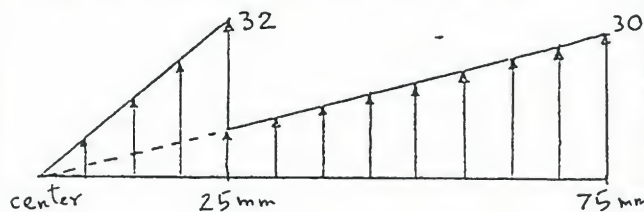
$I_{PAL} = \frac{\pi}{2} [(0.075)^4 - (0.025)^4] = 4.91 \times 10^{-5} \text{ m}^4$

$\tau_{st} = G_{st} \frac{1}{3} \gamma_{max}$ ,  $\tau_{AL} = G_{AL} \gamma_{max}$

$T = 20 = \frac{1}{0.075} \left[ \left( 83 \times 10^9 \times 6.14 \times 10^{-7} \times \frac{1}{3} \right) + \left( 26 \times 10^9 \times 4.91 \times 10^{-5} \right) \right] \gamma_{max}$

$\gamma_{max} = 1.16 \times 10^{-6} \text{ rad}$

$\tau_{st} = 32 \text{ kPa}$ ,  $\tau_{AL} = 30 \text{ kPa}$



(b)  $L = 1 \text{ m}$

$$\phi = \frac{32 \times 1}{83 \times 10^6 \times 25 \times 10^{-3}} + \frac{30 \times 1}{26 \times 10^6 \times 75 \times 10^{-3}}$$
  

$$= 3.08 \times 10^{-5}$$

$k_t = \frac{T}{\phi} = \frac{20}{3.08 \times 10^{-5}} = 6.49 \times 10^5 \text{ (kN.m/rad)}$

$f_t = \frac{1}{k_t} = 1.54 \times 10^{-6} \text{ (rad/kN.m)}$

6-30  $\phi_s = \phi_t$   $\frac{T_s L}{G I_{ps}} = \frac{T_t L}{G I_{pt}}$

$\frac{T_s}{T_t} = \frac{I_{ps}}{I_{pt}} = \frac{\pi (0.025)^4 / 32}{2\pi (0.025)^3 (0.002)} = 0.195$

$T_s + T_t = T$

$0.195 T_t + T_t = T$ ,  $T_t = 0.837 T$

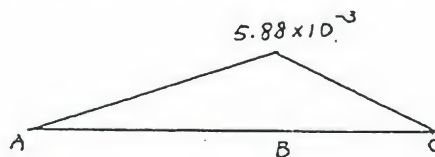
6-31

$T_s = (1 - 0.837) \times 200 = 33 \text{ N.m}$

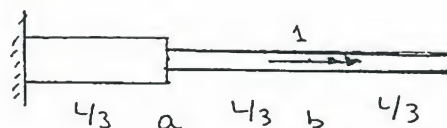
6-32  $T_1 = \frac{\phi \times 12 \times 10^3 \times 2\pi}{15} = 5 \times 10^3 \phi \text{ (K-in)}$

$T_2 = \frac{\phi \times 12 \times 10^3 \times \pi}{10 \times 2} = 1.8 \times 10^3 \phi \text{ (K-in)}$

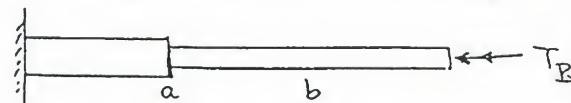
$T_1 + T_2 + T = 0 \Rightarrow \phi = 5.88 \times 10^{-3} \text{ (rad)}$



6-33 (a) unit torque applied at b



$$\phi_{end} = \frac{L}{3 I_{p2} G} + \frac{L}{3 I_{p1} G} = \frac{4}{3} \frac{L}{3 I_{p2} G}$$



$$\phi_{end} = \frac{2}{3} \frac{L T_B}{G I_{p2}} + \frac{L}{3} \frac{T_B}{G I_{p1}} = \frac{7}{3} \frac{L T_B}{3 I_{p2} G}$$

$\therefore T_B = 4/7$  and  $T_A = 3/7$

$\phi_{ab} = \frac{3}{7} \cdot \frac{L}{3 G I_{p1}} = \frac{L}{7 G I_{p1}}$

(b) unit torque applied at a



$$\phi_{end} = \frac{L}{3 G I_{p1}} = \frac{7}{3} \frac{T'_B L}{3 G I_{p2}}$$

$T'_B = 1/7$

$$\phi_{ba} = \frac{L}{7 \times 3 G I_{p2}} = \frac{L}{7 G I_{p1}}$$

$\therefore \phi_{ab} = \phi_{ba} = \frac{L}{7 G I_{p1}}$

6-34 Flexibility,  $f_t = \phi_{aa}$  due to unit torque at the location where the disk is attached

$$\phi_{aa} = \frac{T_A (0.2)}{G I_p} = \frac{T_B (0.4)}{G I_p}$$

$$\therefore T_A = 2 T_B, \quad T_B = 1/3 \text{ and } T_A = 2/3$$

$$\phi_{aa} = \frac{2}{3} \frac{(0.2)}{G I_p} = f_t$$

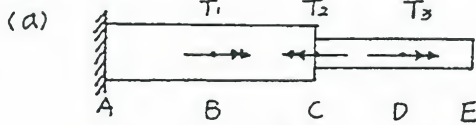
$$k_t = 1/f_t = \frac{3 G I_p}{0.4} = 7.5 G I_p$$

$$\text{Frequency: } f = \frac{1}{2\pi} \sqrt{\frac{k_t}{I_m}} = \frac{1}{2\pi} \sqrt{\frac{k_t}{m R^2/2}}$$

$$f = \frac{1}{2\pi} \sqrt{\frac{7.5 G \pi (0.020)^4}{2 \times 50 \frac{R^2}{2}}}$$

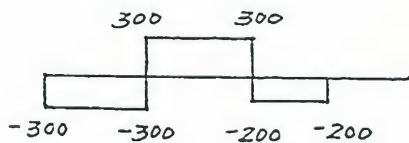
$$\therefore f = 4.37 \times 10^{-5} \frac{\sqrt{G}}{R}$$

6-35

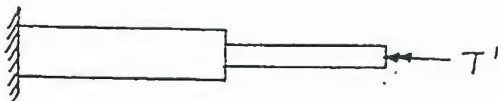


$$J_1 = \frac{\pi}{2} \left( \frac{2.83}{2} \right)^4, \quad J_2 = \frac{\pi}{2} \left( \frac{2.38}{2} \right)^4$$

from Table 1 of Appendix  $G = 3.75 \text{ Ksi}$



$$\phi_{end} = \frac{(-300+300)}{J_1 G} \times 30 + \frac{200 \cdot 25}{J_2 G} = 0.423$$



$$\frac{-T' L_1}{J_1 G} + \frac{-T' L_2}{J_2 G} + 0.423 = 0$$

$$T' = 62.4 \text{ (lb-in)}$$

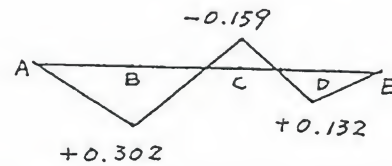
$$(b) \phi_{AB} = -\left( \frac{-300 \times 30}{J_1 G} + \frac{62.4 \times 30}{J_1 G} \right) = 0.302 \text{ (rad)}$$

$$\phi_{BC} = -\left( \frac{300 \times 30}{J_1 G} + \frac{62.4 \times 30}{J_1 G} \right) = -0.461 \text{ (rad)}$$

$$\phi_{CD} = -\left( \frac{-200 \times 25}{J_2 G} + \frac{62.4 \times 25}{J_2 G} \right) = 0.291 \text{ (rad)}$$

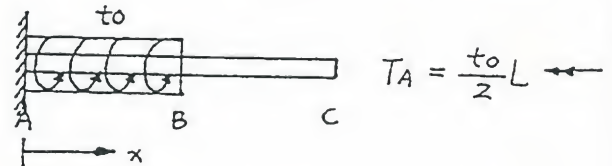
$$\phi_{DE} = -\left( \frac{62.4 \times 25}{J_2 G} \right) = -0.132 \text{ (rad)}$$

$$\text{check } \phi_{AE} = \phi_{AB} + \phi_{BC} + \phi_{CD} + \phi_{DE} = 0 \quad \text{O.K.}$$



6-36

(a) solved by forced method



$$T(x) = -\frac{toL}{2} + to x, \quad x \leq \frac{L}{2}$$

$$\phi_{AC} = -\int_0^{\frac{L}{2}} \frac{T(x)}{JG} dx = +\frac{toL^2}{8JG}$$



$$\frac{T_c L}{JG} + \phi_{AC} = 0 \Rightarrow T_c = \frac{toL}{8}$$

$$(b) \phi_x = -\int_0^x \frac{T(x)}{JG} dx - \frac{T_c x}{JG} \quad x \leq \frac{L}{2}$$

$$= \frac{1}{JG} \left( \frac{3toL}{8} x - \frac{to}{2} x^2 \right)$$

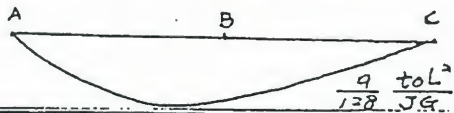
$$\text{Let } \frac{d\phi}{dx} = 0 \Rightarrow x = \frac{3}{8} L$$

$$\phi_x = \frac{toL^2}{8JG} - \frac{T_c x}{JG} \quad x > \frac{L}{2}$$



G-36 (contd.)

$x$	0	$\frac{L}{8}$	$\frac{L}{4}$	$\frac{3L}{8}$	$\frac{L}{2}$	$\frac{3L}{4}$	$L$
$\phi \cdot \frac{JG}{t_0 L^2}$	0	$\frac{5}{128}$	$\frac{1}{16}$	$\frac{9}{128}$	$\frac{1}{16}$	$\frac{1}{32}$	0



G-37 (a)  $\frac{T_B L}{JG} = \frac{t_0 L^2}{3JG}$   $T_B = \frac{t_0 L}{3}$  ←

(b)  $\phi_x = \frac{-1}{JG} \int_0^x T(x) dx + \frac{t_0 L}{3JG} x$   
 $= \frac{-1}{JG} \left( \frac{t_0 L}{6} x + \frac{t_0 x^3}{6L} \right)$

$\frac{d\phi}{dx} = 0 \Rightarrow x = \frac{L}{\sqrt{3}}$

check  $x = L$ ,  $\phi_L = 0$  O.K.

G-38

$JG\phi'' = -\frac{t_0}{L}x$

$JG\phi = -\frac{t_0}{6L}x^3 + C_1x + C_2$

$\phi(0) = C_2 = 0$

$T = JG\phi' = -\frac{t_0}{2L}x^2 + C_1$

$T(L) = 0 \Rightarrow C_1 = \frac{t_0 L}{2}$

$\phi(L) = -\frac{t_0 L^2}{6JG} + \frac{t_0 L^2}{2} = \frac{t_0 L^2}{3}$

G-39

$JG\phi'' = -440 + \frac{240}{2}x$

$JG\phi' = T = -440x + 60x^2 + C_1$

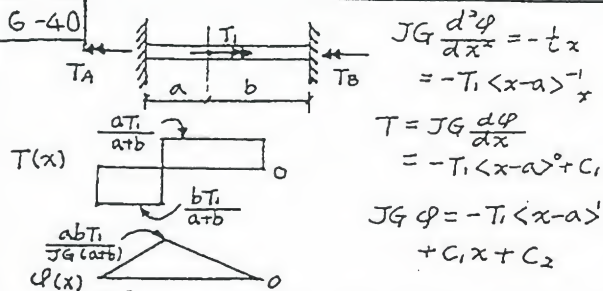
$T_{end} = 0 \Rightarrow C_1 = 0$

$JG\phi = -220x^2 + 20x^3 + C_2$

$\phi(2) = 0 \Rightarrow C_2 = 720$

$\phi_{end} = \phi(0) = \frac{720}{JG}$

G-40



$JG \frac{d^2\phi}{dx^2} = -tx$   
 $= -T_1 \langle x-a \rangle^{-1}$

$T = JG \frac{d\phi}{dx}$   
 $= -T_1 \langle x-a \rangle^0 + C_1$

$JG\phi = -T_1 \langle x-a \rangle^1 + C_1x + C_2$

Since  $\phi(0) = 0$ ,  $C_2 = 0$ ; also since

$\phi(L) = \phi(a+b) = 0$ ,  $C_1 = (b/L)T_1$

$T(0) = C_1 = T_A = \frac{b}{a+b}T_1$  →

$T(L) = T_B = -\frac{a}{a+b}T_1$  ←

G-41

(a)  $JG\phi'' = -t_0$   $x \leq \frac{L}{2}$

$JG\phi' = -t_0x + C_1$

$JG\phi = -\frac{t_0}{2}x^2 + C_1x + C_2$

$\phi(0) = 0 \Rightarrow C_2 = 0$

$JG\phi'(\frac{L}{2}) = -\frac{t_0 L}{2} + C_1$

$JG\phi(\frac{L}{2}) = -\frac{t_0 L^2}{8} + C_1 \frac{L}{2}$

(b)  $JG\phi'' = 0$   $x \geq \frac{L}{2}$

$JG\phi = C_1'x + C_2'$

$\phi(L) = 0 \Rightarrow C_2' = -C_1'L$

$JG\phi'(\frac{L}{2}) = C_1'$

$JG\phi(\frac{L}{2}) = -\frac{C_1'L}{2}$

$C_1' = -\frac{t_0 L}{2} + C_1$

$\frac{t_0 L^2}{4} - \frac{C_1}{2}L = -\frac{t_0 L^2}{8} + C_1 \frac{L}{2}$

$C_1 = \frac{3}{8}t_0 L$ ,  $C_1' = \frac{-1}{8}t_0 L$

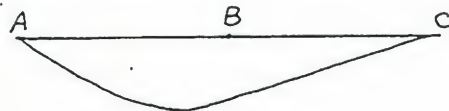
$T_A = JG\phi'(0) = \frac{3}{8}t_0 L$  ←

$T_B = JG\phi'(L) = \frac{1}{8}t_0 L$  ←

(angle of twist diagram  
on the next page)

6-41 (contd.)

$x$	0	$\frac{L}{8}$	$\frac{L}{4}$	$\frac{3L}{8}$	$\frac{L}{2}$	$\frac{3L}{4}$	$L$
$\frac{JG}{t_0 L^2} \phi$	0	$\frac{5}{128}$	$\frac{1}{16}$	$\frac{9}{128}$	$\frac{1}{16}$	$\frac{1}{32}$	0



$$\frac{9}{128} \frac{t_0 L^2}{JG}$$

6-42

$$(a) J_1 = 98.2 \times 10^{-3} d^4, J_2 = 31.06 \times 10^{-3} d^4$$

$$\tau_{1max} = 5.09 \frac{T_1}{d^3}, \tau_{2max} = 12.07 \frac{T_1}{d^3}$$

$$V_1 = \pi \left(\frac{d}{2}\right)^2 \times a = 785.4 \times 10^{-3} d^2 a$$

$$V_2 = \pi \left(\frac{3}{4} \times \frac{d}{2}\right)^2 \times a = 442 \times 10^{-3} d^2 a$$

$$\frac{T_1 \phi}{2} = \frac{\tau_{1max}^2}{2G} \times \frac{1}{2} V_1 + \frac{\tau_{2max}^2}{2G} \times \frac{1}{2} V_2$$

$$\phi = 42.37 \frac{T_1}{d^4 G}$$

(b) check by eq. 6-16

$$\phi = \frac{T_1 a}{J_1 G} + \frac{T_1 a}{J_2 G}$$

$$= \frac{T_1 a}{d^4 G} \left( \frac{32}{\pi} + \frac{32 \times 4^2}{\pi \times 3^2} \right)$$

$$= 42.4 \frac{T_1 a}{d^4 G} \quad \text{O.K.}$$

6-43

$$T = \sum F \cdot r = \sum (\tau A) r$$

$$(a) T = 8 \left[ (10.5) \pi \left(\frac{3}{8}\right)^2 \right] 5$$

$$= 185.55 \text{ k-in}$$

$$(b) T = \frac{63000}{N} H_p$$

$$H_p = \frac{300 (185.55 \times 10^3)}{63000} = 883.6$$

6-44

$$F = A \tau_{all}$$

$$F_o = 0.2 (16) = 3.2 \text{ k}$$

$$F_i = \frac{5}{8} [0.5 (16)] = 5 \text{ k}$$

$$T = \sum F r$$

$$T = 6 (3.2 \times 4 + 5 \times 2.5) = 151.80 \text{ k-in}$$

6-45

$$\tau = \frac{T c}{I_p} = \frac{800 (0.010)}{\frac{\pi}{2} (0.010)^4} = 509 \text{ MPa}$$

6-46

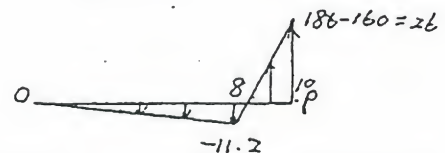
$$(a) T = \int_0^8 2\pi \left(\frac{160}{8}\right) \rho^3 d\rho + \int_8^{10} 160 \cdot 2\pi \rho^2 d\rho$$

$$= 40 \pi \frac{\rho^4}{4} \Big|_0^8 + 320 \pi \frac{\rho^3}{3} \Big|_8^{10}$$

$$= 292210 \text{ (N}\cdot\text{mm)}$$

$$= 292.9 \text{ (N}\cdot\text{m)}$$

$$(b) \tau_{max} = \frac{292210 \times 10}{\frac{\pi}{2} (10)^4} = 186 \text{ (MPa)}$$



6-47

$$\frac{\gamma_{max}}{10} = \frac{0.25}{1000} \Rightarrow \gamma_{max} = 0.0025$$

$$\rho_e = \frac{0.002}{0.0025} \times 10 = 8 \text{ (cm)}$$

from prob. 4-38 we get

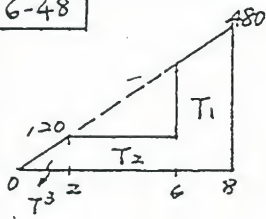
$$T = 292210 \text{ N}\cdot\text{mm}$$

$$\phi_R = \frac{292210 \times 1000}{\frac{\pi}{2} (10^4) \times 80 \times 10^3} = 0.232$$

$$\Delta \phi = 0.25 - 0.232 = 0.018 \text{ (rad)}$$

$$= 1.0 \text{ (degree)}$$

6-48



$$p_e = \frac{120}{480} \times 8 = 2$$

$$T_1 = 480 \times \frac{\pi}{2} \times \frac{(8^4 - 6^4)}{8} = 263894$$

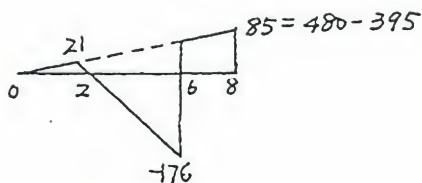
$$T_3 = 120 \times \frac{\pi}{2} \times \frac{2^4}{2} = 1508$$

$$T_2 = \int_2^6 120 \times 2\pi p^2 dp = 52276$$

$$T = T_1 + T_2 + T_3 = 317678 \text{ N-mm} = 318 \text{ N-m}$$

6-49

$$(a) \tau_{\max} = \frac{317678 \times 8}{\frac{\pi}{2} \times 8^4} = 395 \text{ (MPa)}$$



$$(b) \text{ initially } \theta_i = \frac{T_1}{JG} = 5.0 \times 10^{-4} \text{ rad}$$

$$\text{release } \theta_r = \frac{T}{JG} = 4.11 \times 10^{-4} \text{ rad}$$

$$\Delta\theta = (5 - 4.11) \times 10^{-4} = 0.89 \times 10^{-4} \text{ rad}$$

6-50

$$\text{Area} = \pi(5)^2 = 78.54 \text{ cm}^2$$

$$\text{Rectangle: } a = 2.5 \text{ cm, } b = 31.42 \text{ cm}$$

$$\text{Square: } s = \sqrt{78.54} = 8.86 \text{ cm}$$

$$\frac{\tau_c}{\tau_s} = \frac{Tc/J}{T/ds s^3} = \frac{5(8.86)^3(.208)}{(\pi/2)(5)^4} = 0.74$$

$$\frac{\tau_R}{\tau_s} = \frac{T/drba^2}{T/ds s^3} = \frac{(8.86)^3(.208)}{(0.333)31.42(2.5)^3} = 2.21$$

$$\tau_s : \tau_c : \tau_R = 1 : 0.74 : 2.21$$

$$\frac{\phi_c}{\phi_s} = \frac{TL/JG}{TL/\beta_s s^4 G} = \frac{0.141(8.86)^4}{(\pi/2)(5)^4} = 0.886$$

6-50 (contd.)

$$\frac{\phi_R}{\phi_s} = \frac{\beta_s s^4}{\beta_R a^3 b} = \frac{0.141(8.86)^4}{(0.333)(2.5)^3(31.42)} = 5.32$$

$$\phi_s : \phi_c : \phi_R = 1 : 0.886 : 5.32$$

6-51

$$\frac{\tau_{\text{slot}}}{\tau} = \frac{\frac{T}{0.33(2\pi R t^3)}}{\frac{TR}{2\pi R^3 t}} = \frac{3R}{t}$$

$$\frac{T_{\text{slot}}}{T} = \frac{\phi_G(0.33 \times 2\pi R t^3)}{\phi_G(2\pi R^3 t)} = \frac{t^2}{3R^2}$$

6-52

$$4 T_{\text{bar}} + T_{\text{tube}} = T$$

$$\text{Bar: } b = 2, t = 1 \quad b/t = 2 \Rightarrow \alpha = 0.246$$

$$\beta = 0.229$$

$$\frac{T_{\text{tube}}}{T_{\text{bar}}} = \frac{\pi/2(2^4 - 1.5^4)}{0.229 \times 2 \times 1^3} = 37.5$$

$$4 T_{\text{bar}} + 37.5 T_{\text{bar}} = T$$

$$T_{\text{bar}} = 0.0241 T$$

$$T_{\text{tube}} = 0.9036 T$$

$$T_{\text{tube}} = \frac{\tau I_p}{c} = 10 \frac{\pi}{2} (2^4 - 1.5^4) \cdot \frac{1}{2}$$

$$= 85.9 \text{ k-in}$$

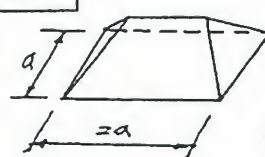
$$T = T_{\text{tube}}/0.9036 = 95.1 \text{ k-in}$$

$$T_{\text{bar}} = \tau \cdot \alpha b t^2 = 10(0.246) \times 2 \times 1^2 = 4.92 \text{ k-in}$$

$$T = T_{\text{bar}}/0.0241 = 204.1 \text{ k-in}$$

$$\therefore \text{Max } T = 95.1 \text{ k-in}$$

6-53



$$\tau_{yp} = \frac{h}{a/2}; \quad h = \frac{a}{2} \tau_{yp}$$

$$V = \frac{1}{2} h a^2 + \frac{1}{3} h a^2$$

$$= \frac{1}{2} \frac{a}{2} \tau_{yp} \cdot a^2 + \frac{1}{3} \frac{a}{2} \tau_{yp} a^2$$



6-53 (contd.)

$$V = \frac{a^3}{4} \tau_{yp} + \frac{a^3}{6} \tau_{yp} = \frac{5a^3 \tau_{yp}}{12}$$

$$T_{ult.} = 2V = 2 \times \frac{5a^3 \tau_{yp}}{12} = \frac{5}{6} a^3 \tau_{yp}$$

6-54

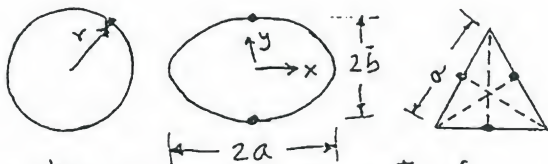
$$\frac{b}{t} = \frac{120}{9} = 13.3 \Rightarrow \beta \approx 0.312$$

$$J_0 = 2 \times \frac{\pi}{2} (20^4 - 12^4) + 0.312 \times 120 \times 9^3$$

$$= 465 \times 10^3 \text{ mm}^4$$

6-55

(a) Membrane analogy



Max stress occurs at point of max slope of membrane over these shapes

Circle: max stress around the circumference

Ellipse: max stress at  $x=0, y=\pm b$

Equilateral triangle: max stress at mid-side locations

(b)  $T = 1000 \text{ N}\cdot\text{m}$

$$r = b = 0.015 \text{ m}, a = 0.020 \text{ m}$$

Let  $G = 84 \text{ GPa}$  for steel

Circle:  $\tau_{max} = \frac{Tc}{I_p} = \frac{2T}{\pi r^3}$

$$\tau_{max} = \frac{2 \times 1000}{\pi (0.015)^3} = 188.6 \times 10^6 \text{ Pa}$$

$$\phi = \frac{TL}{I_p G} = \frac{2T}{\pi r^4} \frac{L}{G} = \frac{2 \times 1000 \times L}{\pi (0.015)^4 84 \times 10^9}$$

$$\phi = 0.15 L$$

Ellipse:  $\tau_{max} = \frac{2T}{\pi ab^2} = \frac{2 \times 1000}{\pi (0.020)(0.015)^2}$

$$\tau_{max} = 141.5 \times 10^6 \text{ Pa}$$

$$\phi = \frac{a^2 + b^2}{\pi a^3 b^3} \frac{TL}{G} = \frac{(0.02^2 + 0.015^2) 1000 L}{\pi (0.02)^3 (0.015)^3 84 \times 10^9}$$

$$\phi = 0.088 L$$

Equilateral Triangle:

$$\tau_{max} = \frac{20T}{a^3} = \frac{20 \times 1000}{0.02^3} = 2.5 \times 10^9 \text{ Pa}$$

$$\phi = \frac{80TL}{a^4 \sqrt{3} G} = \frac{80 \times 1000 L}{(0.02)^4 \sqrt{3} 84 \times 10^9}$$

$$\phi = 3.44 L$$

6-56

$$A = 1200 \text{ mm}^2 = 0.0012 \text{ m}^2$$

Circle:  $A = \pi r^2 = 0.0012, r = 0.01954 \text{ m}$

$$\tau_{max} = \frac{2T}{\pi r^3} = \frac{2T}{A r} = \frac{2 \times 1000}{(0.0012)(0.01954)}$$

$$\tau_{max} = 85.3 \times 10^6 \text{ Pa or } 85.3 \text{ MPa}$$

Ellipse:  $A = \pi ab$

$$\text{Let } b = 10 \text{ mm} = 0.010 \text{ m}$$

$$\tau_{max} = \frac{2T}{\pi ab^2} = \frac{2T}{A \cdot b} = \frac{2 \times 1000}{(0.0012)(0.010)}$$

$$\tau_{max} = 166.7 \times 10^6 \text{ Pa or } 166.7 \text{ MPa}$$

Equilateral triangle:

$$A = \frac{1}{2} a \cdot h = \frac{1}{2} a \cdot a \sin 60^\circ = \frac{\sqrt{3}}{4} a^2$$

$$A = \frac{\sqrt{3}}{4} a^2 = 0.0012, a = 0.0526 \text{ m}$$

$$\tau_{max} = \frac{20T}{a^3} = \frac{20 \times 1000}{(0.0526)^3}$$

$$\tau_{max} = 137 \times 10^6 \text{ Pa or } 137 \text{ MPa}$$

6-57

For the circle, ellipse and triangle to have the same torsional rigidity:

$$I_p \text{ circle} = K_{\text{ellipse}} = K_{\text{triangle}}$$

$$\frac{\pi r^4}{2} = \frac{\pi a^3 b^3}{(a^2 + b^2)} = \frac{a^4 \sqrt{3}}{80}$$

6-57 (contd.)

Triangle & circle:  $\frac{a^4 \sqrt{3}}{80} = \frac{\pi r^4}{2}$

$$a^4 = \frac{40\pi}{\sqrt{3}} r^4$$

$$a = 2.919 r$$

Ellipse & circle:  $\frac{\pi a^3 b^3}{a^2 + b^2} = \frac{\pi r^4}{2}$

Say, e.g., that  $b = a/2$

Then,  $\frac{\pi a^3 \cdot a^3/8}{a^2 + a^2/4} = \frac{\pi r^4}{2}$

$$a = 1.495 r$$

and  $b = a/2 = 0.748 r$

6-58

$$A = \pi(12)^2 = 452.4 \text{ in}^2$$

$$q = \frac{T}{2A} = \frac{1000}{2 \times 452.4} = 1.11 \text{ lb/in}$$

$$\tau_{\max} = \frac{q}{t_{\min}} = \frac{1.11}{0.1} = 11.1 \text{ psi}$$

$$\theta = \frac{T}{4A^2 G} \oint \frac{ds}{t} = \frac{1000}{4(452.4)^2 G} \left( \frac{12\pi}{0.1} + \frac{12\pi}{0.2} \right)$$

$$= \frac{1000 \times 180\pi}{4 \times 452^2 G}$$

$$= \frac{0.691}{G} \text{ rad/in}$$

6-59

$$T = 2 \textcircled{A} q = 2 \textcircled{A} \tau_{\max} t$$

$$= 2 \times \left( \frac{\pi}{2} 20^2 + 20 \times 40 + 20 \times 100 \right) \times 10^{-6} \times 20 \times 10^6 \times 0.004$$

$$T = 548.53 \text{ N.m}$$

Since shear flow is constant and  $\tau = q/t$ , increasing the thickness of the inclined plates would reduce  $\tau$  and help reduce stress concentration effect at the sharp corner.

6-60

(a)  $K_{t, \text{bar}} = \beta b t^3 \frac{G}{L}$

$$= 0.229 \times 60 \times 30^3 \frac{G}{L}$$

$$= 370980 \times \frac{G}{L}$$

$$K_{t, \text{box}} = 4A^2 / \oint ds/t \times \frac{G}{L}$$

$$= \frac{4 \times (40 \times 100)^2}{150/3 + 100/4} \times \frac{G}{L}$$

$$= 853333 \frac{G}{L}$$

$$K_{\text{total}} = 1224313 \frac{G}{L}$$

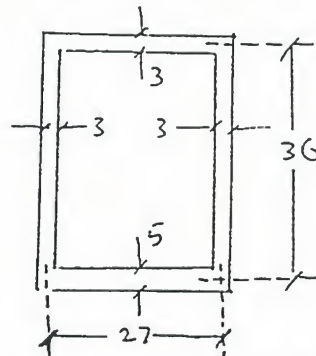
$$T_{\text{bar}} = 0.30 T, T_{\text{box}} = 0.70 T$$

$$\tau_{\text{bar}} = \frac{0.30 T}{0.246 \times 60 \times 30^2} = 3.39 \text{ MPa}$$

$$\tau_{\text{box}} = \frac{0.70 T}{2 \times 40 \times 100 \times 3} = 4.38 \text{ MPa}$$

(b)  $\theta = \frac{\phi}{L} = \frac{T}{K_t L} = \frac{150}{1224313 \times 25} = 4.9 \times 10^{-6} \text{ (rad)}$

6-61



$$\textcircled{A} = (0.027)(0.036) = 9.72 \times 10^{-4}$$

$$\tau = \frac{T}{2 \textcircled{A} t_{\min}} = \frac{T}{2 \times 9.72 \times 10^{-4} (0.003)}$$

Since  $\tau = 120 \times 10^6 \text{ Pa}$ ,

$$T = 120 \times 10^6 \times 2 \times 9.72 \times 10^{-4} \times 0.003$$

$$T = 699.8 \text{ N.m}$$

$$K_t = \frac{4 \textcircled{A}^2 G/L}{\oint ds/t} = \frac{4 (9.72 \times 10^{-4})^2 G/L}{\frac{0.099}{0.003} + \frac{0.027}{0.005}}$$

6-61 (cont'd)

$$k_t = 9.842 \times 10^{-8} \text{ G/L}$$

Flexibility in torsion:

$$f_t = \frac{1}{k_t} = 1.016 \times 10^7 \text{ L/G}$$

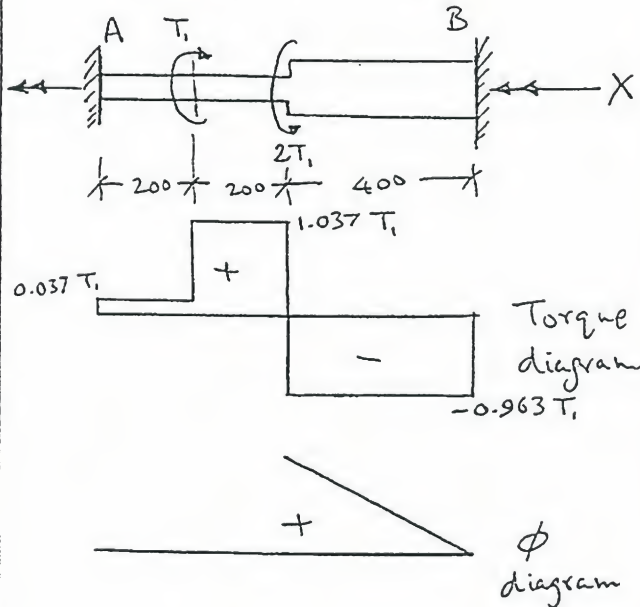
6-62

Solid bar:  $\phi = \frac{TL}{\beta b t^3 G}$

Tube:  $\phi = \frac{TL}{G} \frac{\phi ds/t}{4(A)^2}$

Solid bar:  $\beta b t^3 = 0.141 \times 20 \times 20^3$   
 $= 2.256 \times 10^4$

Tube:  $\frac{4(A)^2}{\phi ds/t} = \frac{4 \times (30 \times 30)^2}{120/1.5} = 4.05 \times 10^4$



Cut at B and reapply torque X:

$$\phi = \frac{2T_1(200)}{G(2.256 \times 10^4)} + \frac{T_1(200)}{G(2.256 \times 10^4)} = \frac{X(400)}{G(2.256 \times 10^4)} + \frac{X(400)}{G(4.05 \times 10^4)}$$

Solving,

$$X = T_B = 0.963 T_1$$

From  $\sum M_x = 0$ ,  $T_A = 0.037 T_1$

Tube:  $T_{max} = \tau_{max} 2(A) t$

$$T_{max} = 40 \times 10^6 \times 2 \times 0.030^2 \times 0.0015$$

$$= 108 \text{ N.m} = 0.963 T_1$$

$$\therefore T_1 = 112.15 \text{ N.m}$$

Square bar:  $T_{max} = \tau_{max} \alpha b t^2$

$$T_{max} = 40 \times 10^6 \times 0.208 \times 0.02 \times 0.02^2$$

$$= 1.331 \text{ N.m} = 0.037 T_1$$

$$\therefore T_1 = 35.98 \text{ N.m}$$

Governs

$$\Delta_{tip} = 0.250 \phi$$

$$\phi = \frac{X}{G} \frac{400}{4.05 \times 10^4} = 0.963 \frac{T_1}{G} (9.885 \times 10^{-3})$$

$$\phi = 9.51 \times 10^{-3} \frac{T_1}{G} \quad [\text{note that } 9.51 \times 10^{-3} \text{ was derived using mm units}]$$

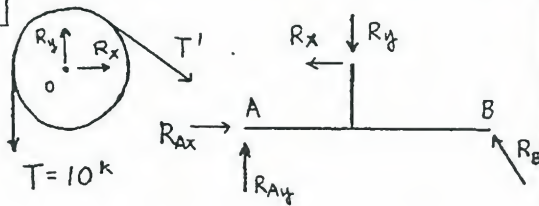
$$\phi = 9.51 \times 10^{-6} \frac{T_1}{G} \quad [\text{using consistent meter units}]$$

$$\Delta_{tip} = 0.25 \times 9.51 \times 10^{-6} \times \frac{35.98}{200 \times 10^{-9}}$$

$$\Delta_{tip} = 0.00043 \text{ m or } 0.43 \text{ mm}$$



7-1



$$\sum M_O = 0 \rightarrow T' = T = 10^k$$

$$R_x = -\frac{2}{\sqrt{5}} T' = -4\sqrt{5}$$

$$R_y = T + \frac{1}{\sqrt{5}} T' = 10 + 2\sqrt{5}$$

$$\begin{cases} R_{Ax} - \frac{1}{\sqrt{2}} R_B = -4\sqrt{5} \\ R_{Ay} + \frac{1}{\sqrt{2}} R_B = 10 + 2\sqrt{5} \\ 10R_{Ay} - 4R_x - 6R_y = 0 \end{cases}$$

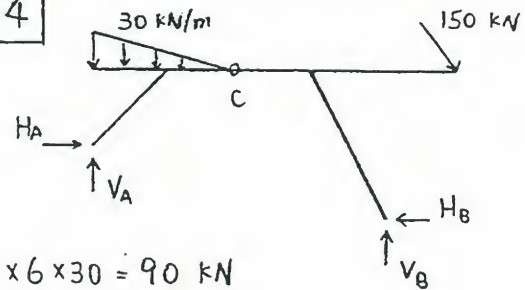
$$\Rightarrow \begin{cases} R_{Ax} = 0.422^k \\ R_{Ay} = 5.106^k \\ R_B = 13.246^k \end{cases}$$

$$\sum M_A = 0$$

$$R_B = \frac{1}{3} (4 \times 30 \times 2 + 25 \times 4) = 113.33 \text{ kN}$$

$$R_A = 4 \times 30 + R_y - R_B = 31.67 \text{ kN}$$

7-4



$$\frac{1}{2} \times 6 \times 30 = 90 \text{ kN}$$

$$\sum M_B = 0 \rightarrow 3H_A + 12V_A = 10 \times 90 - 3 \times 120 - 6 \times 90$$

$$\sum M_C = 0 \rightarrow -3H_A + 6V_A = 4 \times 90$$

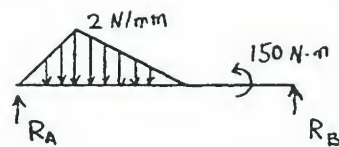
$$\therefore H_A = -80 \text{ kN}, V_A = 20 \text{ kN}$$

$$\sum M_A = 0 \rightarrow 3H_B - 12V_B = -2 \times 90 - 15 \times 120 - 3 \times 90$$

$$\sum M_C = 0 \rightarrow 6H_B - 6V_B = -9 \times 120$$

$$\therefore H_B = 10 \text{ kN}, V_B = 190 \text{ kN}$$

7-2



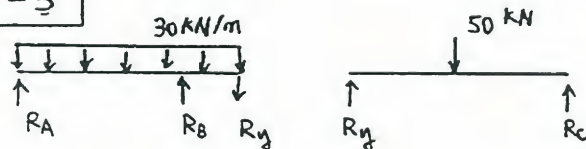
$$\sum M_A = 0$$

$$R_B = \frac{1}{1500} \left( \frac{1}{2} \times 300 \times 2 \times 200 + \frac{1}{2} \times 600 \times 2 \times 500 - 150 \times 10^3 \right)$$

$$= 140 \text{ N}$$

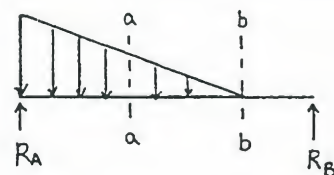
$$\sum F_y = 0, R_A = \frac{1}{2} \times 900 \times 2 - 140 = 760 \text{ N}$$

7-3



$$R_y = R_c = 25 \text{ kN}$$

7-5



$$\sum M_B = 0 \rightarrow R_A = \frac{1}{8} \left( \frac{1}{2} \times 6 \times 8 \times 6 \right) = 18^k$$

$$\sum M_A = 0 \rightarrow R_B = \frac{1}{8} \left( \frac{1}{2} \times 6 \times 8 \times 2 \right) = 6^k$$

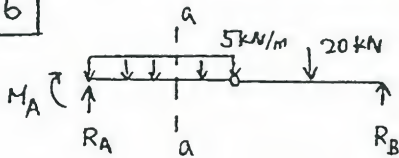
$$V_a = -6 + \frac{1}{2} \times 3 \times 4 = 0$$

$$M_a = 6 \times 5 - \frac{1}{2} \times 3 \times 4 \times 1 = 24 \text{ k-ft } (\circlearrowright)$$

$$V_b = -6^k$$

$$M_b = 6 \times 2 = 12 \text{ k-ft } (\circlearrowright)$$

7-6



$$R_B = 10 \text{ kN}$$

$$R_A = 8 \times 5 + 20 - 10 = 50 \text{ kN}$$

$$M_A = 16 \times 10 = 12 \times 20 - 8 \times 5 \times 4 = -240 \text{ kN}\cdot\text{m}$$

$$V_a = 50 - 5 \times 5 = 25 \text{ kN}$$

$$M_a = -240 + 5 \times 50 - 5 \times 5 \times 2.5 = -52.5 \text{ kN}\cdot\text{m}$$

$$R_x = 4 - 2 = 2 \text{ kN}, \quad R_y = 2\sqrt{3} \text{ kN}$$

$$V_A = \frac{1}{2000} (4 \times 750) = 1.5 \text{ kN}$$

$$V_B = -1.5 \text{ kN}, \quad H_B = 4 \text{ kN}$$

$$P_a = 0 \text{ kN}, \quad V_a = 1.5 \text{ kN}; \quad M_a = 1.5 \text{ kN}\cdot\text{m}$$

$$P_b = 2 \text{ kN}, \quad V_b = 1.5 - 2\sqrt{3} = -1.96 \text{ kN}$$

$$M_b = 1.5 \times 1 - 2 \times 0.5 = 0.5 \text{ kN}\cdot\text{m}$$

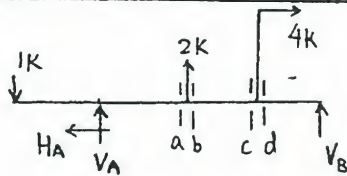
$$P_c = 2 \text{ kN}, \quad V_c = -1.96 \text{ kN}$$

$$M_c = 1.5 \times 1.57735 - 2 \times 0.5 - 2\sqrt{3} \times 0.57735 = -0.63 \text{ kN}\cdot\text{m}$$

$$P_d = 4 \text{ kN}, \quad V_d = 1.5 \text{ kN}$$

$$M_d = -1.5 \times 0.42265 = -0.63 \text{ kN}\cdot\text{m}$$

7-7



$$H_A = 4 \text{ k}, \quad V_A = \frac{1}{10} (14 - 2 \times 6 - 4 \times 5) = -1.8$$

$$V_B = 1 - 2 + 1.8 = 0.8 \text{ k}$$

$$P_a = 4 \text{ k}, \quad V_a = -1 - 1.8 = -2.8 \text{ k}$$

$$M_a = -8 \times 1 - 4 \times 1.8 = -15.2 \text{ k}\cdot\text{ft}$$

$$P_b = 4 \text{ k}, \quad V_b = -2.8 + 2 = -0.8 \text{ k}$$

$$M_b = -15.2 \text{ k}\cdot\text{ft}$$

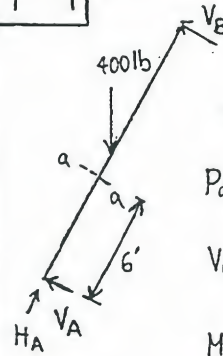
$$P_c = 4 \text{ k}, \quad V_c = -0.8 \text{ k}$$

$$M_c = -11 \times 1 - 7 \times 1.8 + 2 \times 3 = -17.6 \text{ k}\cdot\text{ft}$$

$$P_d = 0 \text{ k}, \quad V_d = -0.8 \text{ k}$$

$$M_d = -17.6 + 5 \times 4 = 2.4 \text{ k}\cdot\text{ft}$$

7-9



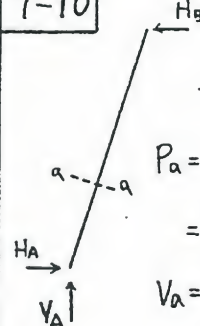
$$V_A = V_B = 400 \times 4 / 4\sqrt{13} = 110.94 \text{ lb}$$

$$P_a = -\frac{3}{\sqrt{13}} 400 = -332.82 \text{ lb}$$

$$V_a = \frac{2}{\sqrt{13}} 400 - 110.94 = 110.94 \text{ lb}$$

$$M_a = 110.94 \times 6 = 665.64 \text{ lb}\cdot\text{ft}$$

7-10



$$V_A = 10 \times \sqrt{5} \times 9.81 = 219.36 \text{ N}$$

$$H_A = H_B = 219.36 \times 0.5 / 2 = 54.84 \text{ N}$$

$$P_a = -\left[\frac{2}{\sqrt{5}} \times (219.36 - 10 \times 0.8 \times 9.81) + \frac{1}{\sqrt{5}} \times 54.84\right]$$

$$= -150.53 \text{ N}$$

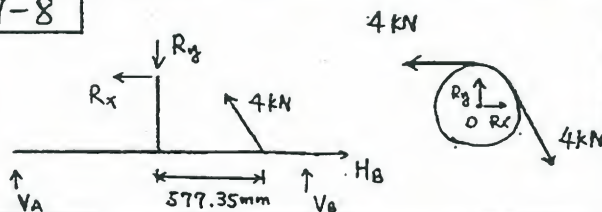
$$V_a = \frac{1}{\sqrt{5}} (219.36 - 10 \times 0.8 \times 9.81) - \frac{2}{\sqrt{5}} \times 54.84$$

$$= 13.95 \text{ N}$$

$$M_a = \frac{0.8}{\sqrt{5}} (219.36 - \frac{1}{2} \times 10 \times 0.8 \times 9.81 - 2 \times 54.84)$$

$$= 25.2 \text{ N}\cdot\text{m}$$

7-8



7-11

$$P_a = \frac{P}{2} (\cos 90^\circ - \sin 90^\circ) = -\frac{P}{2}$$

$$V_a = -\frac{P}{2} (\cos 90^\circ + \sin 90^\circ) = -\frac{P}{2}$$

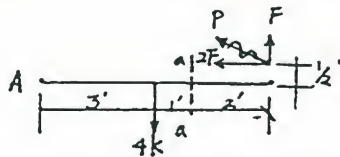
$$M_a = \frac{PR}{2} (0 - 1 - 1) = -PR$$

$$P_b = \frac{P}{2} (\cos 45^\circ - \sin 45^\circ) = 0$$

$$V_b = -\frac{P}{2} (\cos 45^\circ + \sin 45^\circ) = -\frac{\sqrt{2}}{2} P$$

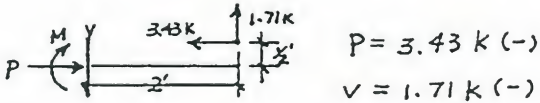
$$M_b = \frac{-PR}{2} (\sin 45^\circ + 1 - \cos 45^\circ) = \frac{-PR}{2}$$

7-12



$$\sum M_a = 0, 6F + 2F(\frac{1}{2}) - 4(3) = 0$$

$$F = 1.71 \text{ k} \uparrow, 2F = 3.43 \text{ k} \leftarrow$$

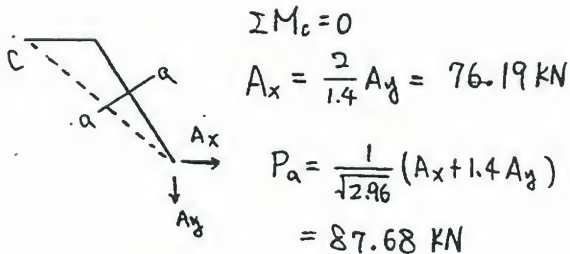


$$M = 3.43(\frac{1}{2}) + 1.71(2) = 5.14 \text{ k-ft (+)}$$

7-13



$$\sum M_B = 0 \rightarrow A_y = \frac{1}{3} (4 \times 40) = 53.33 \text{ kN}$$



$$\sum M_c = 0$$

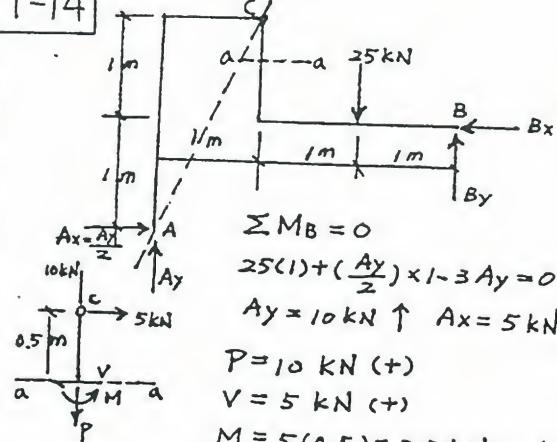
$$A_x = \frac{2}{1.4} A_y = 76.19 \text{ kN}$$

$$P_a = \frac{1}{\sqrt{2.96}} (A_x + 1.4 A_y) = 87.68 \text{ kN}$$

$$V_a = \frac{1}{\sqrt{2.96}} (1.4 A_x - A_y) = 31 \text{ kN}$$

$$M_a = 0.7 A_x - 0.5 A_y = 26.67 \text{ kN}$$

7-14



$$\sum M_B = 0$$

$$25(1) + (\frac{A_y}{2}) \times 1 - 3 A_y = 0$$

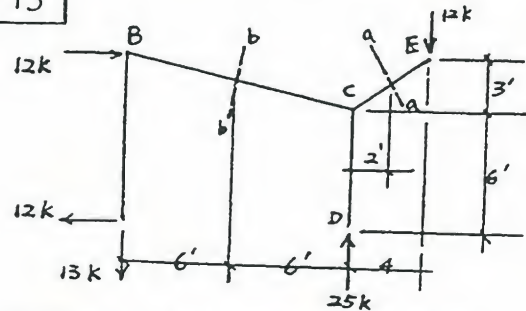
$$A_y = 10 \text{ kN} \uparrow, A_x = 5 \text{ kN} \rightarrow$$

$$P = 10 \text{ kN (+)}$$

$$V = 5 \text{ kN (+)}$$

$$M = 5(0.5) = 2.5 \text{ kN-m (+)}$$

7-15



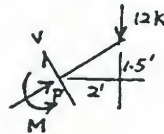
$$\sum M_A = 0, 12(9) + 16(12) - 12(R_D) = 0$$

$$R_D = 25 \text{ k} \uparrow$$

$$\sum F_x = 0, A_x = 12 \text{ k} \leftarrow$$

$$\sum F_y = 0, A_y = 25 - 12 = 13 \text{ k} \downarrow$$

section a-a

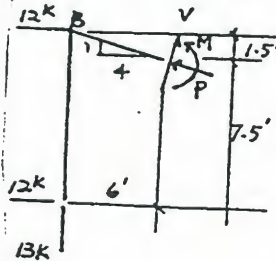


$$P = 12(\frac{3}{5}) = 7.2 \text{ k (-)}$$

$$V = 12(\frac{4}{5}) = 9.6 \text{ k (+)}$$

$$M = 12(2) = 24 \text{ k (-)}$$

section b-b



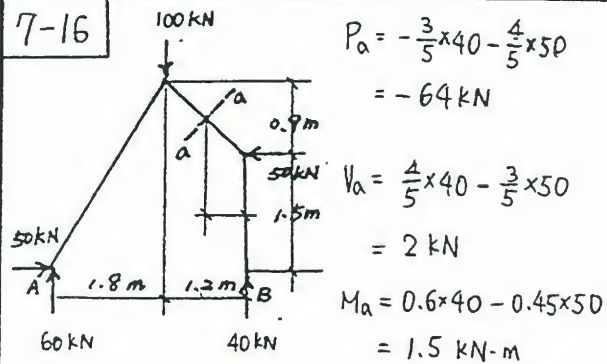
$$P = 13(\frac{1}{\sqrt{17}}) = 3.15 \text{ k (-)}$$

$$V = 13(\frac{4}{\sqrt{17}}) = 12.61 \text{ k (-)}$$

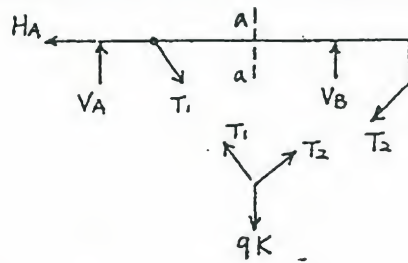
$$M = 12(9) - 13(6) = 30 \text{ k-ft (+)}$$



7-16



7-17



$$\frac{T_1}{\sqrt{5}} = \frac{T_2}{\sqrt{2}}, \quad \frac{2T_1}{\sqrt{5}} + \frac{T_2}{\sqrt{2}} = 9$$

$$T_2 = 3\sqrt{2}, \quad T_1 = 3\sqrt{5}$$

$$\sum M_A = 0 \quad V_B = 6 \text{ (K)} \uparrow$$

$$\sum P = 0 \quad P_a = -3 \text{ (K)}$$

$$\sum V = 0 \quad V_a = -3 \text{ (K)}$$

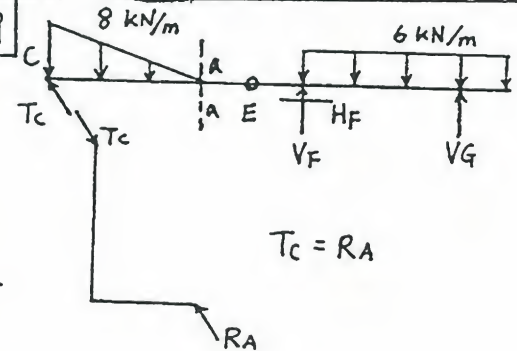
$$\sum M_a = 0 \quad M_a = 6 \text{ k}\cdot\text{ft}$$

$$P_a = \frac{3}{5} T_c = 8 \text{ kN}$$

$$V_a = \frac{(4+8) \times 1.5}{2} - \frac{4}{5} T_c = -1.62 \text{ kN}$$

$$M_a = \frac{4}{5} T_c \times 1.5 - 9 \times \frac{5}{9} \times 1.5 = 8.5 \text{ kN}\cdot\text{m}$$

7-19



$$T_c = R_A$$

$$\sum M_E = 0 \quad T_c = 11.25 \text{ kN}$$

$$P_a = \frac{3}{5} T_c = 6.75 \text{ kN (tension)}$$

$$V_a = 12 - \frac{4}{5} T_c = -3.0 \text{ kN}$$

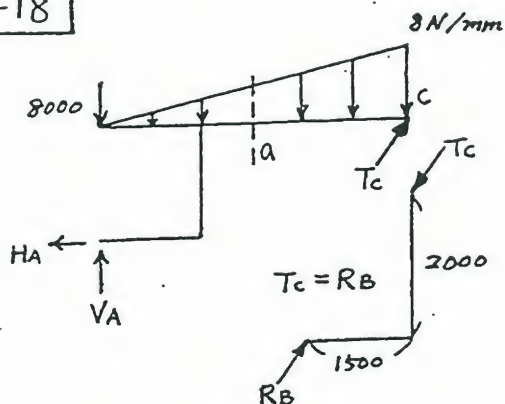
$$M_a = \frac{1}{2} \times 8 \times 3 \times 2 + \frac{4}{5} T_c \times 3 = +3 \text{ kN}\cdot\text{m}$$

$$H_F = P_a = 6.75 \text{ kN} \rightarrow$$

$$\sum M_F = 0 \quad V_G = 15.75 \text{ kN} \uparrow$$

$$\sum V = 0 \quad V_F = 9 \text{ kN} \uparrow$$

7-18

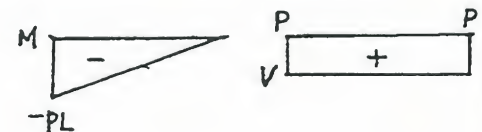


$$\sum M_A = 0 \quad T_c = 40/3 = 13.3 \text{ kN}$$

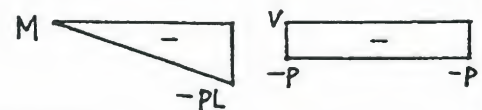
$$\sum H = 0 \quad H_A = \frac{3}{5} T_c = 8 \text{ kN}$$

$$\sum V = 0 \quad V_A = 12 + 8 - \frac{4}{5} T_c = 9.3 \text{ kN}$$

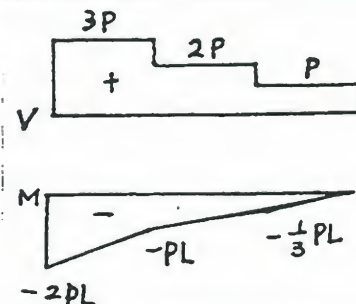
7-20



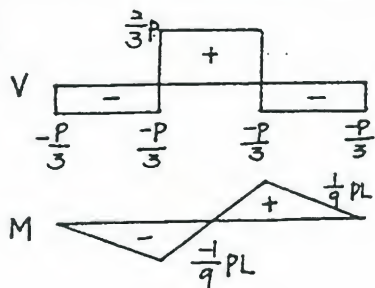
(b)



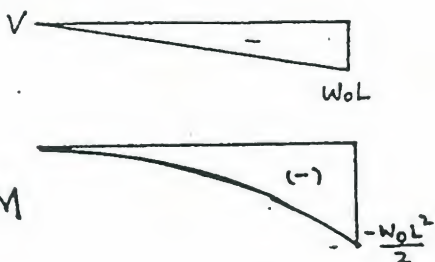
7-21



7-22



7-23



7-24

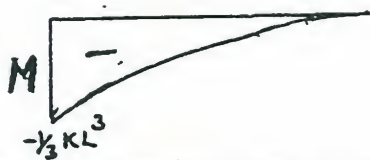
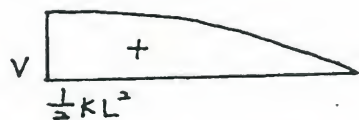
$$V' = -KX \quad V = \frac{-K}{2} X^2 + C$$

$$X=L, V=0 \Rightarrow C = \frac{K}{2} L^2$$

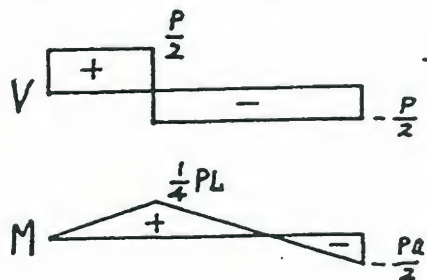
$$M' = \frac{-K}{2} X^2 + \frac{K}{2} L^2$$

$$M = -\frac{K}{6} X^3 + \frac{K}{2} L^2 X + C_1$$

$$X=L, M=0 \Rightarrow C_1 = \frac{-KL^3}{3}$$



7-25



7-26

$$M''(x) = -4X, \quad X \leq 3$$

$$M'(x) = -2X^2 + C_1, \quad X \leq 3$$

$$M'(x) = C_2, \quad X > 3$$

$$C_1 = R_A = \frac{1}{6} \left( \frac{1}{2} \times 3 \times 12 \times 4 \right) = 12$$

$$C_2 = 12 - \frac{1}{2} \times 3 \times 12 = -6$$

$$\text{when } X=0, M(0)=0$$

$$\Rightarrow \begin{cases} M(x) = -\frac{2}{3} X^3 + 12X & X \leq 3 \\ M(x) = -6X + 36 & X > 3 \end{cases}$$

7-27

$$R_A = \frac{1}{9} \left( \frac{1}{2} \times 6 \times 6 \times 2 \right) = 4 \text{ kN}$$

$$M(x) = 4X, \quad X < 3$$

$$M(x) = 4X - \frac{1}{2}(x-3) \times (x-3) \times \frac{1}{3}(x-3)$$

$$= 4X - \frac{1}{6}(x-3)^3, \quad X \geq 3$$

7-28

$$R_A = \frac{1}{6} \left( \frac{1}{2} \times 3 \times 6 \times 5 \right) = \frac{15}{2} \text{ kN}$$

$$M(x) = \frac{15}{2} X - \frac{6}{2} X^2 + \frac{1}{2} X \cdot 2X \cdot \frac{X}{3}$$

$$= \frac{15}{2} X - 3X^2 + \frac{1}{3} X^3, \quad X \leq 3$$

$$M(x) = \frac{15}{2} X - \frac{1}{2} \times 3 \times 6 \times (x-1)$$

$$= 9 - \frac{3}{2} X, \quad X > 3$$

7-29

$$R_A = R_B = \frac{W_0 L}{2}$$

$$M = \frac{W_0 L}{2} X - \frac{W_0}{2} X^2 + M_A$$

7-30

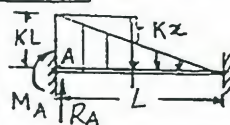
$$M = R_A X - \frac{K}{2} X^2 - \frac{2}{3} X$$

$$M' = R_A - KX - \frac{2}{3} = 0 \quad R_B = R_A - KL^2$$

$$R_A + R_B = \frac{1}{2} KL^2, \quad R_A = \frac{3}{4} KL^2$$

$$R_B = -\frac{1}{4} KL^2$$

7-31



$$M(x) = M_A + R_A x - K L x \frac{x}{2} + \frac{1}{2} x K x \frac{x}{3}$$

$$= M_A + R_A x - \frac{KLx^2}{2} + \frac{Kx^3}{3}$$

7-32

$$M(\theta) = 1000 \times 0.2 \sin \theta = 200 \sin \theta$$

$$P(\theta) = 1000 \times \sin \theta \text{ (tension)}$$

$$V(\theta) = 1000 \cos \theta$$

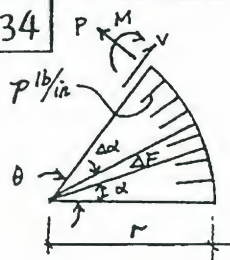
7-33

$$M(\theta) = \frac{-PR}{2} (\sin \theta + 1 - \cos \theta)$$

$$V(\theta) = \frac{-P}{2} (\cos \theta + \sin \theta)$$

$$P(\theta) = \frac{P}{2} (\cos \theta - \sin \theta)$$

7-34

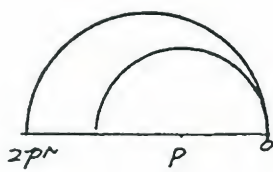


$$\Delta F = p r \Delta \alpha$$

$$\Delta P = p r \Delta \alpha \sin(\theta - \alpha)$$

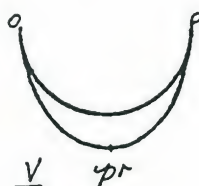
$$\Delta V = -p r \Delta \alpha \cos(\theta - \alpha)$$

$$\Delta M = p r^2 \Delta \alpha \sin(\theta - \alpha)$$



$$P = \int_0^\theta p r \sin(\theta - \alpha) d\alpha$$

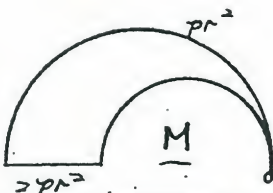
$$P = p r (1 - \cos \theta)$$



$$V = \int_0^\theta -p r \cos(\theta - \alpha) d\alpha$$

$$V = -p r \sin \theta$$

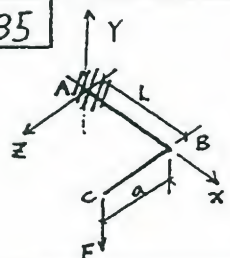
$$= p r \sin(-\theta)$$



$$M = \int_0^\theta p r^2 \sin(\theta - \alpha) d\alpha$$

$$M = p r^2 (1 - \cos \theta)$$

7-35



bar AB:  $V = +F$   
 $M = -F(L-x)$   
 $T = Fa$

bar BC:  $V = +F$   
 $M = -F(a-z)$   
 $T = 0$

7-36

see solution of Prob. 7-24

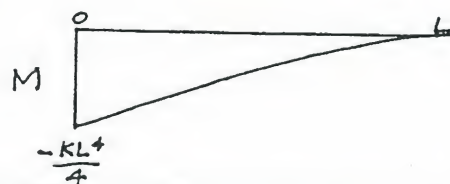
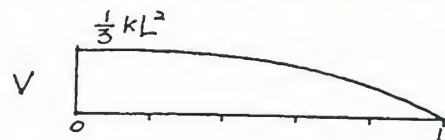
7-37

$$V' = -kx^2, \quad V = -\frac{k}{3}x^3 + C_1$$

$$\text{Since } x=L, V=0 \text{ so } C_1 = \frac{1}{3}kL^3$$

$$M = -\frac{k}{12}x^4 + \frac{1}{3}kL^3x + C_2$$

$$\text{Since } x=L, M=0 \text{ so } C_2 = -\frac{k}{4}L^4$$



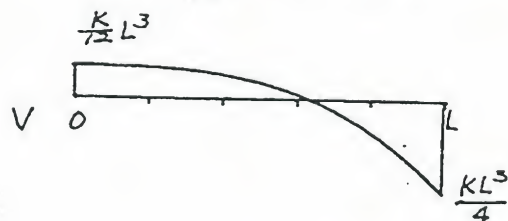
7-38

$$V' = -kx^2, \quad V = -\frac{k}{3}x^3 + C_1$$

$$M = -\frac{k}{12}x^4 + C_1x + C_2$$

$$\text{Since } x=0, L, M=0$$

$$\text{so } C_1 = \frac{k}{12}L^3, \quad C_2 = 0$$



7-39

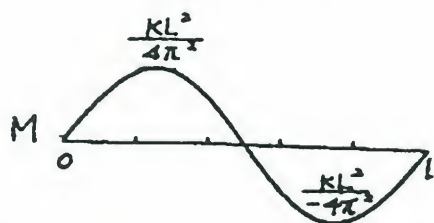
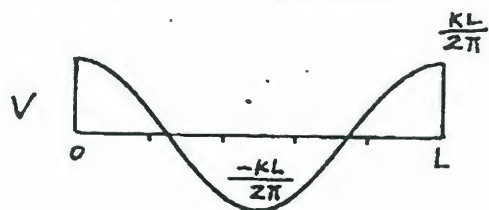
$$V' = K \sin \frac{2\pi}{L} x$$

$$V = -\frac{KL}{2\pi} \cos \frac{2\pi}{L} x + C_1$$

$$M = -\frac{KL^2}{4\pi^2} \sin \frac{2\pi}{L} x + C_1x + C_2$$



since  $x=0, L$   $M=0$   
 so  $C_1=0$ ,  $C_2=0$



7-40

$V=0$ ,  $M=M_1$



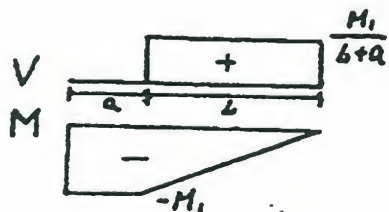
7-41

$V = \frac{-M_1}{L}$   $M = \frac{-M_1}{L} x$



7-42

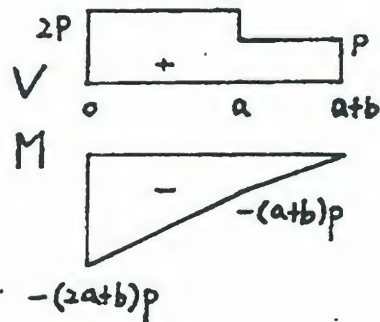
$M = C_1 x + C_2$ ,  $V = C_1$   
 $x=0$ ,  $M=-M_1$ ,  $C_2=-M_1$   
 $x=L+a$ ,  $M=0$ ,  $C_1 = \frac{M_1}{L+a}$



7-43

$V=2p$   $0 \leq x \leq a$   
 $V=p$   $a < x < a+b$

$M_1 = 2px + C_1$   $0 \leq x \leq a$   
 $M_2 = px + C_2$   $a \leq x \leq a+b$   
 $x=a+b$   $M=0 \Rightarrow C_2 = -(a+b)p$   
 $x=a$ ,  $M_1=M_2 \Rightarrow C_1 = -(2a+b)p$



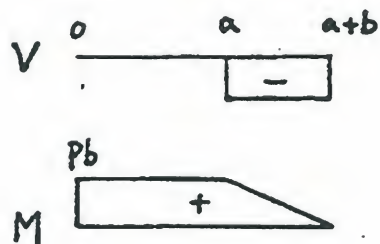
7-44

$V=0$   $0 \leq x \leq a$   
 $V=-p$   $a < x \leq b+a$   
 $M=C_1$   $0 \leq x \leq a$

$M = -px + C_2$   $a < x \leq b+a$

$x=0$   $M=pb = C_1$

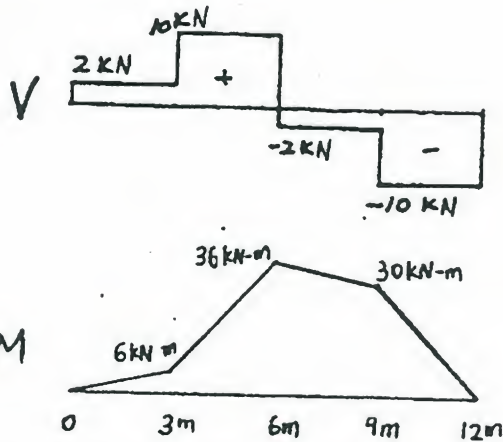
$x=a+b$ ,  $M=0$ ,  $C_2 = p(a+b)$



7-45

$V=2$ ,  $M=2x+C_1$ ,  $0 \leq x \leq 3$   
 $V=10$ ,  $M=10x+C_2$ ,  $3 \leq x \leq 6$   
 $V=-2$ ,  $M=-2x+C_3$ ,  $6 \leq x \leq 9$   
 $V=-10$ ,  $M=-10x+C_4$ ,  $9 \leq x \leq 12$

$$\begin{aligned}
 x=0, \quad M=0, \quad C_1=0 \\
 x=3 \quad M=6 \quad C_2=24 \\
 x=6 \quad M=36 \quad C_3=48 \\
 x=9 \quad M=30 \quad C_4=120 \\
 x=12 \quad M=0 \quad \text{O.K.}
 \end{aligned}$$



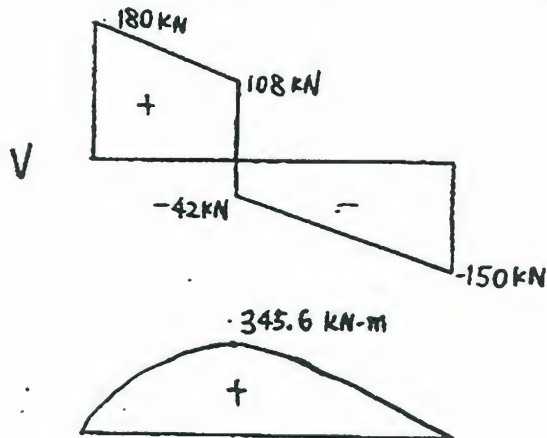
7-46

$$V(x) = -30x + 180 \quad 0 \leq x \leq 2.4$$

$$V(x) = -30x + 30 \quad 2.4 < x \leq 6$$

$$M(x) = -15x^2 + 180x \quad 0 \leq x \leq 2.4$$

$$M(x) = -15x^2 + 30x + 360 \quad 2.4 < x \leq 6$$



7-47

$$V = \frac{-1}{3}P \quad 0 \leq x \leq a$$

$$V = \frac{2}{3}P \quad a < x \leq 2a$$

$$V = -\frac{1}{3}P \quad 2a < x \leq 3a$$

$$M = -\frac{1}{3}Px + C_1 \quad 0 \leq x \leq a$$

$$M = \frac{2}{3}Px + C_2 \quad a \leq x \leq 2a$$

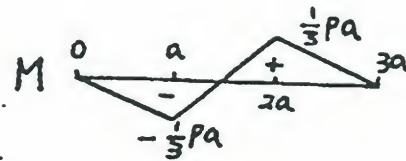
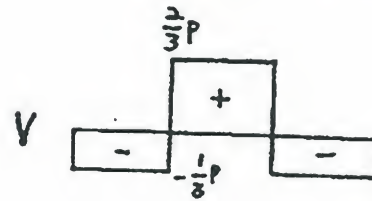
$$M = -\frac{1}{3}Px + C_3 \quad 2a \leq x \leq 3a$$

$$x=0, \quad M=0, \quad C_1=0$$

$$x=a, \quad M = \frac{Pa}{3}, \quad C_2 = -Pa$$

$$x=2a, \quad M = \frac{Pa}{3}, \quad C_3 = Pa$$

$$x=3a, \quad M=0 \quad \text{O.K.}$$



7-48

$$V = -3, \quad M = -3x + C_1 \quad 0 \leq x \leq 4$$

$$V = 10, \quad M = 10x + C_2 \quad 4 \leq x \leq 7$$

$$V = 2, \quad M = 2x + C_3 \quad 7 \leq x \leq 10$$

$$V = -8, \quad M = -8x + C_4 \quad 10 \leq x \leq 16$$

$$V = 4, \quad M = 4x + C_5 \quad 16 \leq x \leq 22$$

$$x=0, \quad M=0, \quad C_1=0$$

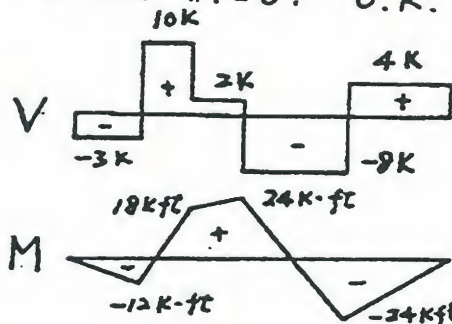
$$x=4, \quad M = -12, \quad C_2 = -52$$

$$x=7, \quad M = 18, \quad C_3 = 4$$

$$x=10, \quad M = 24, \quad C_4 = 104$$

$$x=16, \quad M = -24, \quad C_5 = -88$$

$$x=22, \quad M=0, \quad \text{O.K.}$$



7-49

$$V_0 = \frac{1}{2} \times 9 \times 120 = 540 \text{ kN}$$

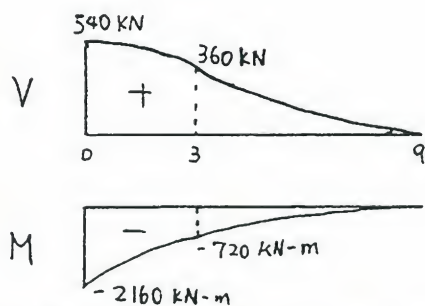
$$M_0 = -\frac{120}{2}(3 \times 2 + 6 \times 5) = -2160 \text{ kN-m}$$

$$V(x) = -20x^2 + 540 \quad 0 \leq x \leq 3$$

$$V(x) = 10(9-x)^2 = 10x^2 - 180x + 810 \quad 3 \leq x \leq 9$$

$$M(x) = -\frac{20}{3}x^3 + 540x - 2160 \quad 0 \leq x \leq 3$$

$$M(x) = \frac{10}{3}x^3 - 90x^2 + 810x - 2430 \quad 3 \leq x \leq 9$$

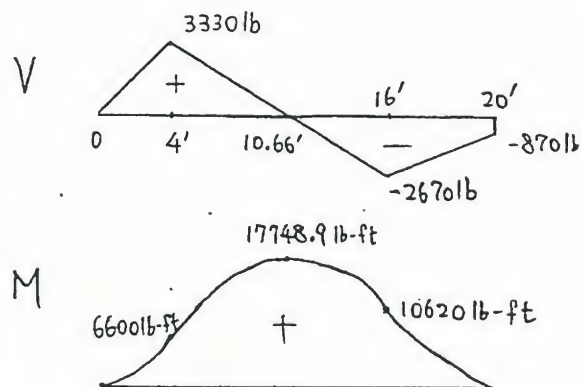


$$V(x) = (-500)(16) + 5330 + 200(x-16) \\ = 200x - 5870 \quad 16 \leq x \leq 22$$

$$M(x) = 416.25x^2 \quad 0 \leq x \leq 4$$

$$M(x) = -250x^2 + 5330x - 10660 \quad 4 \leq x \leq 16$$

$$M(x) = 100x^2 - 5870x + 78940 \quad 16 \leq x \leq 22$$

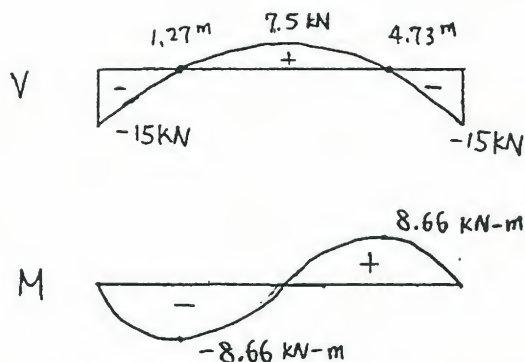


7-50

$$V_0 = -\frac{1}{6} \left( \frac{1}{2} \times 3 \times 15 \times 4 \right) = -15 \text{ kN}$$

$$V(x) = -15 + 15x - \frac{5}{2}x^2$$

$$M(x) = -15x + \frac{15}{2}x^2 - \frac{5}{6}x^3$$



7-52

$$V(x) = -5x + 100 \quad 0 \leq x \leq 20$$

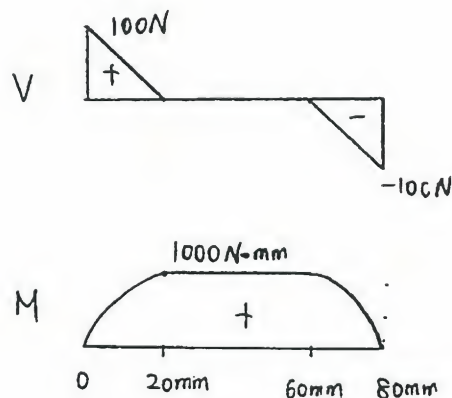
$$V(x) = 0 \quad 20 \leq x \leq 60$$

$$V(x) = -5x + 300 \quad 60 \leq x \leq 80$$

$$M(x) = -2.5x^2 + 100x \quad 0 \leq x \leq 20$$

$$M(x) = 1000 \quad 20 \leq x \leq 60$$

$$M(x) = -2.5x^2 + 300x - 8000 \quad 60 \leq x \leq 80$$



7-51

$$4q_0 \times 20 = 500 \times 12 \times 12 - 300 \times 6 \times 3$$

$$q_0 = 832.5 \text{ kN/m}$$

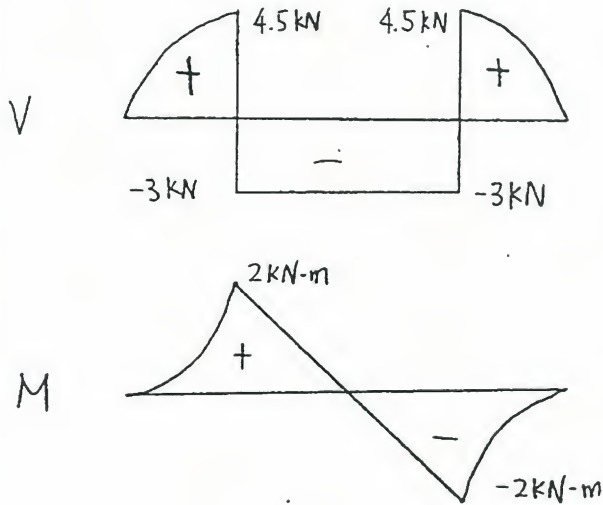
$$V(x) = 832.5x \quad 0 \leq x \leq 4$$

$$V(x) = (832.5)(4) - 500(x-4)$$

$$= -500x + 5330 \quad 4 \leq x \leq 16$$

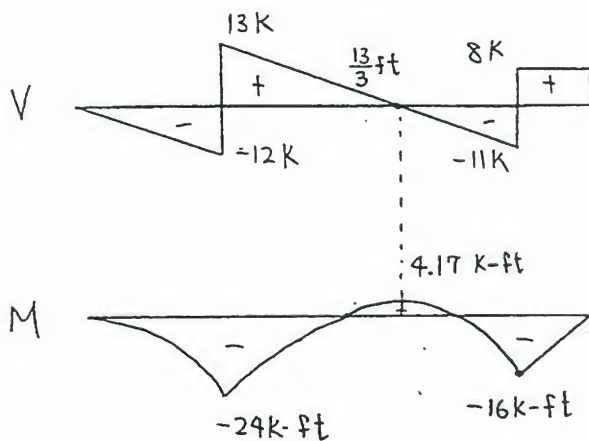


7-53  $R = \frac{1}{3} (\frac{1}{2} \times 1.5 \times 6 \times 5) = 7.5 \text{ kN}$

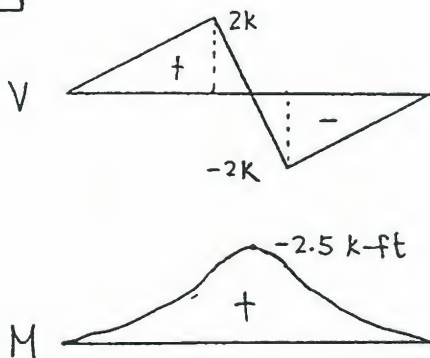


7-54  $R_L = \frac{1}{8} (3 \times 12 \times 6 - 8 \times 2) = 25$

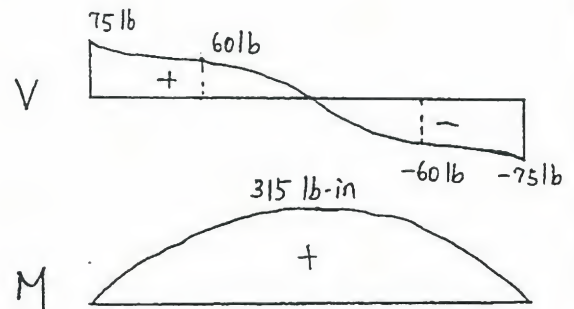
$R_R = \frac{1}{8} (3 \times 12 \times 2 + 8 \times 10) = 19$



7-55

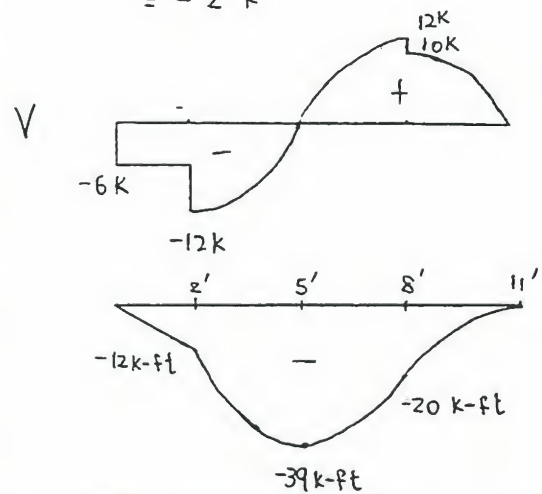


7-56  $R = \frac{1}{2} (3 \times 10 + 3 \times 40) = 75 \text{ lb}$

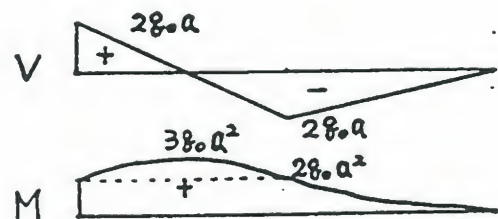


7-57  $R_L = \frac{1}{6} (6 \times 8 - \frac{1}{2} \times 6 \times 8 \times 3 - \frac{1}{2} \times 3 \times 6 \times 2)$   
 $= -7 \text{ k}$

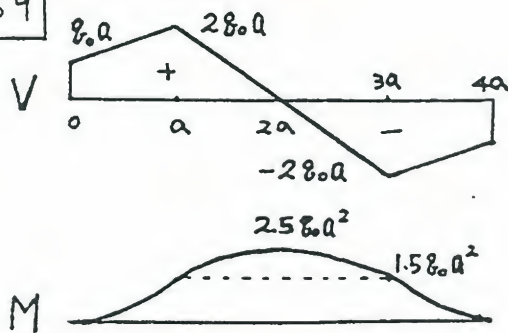
$R_R = \frac{1}{6} (\frac{1}{2} \times 3 \times 6 \times 8 - \frac{1}{2} \times 6 \times 8 \times 3 - 6 \times 2)$   
 $= -2 \text{ k}$



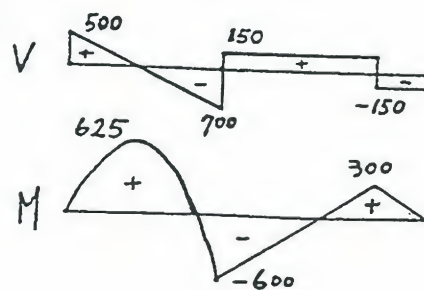
7-58



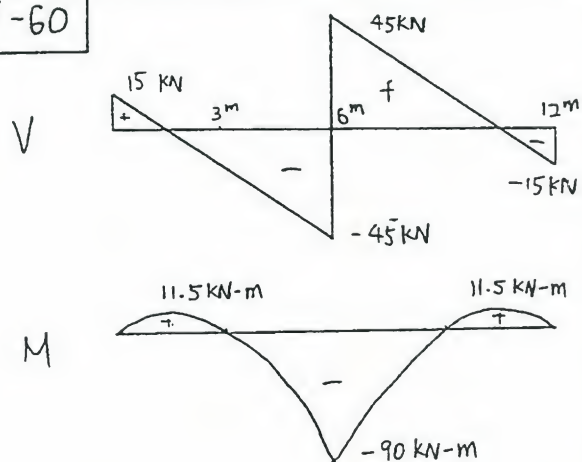
7-59



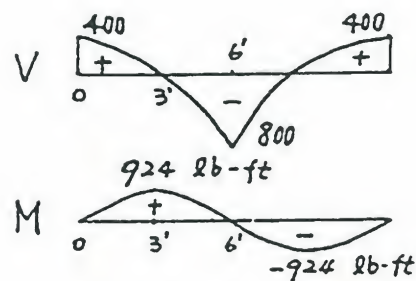
7-63



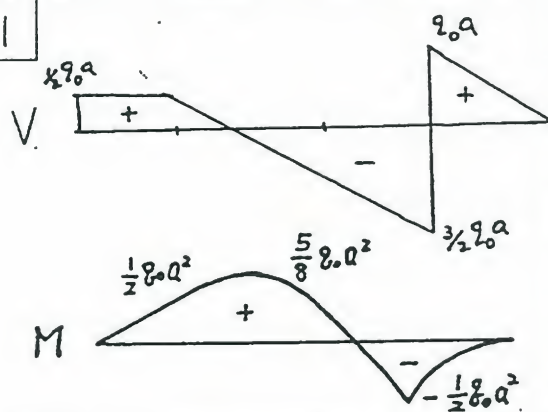
7-60



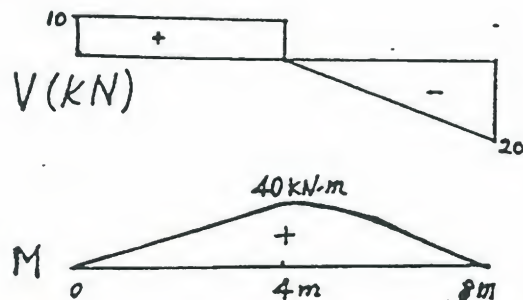
7-64



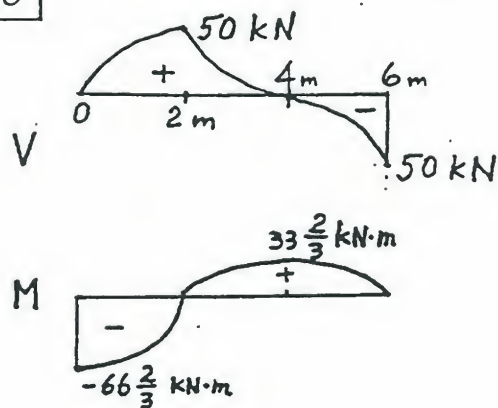
7-61



7-65

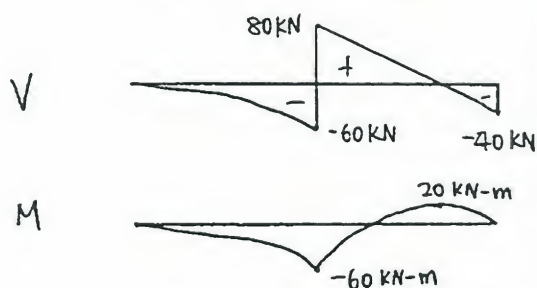


7-66

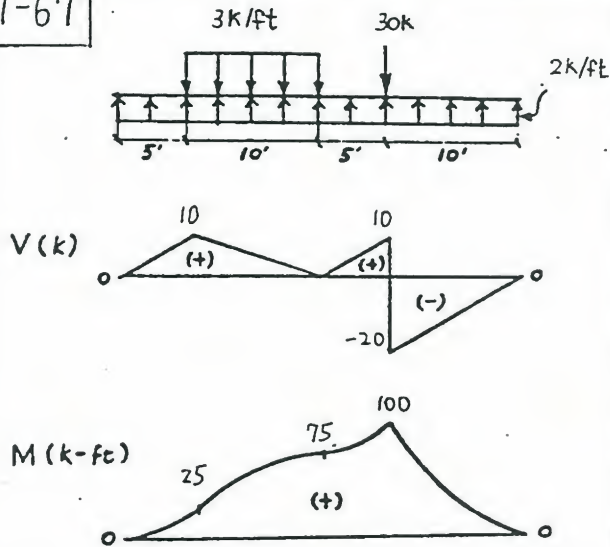


7-62

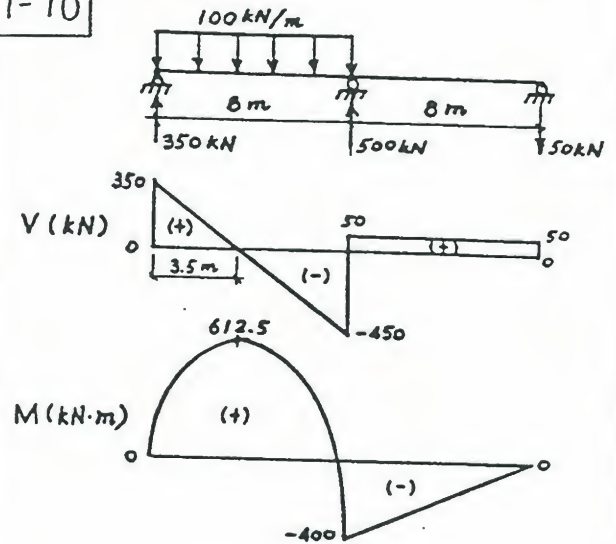
$$R_L = \frac{1}{3} \left( \frac{1}{2} \times 3 \times 40 \times 4 + 3 \times 40 \times 1.5 \right) = 140 \text{ kN}$$



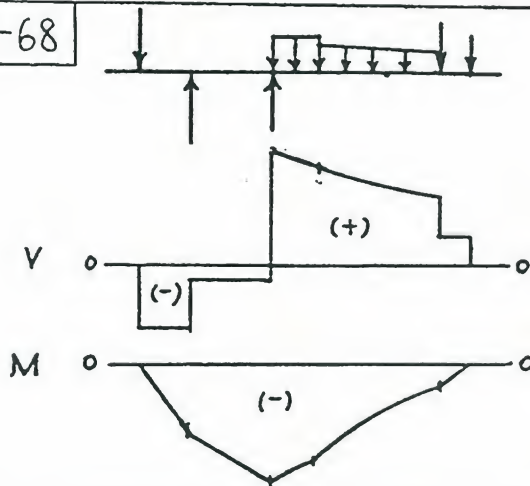
7-67



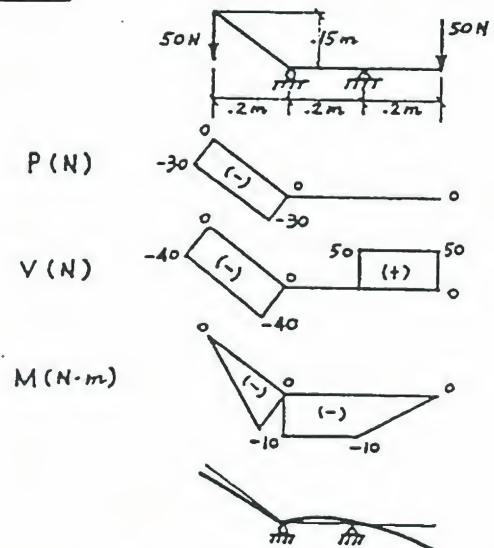
7-70



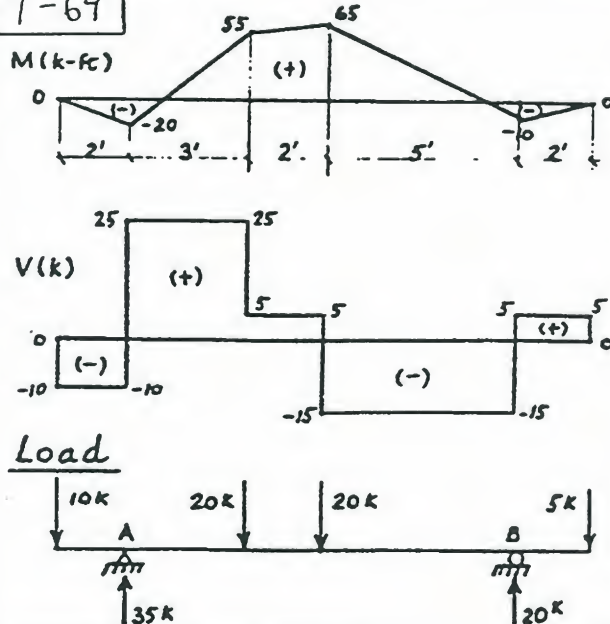
7-68



7-71



7-69



$$S = \frac{M}{\sigma_{all}} = \frac{10000}{90} = 111.11 \text{ mm}^3$$

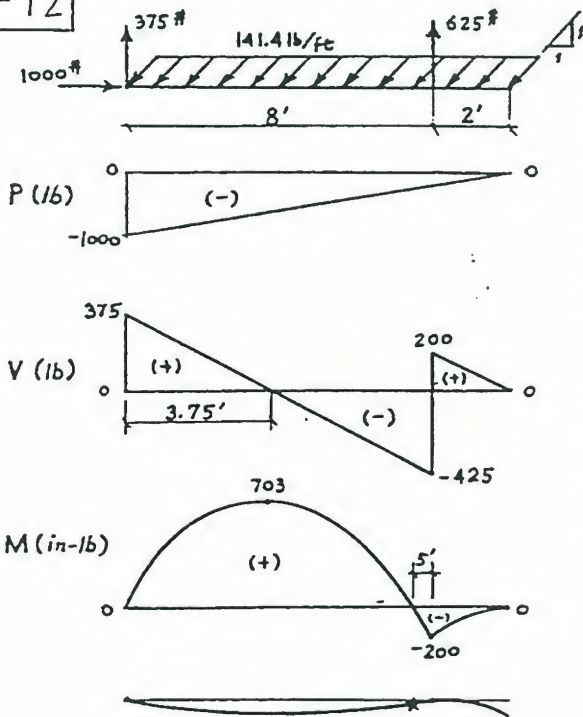
$$S = \frac{\pi R^3}{4} = 111.11, R = 5.21 \text{ mm}$$

use 11 mm diameter solid shaft

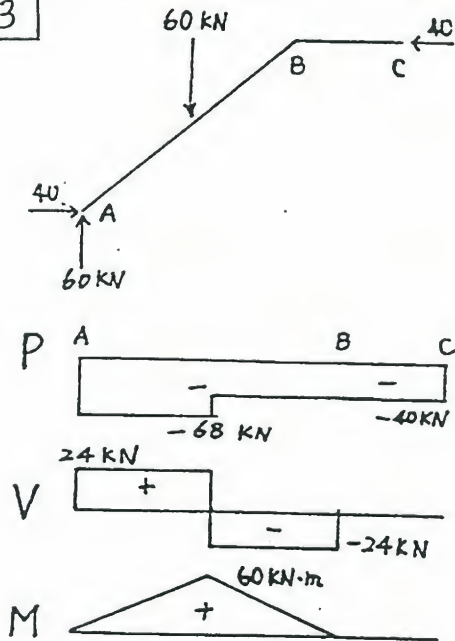
$$\tau_{max} = \frac{4V}{3A} = \frac{4 \times 50}{3 \times (\pi \times 5.5^2)} = 0.70 \text{ MPa} < \tau_{all} \therefore \text{o.k.}$$



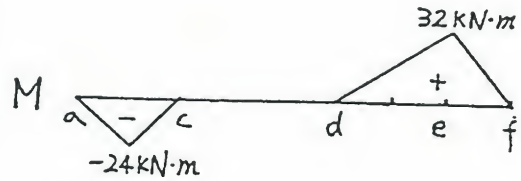
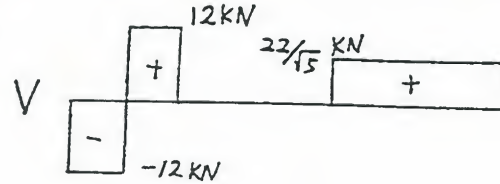
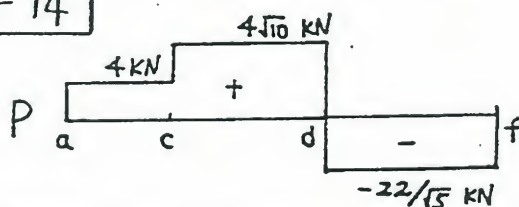
7-72



7-73



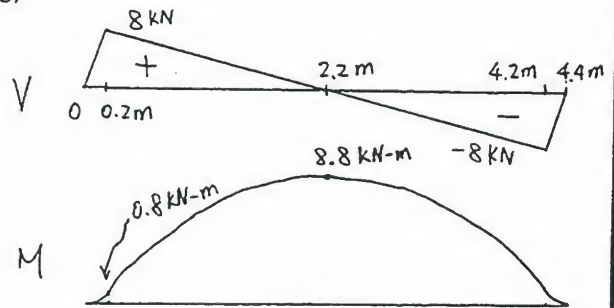
7-74



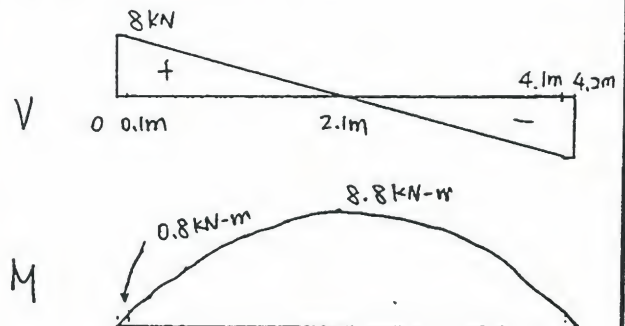
7-75

(a)  $R = 4 \times 4 / 0.4 = 40 \text{ kN/m}$

(b)



(c)  $R' = 4 \times 4 / 2 = 8 \text{ kN}$

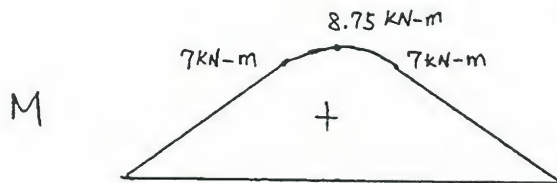
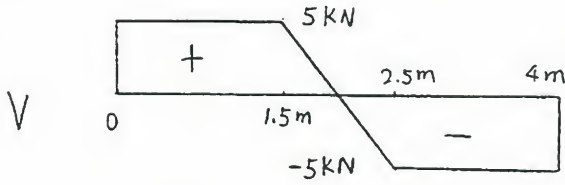


(d) center to center of reaction forces gives a more accurate result

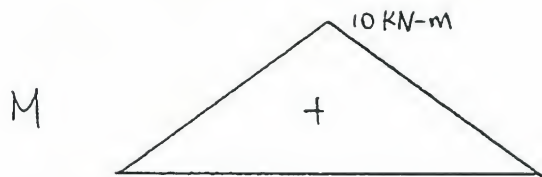
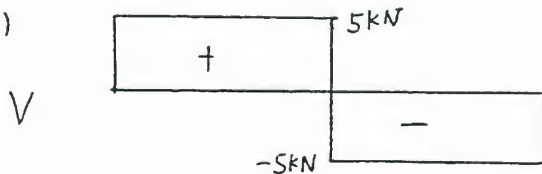
7-76

(a)  $R = 10 \times 1 / 2 = 5 \text{ kN}$

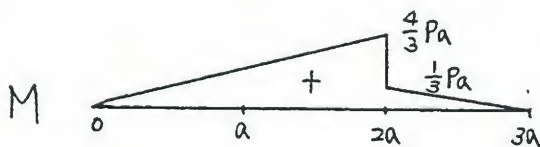
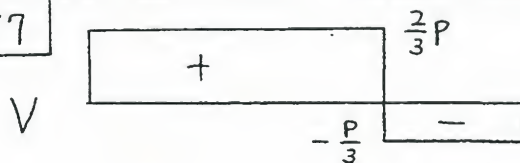
(b)



(c)

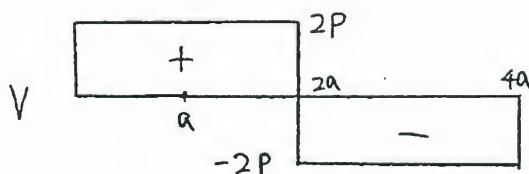


7-77

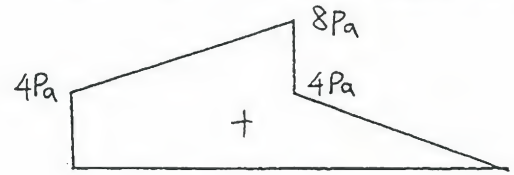


7-78

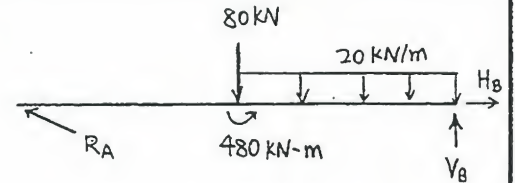
$R_1 = \frac{1}{4a} (-4Pa + 4P \times 3a) = 2P$



M

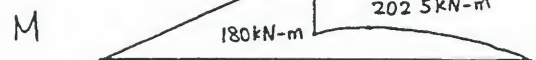
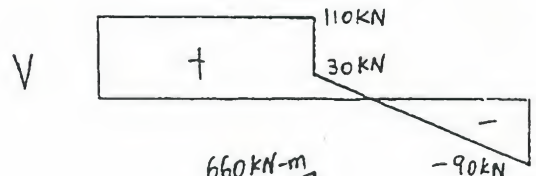
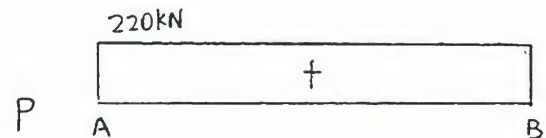


7-79

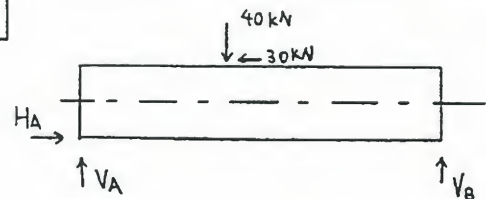


$R_A = \frac{\sqrt{5}}{12} (480 + 80 \times 6 + 20 \times 6 \times 3) = 110 \sqrt{5} \text{ kN}$

$H_B = 220 \text{ kN}, \quad V_B = 90 \text{ kN}$

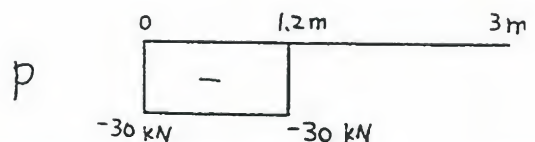


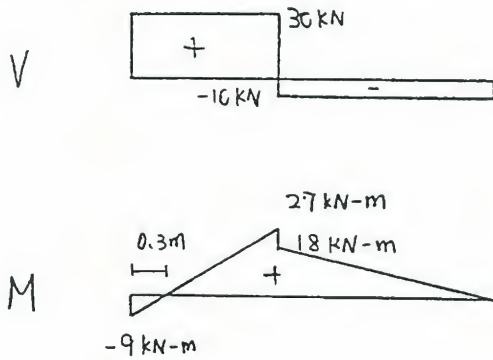
7-80



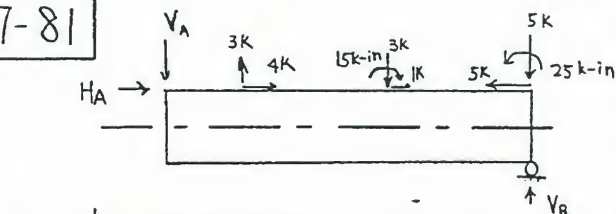
$V_A = \frac{1}{3} (40 \times 1.8 + 30 \times 0.6) = 30 \text{ kN}$

$H_A = 30 \text{ kN}, \quad V_B = 40 - V_A = 10 \text{ kN}$



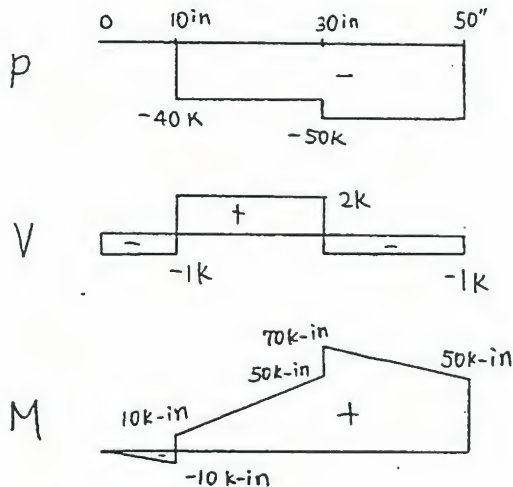


7-81

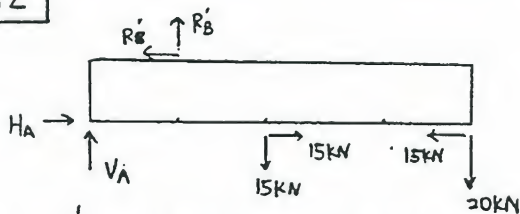


$$V_B = \frac{1}{50} (-25 + 15 + 5 \times 50 + 3 \times 30 - 3 \times 10) = 6 \text{ k}$$

$$V_A = 1 \text{ k}, H_A = 0 \text{ k}$$

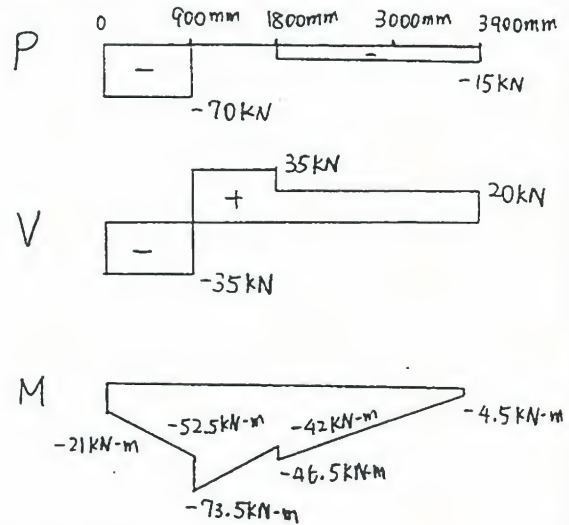


7-82

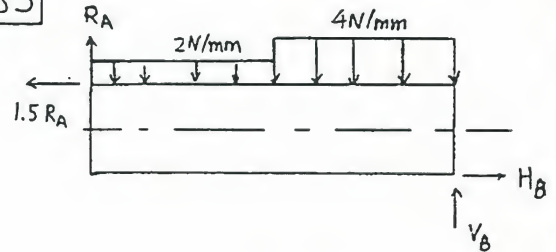


$$R'_B = \frac{1}{(900+600)} (20 \times 3900 + 15 \times 1800) = 70 \text{ kN}$$

$$V_A = -35 \text{ kN}, H_A = 70 \text{ kN}$$

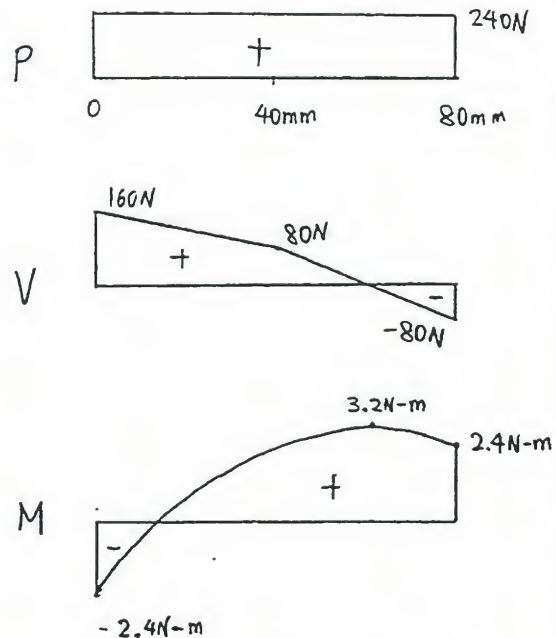


7-83



$$R_A = \frac{1}{80-30} (2 \times 40 \times 60 + 4 \times 40 \times 20) = 160 \text{ N}$$

$$V_B = 2 \times 40 + 4 \times 40 - 160 = 80 \text{ N}, H_B = 240 \text{ N}$$





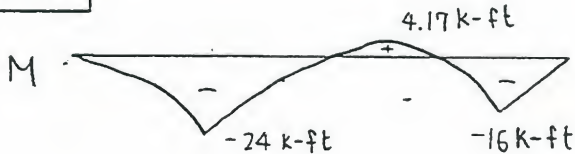
7-84



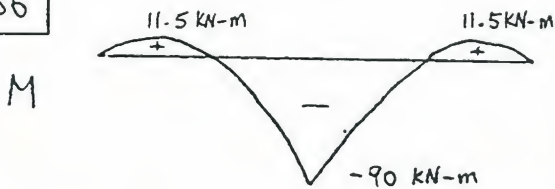
M



7-85



7-86



M



7-87



7-88

$$\frac{d^2 M}{dx^2} = -q_0 \langle x - \frac{L}{2} \rangle^0$$

$$M(x) = -\frac{1}{2} q_0 \langle x - \frac{L}{2} \rangle^2 + C_1 x + C_2$$

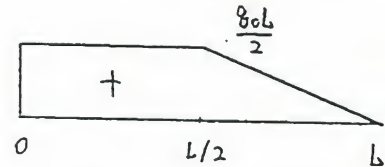
$$V(L) = 0 \rightarrow -q_0 \left(\frac{L}{2}\right) + C_1 = 0 \rightarrow C_1 = \frac{q_0 L}{2}$$

$$M(L) = 0 \rightarrow -\frac{q_0 L^2}{8} + \frac{q_0 L^2}{2} + C_2 = 0 \rightarrow C_2 = -\frac{3q_0 L^2}{8}$$

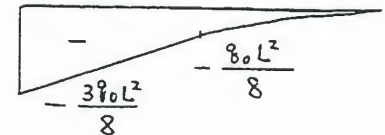
$$M(x) = \frac{q_0 L}{2} \langle x-0 \rangle^1 - \frac{3q_0 L^2}{8} \langle x-0 \rangle^0 - \frac{q_0}{2} \langle x - \frac{L}{2} \rangle^2$$

$$V(x) = \frac{q_0 L}{2} \langle x-0 \rangle^0 - \frac{3q_0 L^2}{8} \langle x-0 \rangle^{-1} - \frac{q_0}{2} \langle x - \frac{L}{2} \rangle^1$$

V



M



7-89

$$\frac{d^2 M}{dx^2} = -\frac{40}{3} \langle x-1 \rangle^1$$

$$M(x) = -\frac{20}{9} \langle x-1 \rangle^3 + C_1 x + C_2$$

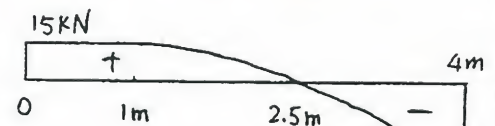
$$M(0) = 0 \rightarrow C_2 = 0$$

$$M(4) = 0 \rightarrow C_1 = 15$$

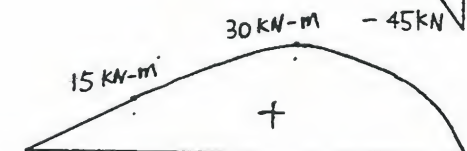
$$M(x) = 15 \langle x-0 \rangle^1 - \frac{20}{9} \langle x-1 \rangle^3$$

$$V(x) = 15 \langle x-0 \rangle^0 - \frac{20}{3} \langle x-1 \rangle^2$$

V



M



7-90

$$\frac{d^2M}{dx^2} = -\frac{100}{3.6} \langle x-0 \rangle^1 + \frac{225}{4.5} \langle x-3.6 \rangle^1$$

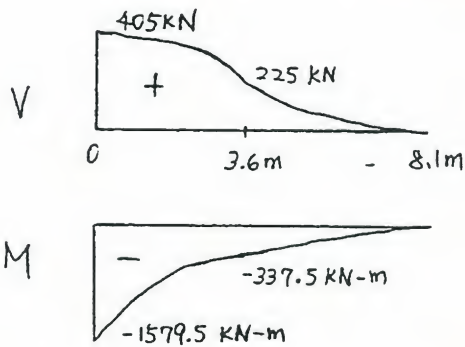
$$M(x) = -\frac{125}{27} \langle x-0 \rangle^3 + \frac{25}{3} \langle x-3.6 \rangle^3 + C_1 x + C_2$$

$$V(8.1) = 0 \rightarrow C_1 = 405$$

$$M(8.1) = 0 \rightarrow C_2 = -1579.5$$

$$M(x) = 405 \langle x-0 \rangle^1 - 1579.5 \langle x-0 \rangle^0 - \frac{125}{27} \langle x-0 \rangle^3 + \frac{25}{3} \langle x-3.6 \rangle^3$$

$$V(x) = 405 \langle x-0 \rangle^0 - 1579.5 \langle x-0 \rangle^{-1} - \frac{125}{9} \langle x-0 \rangle^2 + 25 \langle x-3.6 \rangle^2$$



7-92

$$\frac{d^2M}{dx^2} = -150 \langle x-0 \rangle^0 + 600 \langle x-0 \rangle^1 - 1200 \langle x-0.25 \rangle^1$$

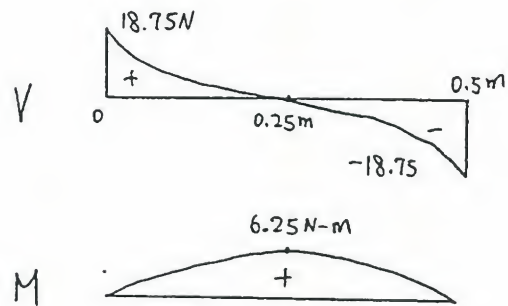
$$M(x) = -75 \langle x-0 \rangle^2 + 100 \langle x-0 \rangle^3 - 200 \langle x-0.25 \rangle^3 + C_1 x + C_2$$

$$M(0) = 0 \rightarrow C_2 = 0$$

$$M(0.5) = 0 \rightarrow C_1 = 18.75$$

$$M(x) = 18.75 \langle x-0 \rangle^1 - 75 \langle x-0 \rangle^2 + 100 \langle x-0 \rangle^3 - 200 \langle x-0.25 \rangle^3$$

$$V(x) = 18.75 \langle x-0 \rangle^0 - 150 \langle x-0 \rangle^1 + 300 \langle x-0 \rangle^2 - 600 \langle x-0.25 \rangle^2$$



7-91

$$\frac{d^2M}{dx^2} = -\frac{81}{4a} \langle x-0 \rangle^1 + C_1 \langle x-0 \rangle^{-1} + C_2 \langle x-3a \rangle^{-1}$$

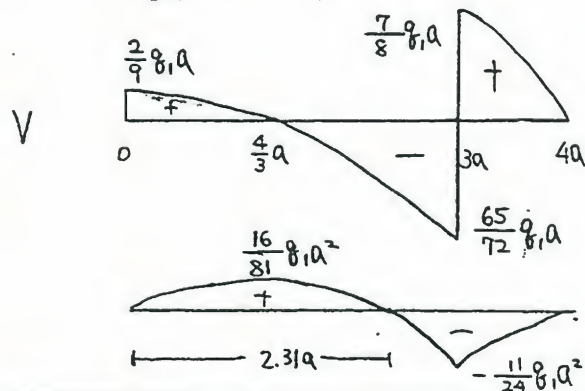
$$M(x) = -\frac{81}{24a} \langle x-0 \rangle^3 + C_1 \langle x-0 \rangle^1 + C_2 \langle x-3a \rangle^1$$

$$V(4a) = 0 \rightarrow -281a + C_1 + C_2 = 0 \quad \left. \begin{array}{l} C_1 = \frac{2}{9} 81a \\ C_2 = \frac{16}{9} 81a \end{array} \right\}$$

$$M(4a) = 0 \rightarrow -\frac{8}{3} 81a + 4C_1 + C_2 = 0$$

$$M(x) = -\frac{81}{24a} \langle x-0 \rangle^3 + \frac{2}{9} 81a \langle x-0 \rangle^1 + \frac{16}{9} 81a \langle x-3a \rangle^1$$

$$V(x) = -\frac{81}{8a} \langle x-0 \rangle^2 + \frac{2}{9} 81a \langle x-0 \rangle^0 + \frac{16}{9} 81a \langle x-3a \rangle^0$$



7-93

$$\frac{d^2M}{dx^2} = -2 \langle x-0 \rangle^{-1} + C_1 \langle x-8 \rangle^{-1} - \langle x-8 \rangle^0 + \langle x-24 \rangle^0 + C_2 \langle x-24 \rangle^{-1} + 2 \langle x-32 \rangle^{-1}$$

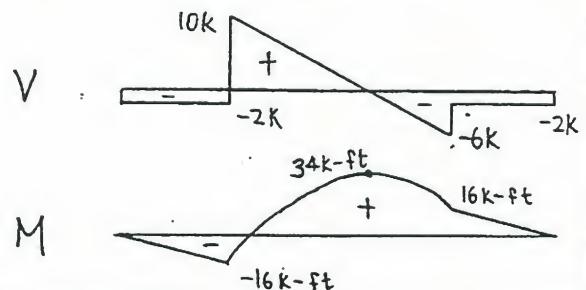
$$M(x) = -2 \langle x-0 \rangle^1 + C_1 \langle x-8 \rangle^1 - \frac{1}{2} \langle x-8 \rangle^2 + \frac{1}{2} \langle x-24 \rangle^2 + C_2 \langle x-24 \rangle^1 + 2 \langle x-32 \rangle^1$$

$$V(32) = 0 \rightarrow -2 + C_1 - 24 + 8 + C_2 + 2 = 0 \quad \left. \begin{array}{l} C_1 = 12 \\ C_2 = 4 \end{array} \right\}$$

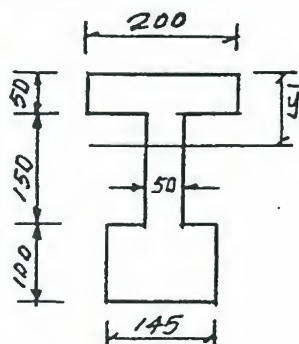
$$M(32) = 0 \rightarrow -64 + 24C_1 - 288 + 32 + 8C_2 = 0$$

$$M(x) = -2 \langle x-0 \rangle^1 + 12 \langle x-8 \rangle^1 - \frac{1}{2} \langle x-8 \rangle^2 + \frac{1}{2} \langle x-24 \rangle^2 + 4 \langle x-24 \rangle^1 + 2 \langle x-32 \rangle^1$$

$$V(x) = -2 \langle x-0 \rangle^0 + 2 \langle x-8 \rangle^0 - \langle x-8 \rangle^1 + \langle x-24 \rangle^1 + 4 \langle x-24 \rangle^0 + 2 \langle x-32 \rangle^0$$



8-1



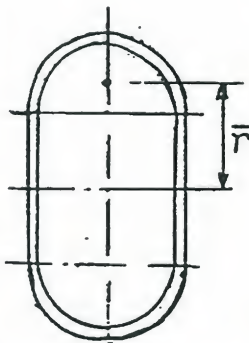
$$A = 145 \times 100 + 150 \times 50 + 200 \times 50 = 32000 \text{ mm}^2$$

$$\bar{y} = \frac{1}{32000} (145 \times 100 \times 250 + 50 \times 150 \times 125 + 200 \times 50 \times 25) = 150.4 \text{ mm} = 0.15 \text{ m}$$

$$I = \frac{1}{12} \times 0.145 \times 0.1^3 + 0.145 \times 0.1 \times 0.1^2 + \frac{1}{12} \times 0.05 \times 0.15^3 + 0.15 \times 0.05 \times 0.025^2 + \frac{1}{12} \times 0.2 \times 0.05^3 + 0.05 \times 0.2 \times 0.125^2 = 3.34 \times 10^{-4} \text{ m}^4$$

$$M = \frac{\sigma_w I}{C} = \frac{165 \times 10^3 \times 3.34 \times 10^{-4}}{0.15} = 367 \text{ KN}\cdot\text{m}$$

8-2



From Table 2 of Appendix:

$$\bar{r} = \frac{2}{\pi} R_{avg} + \frac{0.125}{2} = 0.094 \text{ m}$$

$$I_o \approx 0.095 \pi R_{avg}^3 t = 4.66 \times 10^{-8} \text{ m}^4$$

$$A = \pi R_{avg} t = 1.96 \times 10^{-4} \text{ m}^2$$

$$I = 2 \times \frac{1}{12} \times 0.00125 \times 0.125^3 + 2(4.66 \times 10^{-8} + 1.96 \times 10^{-4} \times 0.094) = 3.73 \times 10^{-5} \text{ m}^4$$

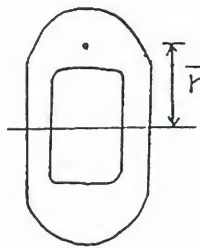
$$M = \frac{\sigma_w I}{C} = \frac{165 \times 10^3 \times 3.75 \times 10^{-5}}{0.1125} = 55 \text{ KN}\cdot\text{m}$$

8-3

$$\bar{r} = \frac{4(0.008)}{3\pi} + \frac{0.016}{2} = 1.14 \times 10^{-2} \text{ m}$$

$$I_o = 0.11 R^4 = 0.11 (0.008)^4 = 4.51 \times 10^{-10} \text{ m}^4$$

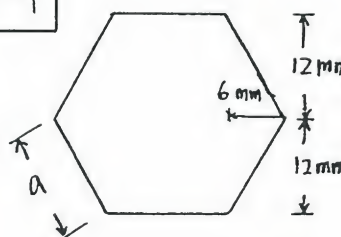
$$A = \frac{\pi R^2}{2} = 1.005 \times 10^{-4} \text{ in}^2$$



$$I = \frac{2}{12} (0.003) (0.016)^3 + 2 \times 4.51 \times 10^{-10} + 2 (1.005 \times 10^{-4}) (1.14 \times 10^{-2})^2 = 2.91 \times 10^{-8} \text{ m}^4$$

$$M = \frac{\sigma_w I}{C} = \frac{(165 \times 10^6) (2.91 \times 10^{-8})}{(0.016)} = 300 \text{ N}\cdot\text{m}$$

8-4



$$a = \frac{12}{\sin 60^\circ} = 13.86 \text{ mm}$$

$$I = \frac{1}{12} (13.86) (24)^3 + \frac{4}{36} (6) (12)^3 + 4 (\frac{1}{2} \times 6 \times 12) (4)^2 = 19422 \text{ mm}^4$$

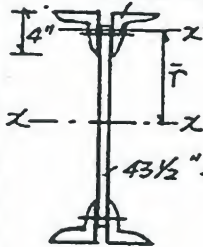
$$M = \frac{\sigma_w I}{C} = \frac{(165 \times 10^6) (19422 \times 10^{-12})}{(12 \times 10^{-3})} = 267 \text{ N}\cdot\text{m}$$

8-5

$$I_x = 6.27 \text{ in}^4, A = 4.75 \text{ in}^2$$

$$\bar{r} = 22 - 0.987 = 21.01 \text{ in}$$

6" x 4" x 1/2" angles

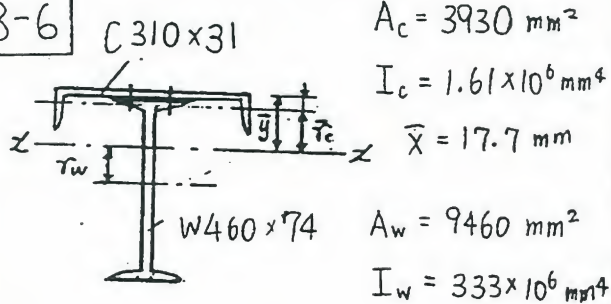


$$I_x = 4(6.27 + 4.75 \times 21.01^2) + \frac{1}{12} \times \frac{3}{8} \times 43.5^3 = 10984 \text{ in}^4$$

$$M = \frac{\sigma_w I}{C} = \frac{24 \times 10984}{22} \times \frac{1}{12} = 1000 \text{ K}\cdot\text{ft}$$



8-6



$$\bar{y} = \frac{(3930)(17.7) + (9460)(7.16 + 457/2)}{(3930 + 9460)}$$

$$= 171.7 \text{ mm (from top)}$$

$$I = 333 \times 10^6 + (9460)(171.7 - 235.66)^2 + 1.61 \times 10^6 + (3930)(171.7 - 17.7)^2 = 467 \times 10^6 \text{ mm}^4$$

$$M = \frac{\sigma_w I}{c} = \frac{(165 \times 10^3)(467 \times 10^{-6})}{(292.5 \times 10^{-3})} = 263 \text{ kN} \cdot \text{m}$$

8-7

	$A(\text{mm}^2)$	$\bar{y}(\text{mm})$	$A\bar{y}(\text{mm}^3)$
①	2400	10	24000
②	4480	160	716800
③	4000	310	1240000

$$\bar{y} = \frac{24000 + 716800 + 1240000}{2400 + 4480 + 4000} = 182 \text{ mm}$$

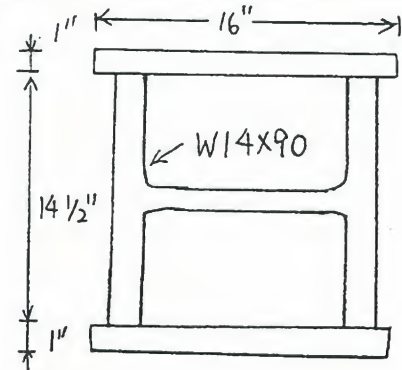
$$I = \left[ \frac{1}{12} (120)(20)^3 + (2400)(172)^2 \right] + \left[ \frac{1}{12} (16)(280)^3 + (4480)(22)^2 \right] + \left[ \frac{1}{12} (200)(20)^3 + (4000)(128)^2 \right] = 168188587 \text{ mm}^4$$

$$M = \frac{\sigma_w I}{c} = \frac{(165 \times 10^3)(168188587 \times 10^{-12})}{(182 \times 10^{-3})} = 152 \text{ kN} \cdot \text{m}$$

8-8

W14 x 90

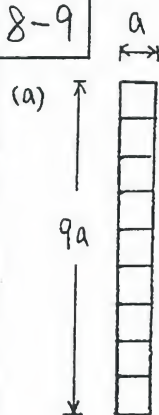
$$I_{yy} = 362$$



$$I = 362 + 2 \left[ \frac{1}{12} (16)(1)^3 + (16)(7.75)^2 \right] = 2286.7$$

$$M = \frac{\sigma_w I}{c} = \frac{(24)(2286.7)}{(8.25)} = 6652 \text{ k-in}$$

8-9



horizontal axis:

$$I = \frac{1}{12} (a)(9a)^3 = 60.75a^4$$

$$I/c = \frac{60.75a^4}{4.5a} = 13.5a^3$$

vertical axis:

$$I = \frac{1}{12} (9a)(a^3) = 0.75a^3$$

$$I/c = \frac{0.75a^4}{0.5a} = 1.5a^3$$

(b)



horizontal axis:

$$I = \frac{1}{12} (a)(3a)^3 + 2 \left[ \frac{1}{12} (3a)(a)^3 + (3a^2)(2a)^2 \right] = 16.75a^4$$

$$I/c = \frac{16.75a^4}{2.5a} = 6.7a^3$$

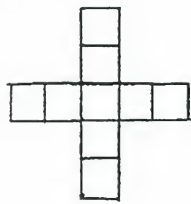
vertical axis:

$$I = \frac{1}{12} (3a)(a)^3 + 2 \left[ \frac{1}{12} a(3a)^3 \right] = 4.75a^4$$

$$I/c = \frac{4.75a^4}{1.5} = 3.17a^3$$

(c)

both axes:



$$I = \frac{1}{12}(a)(5a)^3 + \frac{1}{12}(4a)(a)^3$$

$$= 10.75a^4$$

$$I/C = \frac{10.75a^4}{2.5a} = 4.3a$$

8-10

$$S_x = \frac{I_x}{C_x}, \quad S_y = \frac{I_y}{C_y}$$

$$S310 \times 74: S_x = \frac{127 \times 10^6}{305/2} = 833 \times 10^3 \text{ mm}^3$$

$$S_y = \frac{6.53 \times 10^6}{139/2} = 94 \times 10^3 \text{ mm}^3$$

$$W360 \times 134: S_x = \frac{415 \times 10^6}{356/2} = 2330 \times 10^3 \text{ mm}^3$$

$$S_y = \frac{151 \times 10^6}{369/2} = 818 \times 10^3 \text{ mm}^3$$

$$C380 \times 50: S_x = \frac{131 \times 10^6}{381/2} = 688 \times 10^3 \text{ mm}^3$$

$$S_y = \frac{3.38 \times 10^6}{(86.4-20)} = 50.9 \times 10^3 \text{ mm}^3$$

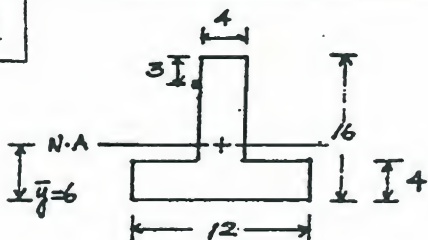
8-11

$$\sigma = E\epsilon = (200 \times 10^3)(0.0002) = 40 \text{ MPa}$$

$$M = \sigma S = (40 \times 10^3)(1460 \times 10^{-6}) = 58.4 \text{ kN}\cdot\text{m}$$

$$W = \frac{2M}{L^2} = \frac{2(58.4)}{(1)^2} = 116.8 \text{ kN}$$

8-12



$$\sigma_A = E\epsilon_A = (200 \times 10^9)(50 \times 10^{-3}) = 10^{10} \text{ Pa}$$

$$M_A = \frac{\sigma_A I}{C} = \frac{(10^{10})(2176 \times 10^{-9})}{(7 \times 10^{-3})} = 3108.6 \text{ N}\cdot\text{m}$$

and  $M_A = 0.3P$  from the simply supported beam

$$P = \frac{M_A}{0.3} = 10.36 \text{ kN}$$

8-13

$$\bar{y} = \frac{4 \times 6 \times 3 - 3 \times 2 \times 3.5}{4 \times 6 - 3 \times 2} = 2.83"$$

$$M_C = \frac{\sigma I}{C} = \frac{15 \times 65.5}{3.17} = 310 \text{ k}\cdot\text{in}$$

$$M_T = \frac{\sigma I}{C} = \frac{10 \times 65.5}{2.83} = 231 \text{ k}\cdot\text{in}$$

8-14

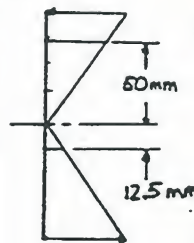
$$\bar{y} = \frac{(80 \cdot 120 \cdot 60) - (\pi \cdot 30^2 \cdot 70)}{(80 \cdot 120) - (\pi \cdot 30^2)} = 55.83 \text{ mm}$$

$$M_C = \frac{\sigma_c I}{C} = \frac{(150 \times 10^6)(10.483 \times 10^{-6}) \times 10^{-3}}{0.06417} = 24.5 \text{ kN}\cdot\text{m}$$

$$M_T = \frac{\sigma_T I}{C} = \frac{(100 \times 10^6)(10.483 \times 10^{-6}) \times 10^{-3}}{0.05583} = 18.8 \text{ kN}\cdot\text{m}$$

8-15

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(100)(150)^3(10^{-12}) = 2.81 \times 10^{-5} \text{ m}^4$$



$$(a) C = \bar{\sigma}_{ave} A = \frac{M_{yave}}{I} A = \frac{(1.6 \times 10^{-3})(5 \times 10^{-3})(0.1)(0.05)}{2.81 \times 10^{-5}}$$

$$= 0.142 \text{ MN} = 142 \text{ kN}$$

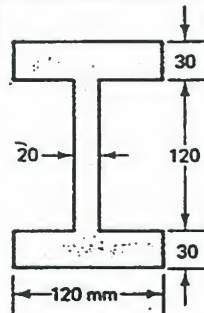
$$(b) T = \frac{(1.6 \times 10^{-3})(1.25 \times 10^{-3})(0.05)(0.05)}{2.81 \times 10^{-5}}$$

$$= 8.90 \times 10^{-3} \text{ MN} = 8.90 \text{ kN}$$

8-16

$$F = \int_{A_1} \sigma dA = \int_{A_1} \frac{My}{I} dA = \frac{M}{I} \int_{A_1} y dA = \frac{MQ}{I}$$

8-17



$$I = \frac{1}{12} \times 20 \times 120^3$$

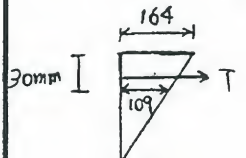
$$+ \frac{1}{12} \times 120 \times 30^3$$

$$+ (120 \times 30)(75)^2$$

$$= 43.92 \times 10^6 \text{ mm}^4$$



$$\sigma = \frac{Mc}{I} = \frac{(80 \times 10^6)(90)}{43.92 \times 10^6} = 164 \text{ N/mm}^2$$



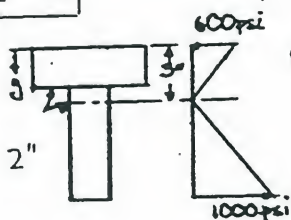
$$T = (30 \times 120) \left( \frac{164 + 109}{2} \right) + (60 \times 20 \times \frac{109}{2}) = 556800 \text{ N}$$

$$e = \frac{1}{(556800)} \left[ \frac{1}{3} (60 \times 20 \times 109 \times 60) + \frac{1}{2} (30 \times 120) (164 - 109) (80) + (30 \times 120 \times 137 \times 75) \right]$$

$$= 71.8 \text{ mm}$$

$$T \cdot e = (556.8)(71.8 \times 10^{-3}) = 80 \text{ kN} \cdot \text{m}$$

8-18



$$\bar{y} = \frac{\sum Ay}{\sum A} = \frac{2(6)(145)}{2(2)(6)} = 3"$$

$$(a) \sigma_t = \frac{Mc_t}{I} = \frac{(2270)(12)(3)}{(136)} = 600 \text{ psi}$$

$$\sigma_b = \frac{5}{3} \sigma_t = 1000 \text{ psi}$$

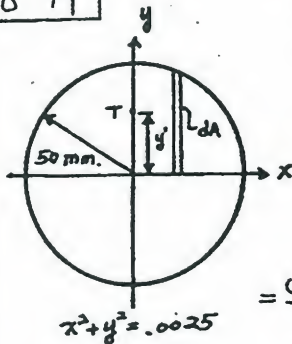
$$(b) = \sum \sigma A \bar{y} \leq A$$

$$= \frac{1}{2} (600 + 200)(12) + \frac{1}{2} (200)(2)(1) = 5000 \text{ lb.}$$

$$(c) T = \frac{1}{2} (1000)(5)(2) = 5000 \text{ lb.}$$

same as b).

8-19



$$dT = \frac{\sigma}{2} dA$$

$$T = \int \frac{\sigma}{2} dA = 2 \int_0^{0.05} \frac{M}{2I} y dx$$

$$= \frac{M}{I} \int_0^{0.05} (0.025 - x^2) dx$$

$$= \frac{M}{I} \left( 0.025x - \frac{x^3}{3} \right) \Big|_0^{0.05}$$

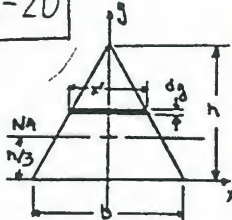
$$= \frac{(3500 \times 10^{-3})}{\frac{\pi(-0.05)^4}{4}} \left[ 0.025(0.05) - \frac{0.05^3}{3} \right]$$

$$= 59.4 \text{ kN}$$

$$M = T(2y')$$

$$y' = \frac{3.5}{2(59.4)} = 0.29 \text{ m} = 29 \text{ mm}$$

8-20



$$I_{xx} = I_o + Ad^2$$

$$I_{xx} = \int y^2 dA$$

$$dA = x'dy = b \left( \frac{h-y}{h} \right) dy$$

$$= b \left( 1 - \frac{y}{h} \right) dy$$

$$I_{xx} = b \int_0^h y^2 \left( 1 - \frac{y}{h} \right) dy$$

$$= b \left( \frac{1}{3} y^3 - \frac{1}{4h} y^4 \right) \Big|_0^h$$

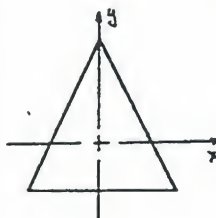
$$= b \left( \frac{1}{3} h^3 - \frac{1}{4} h^3 \right) = \frac{1}{12} bh^3$$

$$I_{xx} = I_o + Ad^2$$

$$I_o = I_{xx} - Ad^2$$

$$= \frac{1}{12} bh^3 - \left( \frac{1}{3} bh \right) \left( \frac{h}{3} \right)^2$$

$$= \frac{1}{12} bh^3 - \frac{1}{18} bh^3 = \frac{1}{36} bh^3$$



$$T = \int \sigma dA$$

$$dA = x'dy = b \left( \frac{2}{3} - \frac{y}{h} \right) dy$$

$$T = \int \left( \frac{My}{I} \right) b \left( \frac{2}{3} - \frac{y}{h} \right) dy$$

$$= \frac{36M}{h^3} \int_0^{\frac{2}{3}h} y \left( \frac{2}{3} - \frac{y}{h} \right) dy$$

$$= \frac{36M}{h^3} \left( \frac{1}{3} y^2 - \frac{1}{3h} y^3 \right) \Big|_0^{\frac{2}{3}h}$$

$$= \frac{16M}{9h} = \frac{16(4000)(10^{-6})}{9(-15)}$$

$$= 47.4 \text{ kN}$$

$$y_T = \frac{\int \sigma y dA}{\int \sigma dA}$$

$$\int \sigma y dA = \frac{Mb}{I} \int_0^{\frac{2}{3}h} y^2 \left( \frac{2}{3} - \frac{y}{h} \right) dy$$

$$= \frac{36M}{h^3} \left( \frac{2}{9} y^3 - \frac{1}{4h} y^4 \right) \Big|_0^{\frac{2}{3}h}$$

$$= \frac{16}{27} M$$

$$y_T = \frac{16(4000)}{27(47.4)} = 50 \text{ mm (from NA)}$$



$$C = T = 47.4 \text{ kN (from equilibrium)}$$

$$M = T (\text{Lever arm})$$

$$\text{lever arm} = y_T + y_C = \frac{M}{T}$$

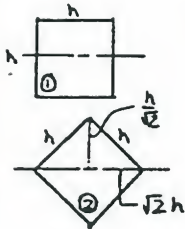
$$y_C = \frac{M}{T} - y_T = \frac{4000}{47.4 \times 10^3} - 0.05$$

$$= 34.4 \text{ mm from NA}$$

8-21

$$M = \frac{\sigma I}{C}$$

$$M_1 = \frac{\frac{1}{12} h h^3 \sigma}{\frac{h}{2}} = \frac{1}{6} h^3 \sigma$$



$$I_z = 2 \left[ \frac{1}{36} (\sqrt{2}h) \left( \frac{h}{\sqrt{2}} \right)^3 + \frac{1}{2} (\sqrt{2}h) \left( \frac{h}{\sqrt{2}} \right) \left( \frac{h}{3\sqrt{2}} \right)^2 \right] = \frac{h^4}{12}$$

$$M_2 = \frac{\frac{1}{12} h^4 \sigma}{\frac{h}{\sqrt{2}}} = \frac{\sqrt{2}}{12} h^3 \sigma$$

$$\frac{M_1}{M_2} = \frac{\frac{1}{6} h^3 \sigma}{\frac{\sqrt{2}}{12} h^3 \sigma} = \frac{2}{\sqrt{2}} = \sqrt{2}$$

Tension at top and compression at bottom  $\rightarrow$  Negative moment

$$\frac{\epsilon_{top} + 0.00115}{0.00125 + 0.00115} = \frac{150}{125}$$

$$\epsilon_{top} = 0.00038$$

$$\sigma_{top} = E \epsilon_{top} = (200 \times 10^3)(0.00038) = 76 \text{ kPa (tension)}$$

$$\frac{\epsilon_{bottom} + 0.000125}{0.000125 + 0.00115} = \frac{160}{125}$$

$$\epsilon_{bottom} = 0.001507$$

$$\sigma_{bottom} = E \epsilon_{bottom} = (200 \times 10^3)(0.001507) = 301.4 \text{ kPa (compression)}$$

8-27

$$I = \frac{2}{12} [(1-k)a\sqrt{2}] \left[ \frac{\sqrt{2}}{2} (1-k)a \right]^3 + 2 \int_0^{\frac{\sqrt{2}}{2} (1-k)a} \frac{\sqrt{2}}{2} (1-k)x \cdot \sqrt{2} k a \cdot dx$$

$$= \frac{1}{12} (1+3k)(1-k)^3 a^4$$

$$S = \frac{I}{C} = \frac{\sqrt{2}}{6} (1+3k)(1-k)^3 a^3$$

$$\frac{ds}{dk} = \frac{\sqrt{2}}{6} a^3 [9k^2 - 10k + 1] = 0 \quad k = \frac{1}{9} \text{ or } 1$$

$$k = \frac{1}{9} \therefore S' = \frac{\sqrt{2}}{6} a^3 \left( \frac{256}{243} \right)$$

$$k = 0 \quad S = \frac{\sqrt{2}}{6} a^3$$

$$\frac{\sigma'}{\sigma} = \frac{S}{S'} = \frac{243}{256} = 95\%$$

$$\therefore 1 - \sigma'/\sigma = 5\%$$

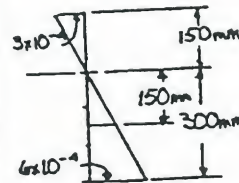
8-23

$$\epsilon_{AB} = \frac{0.025}{200} = 0.000125$$

$$\epsilon_{CD} = \frac{-0.23}{200} = -0.00115$$

8-24

$$C_b = 450 \left( \frac{6}{9} \right) = 300 \text{ mm}$$



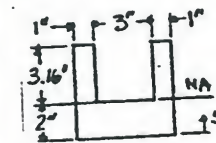
$$P = \epsilon_{ave} EA$$

$$= \left( \frac{1}{2} \right) (6 \times 10^{-4}) (2 \times 10^5) (1 \times 15)$$

$$= 9 \text{ MN}$$

$$= 900 \text{ kN}$$

8-25



$$\bar{y} = \frac{\sum Ay}{\sum A}$$

$$= \frac{2(3 \times 1) + 2(1)(5.16) \left( \frac{5.16}{2} \right)}{2(3) + 2(1)(5.16)}$$

$$= 2"$$

$$I = \frac{1}{12} (5)(2)^3 + 2(5)(1)^2 + 2 \left( \frac{1}{12} (1) \right)$$

$$3.16^3 + 2(1)(2.16) \left( \frac{3.16}{2} \right)^2$$

$$= 34.4 \text{ in}^4$$

$$\sigma = \epsilon E = \frac{My}{I}$$

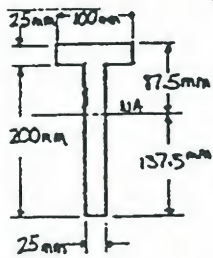
$$= \frac{16 Py}{I}$$

$$P = \frac{\epsilon EI}{16y} = \frac{(900 \times 10^{-6})(30 \times 10^6)(10^{-3})(34.4)}{16(2.16)}$$

$$= 26.9 \text{ k}$$

8-26

$$\bar{y} = \frac{100(25)(12.5) + 200(25)(125)}{(100)(25) + (200)(25)} = 87.5 \text{ mm from top}$$



$$I = \frac{1}{12} (100)(25)^3 + (100)(25)(87.5)^2 + \frac{1}{12} (25)(175)^3 + (25)(175)(87.5)^2 = 3.79 \times 10^{-5} \text{ m}^4$$

$$M = \left(\frac{2}{3}F\right)(1) = \frac{2}{3}F \text{ KN}\cdot\text{m}$$

for F acting up!

$$\sigma_t = \frac{Mc}{I} = \frac{(0.667F)(0.0875)}{(3.79 \times 10^{-5})} = \frac{20000}{1.5}$$

$$\sigma_b = \frac{(0.667F)(0.1375)}{3.79 \times 10^{-5}} = \frac{40000}{1.5} \rightarrow F = \frac{F = 8.66 \text{ kN}}{11.0 \text{ kN} > 8.66}$$

for F acting down:

$$\sigma_t = \frac{(0.667F)(0.0875)}{3.79 \times 10^{-5}} = \frac{40000}{1.5} \rightarrow F = 17.3 \text{ kN}$$

$$\sigma_b = \frac{(0.667F)(0.1375)}{3.79 \times 10^{-5}} = \frac{20000}{1.5} \rightarrow F = 5.51$$

8-27

$$n = \frac{E_T}{E_C} = 1.5 \quad \Sigma M_{NA} = 0$$

$$225\bar{y}\left(\frac{\bar{y}}{2}\right) = 150(300 - \bar{y})\left(\frac{300 - \bar{y}}{2}\right)$$

$$\bar{y} = 135 \text{ mm}, (300 - \bar{y}) = 165 \text{ mm}$$

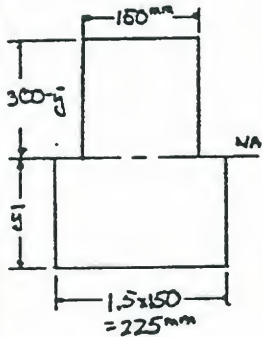
$$I = \frac{1}{3} (150)(165)^3 + \frac{1}{3} (225)(135)^3$$

$$= 4.09 \times 10^{-4} \text{ m}^4$$

$$\sigma_t = 1.5(240000)(10^{-6})(135)$$

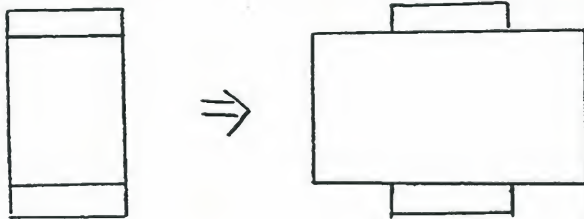
$$(4.09 \times 10^{-4}) = 119 \text{ MN}\cdot\text{m}^2$$

$$\sigma_c = \frac{(240000)(10^{-6})(165)}{(4.09 \times 10^{-4})} = 96.8 \text{ MN/m}^2$$



8-28

Alternative Solution



$$n = \frac{E_s}{E_b} = \frac{200}{86} = 2.3256$$

$$I = \frac{1}{12} \left( \frac{200}{86} \times 40 \right) (40)^3 + 2 \left[ \frac{1}{12} (40)(10)^3 + (400)(25)^2 \right] = 1002791 \text{ mm}^4$$

$$M_1 = \frac{\sigma_{\max} I}{c_1} = \frac{\frac{40 \times 10^6}{2.3256} (1002791 \times 10^{-12})}{(20 \times 10^{-3})} = 862$$

$$M_2 = \frac{\sigma_{\max} I}{c_2} = \frac{(40 \times 10^6) (1002791 \times 10^{-12})}{(30 \times 10^{-3})} = 1337$$

$$\therefore M_{\max} = M_1 = 862 \text{ N}\cdot\text{m}$$

8-29

$$n = \frac{E_s}{E_b} = \frac{200}{86} = 2.3256$$

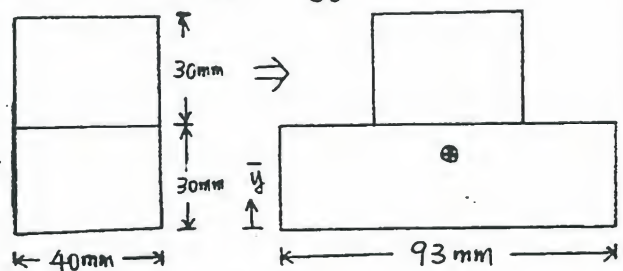
$$I = \frac{1}{12} (20 + 30 \times \frac{200}{86}) (60)^3 = 1615814 \text{ mm}^4$$

$$M_{\max} = \frac{\sigma_{\max} I}{c} = \frac{\frac{40 \times 10^6}{2.3256} (1615814 \times 10^{-12})}{(30 \times 10^{-3})}$$

$$= 926.4 \text{ N}\cdot\text{m} \quad \text{Alternative Solution}$$

8-30

$$n = \frac{E_s}{E_b} = \frac{200}{86} = 2.3256$$



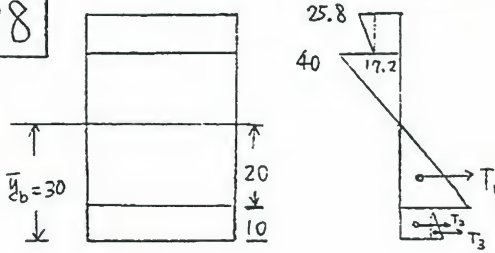
$$\bar{y} = \frac{(93 \times 30 \times 15) + (40 \times 30 \times 45)}{(93 \times 30) + (40 \times 30)} = 24 \text{ mm}$$

$$I = \frac{1}{12} (93)(30)^3 + (93 \times 30)(9)^2 + \frac{1}{12} (40)(30)^3 + (40 \times 30)(21)^2 = 1054440 \text{ mm}^4$$

Alternative Solution



8-28



Since the cross section is symmetry about the horizontal axis,  $\bar{y}_b = 30 \text{ mm}$   
 The maximum stress in steel is  $40 \text{ MPa}$   
 The maximum stress in brass

$$\sigma'_b = 40 \left( \frac{30}{20} \right) \left( \frac{86}{200} \right) = 25.8 \text{ MPa} < 40 \text{ MPa}$$

The stress in brass at contact surface

$$\sigma_b'' = 25.8 \left( \frac{20}{30} \right) = 17.2 \text{ MPa}$$

The flexural strength

$$M = 2 \left[ \left( \frac{1}{2} \times 40 \times 20 \times 40 \right) \times \frac{0.04}{3} + (40 \times 10 \times 17.2) \times 0.025 + \frac{1}{2} \times 40 \times 10 \times (25.8 - 17.2) \times (0.02 + \frac{0.02}{3}) \right] \\ = 862.4 \text{ N}\cdot\text{m}$$

8-29

Since the cross section is symmetry about the horizontal axis, the neutral axis located at middle.

The maximum stress in steel is  $40 \text{ MPa}$

The maximum stress in brass

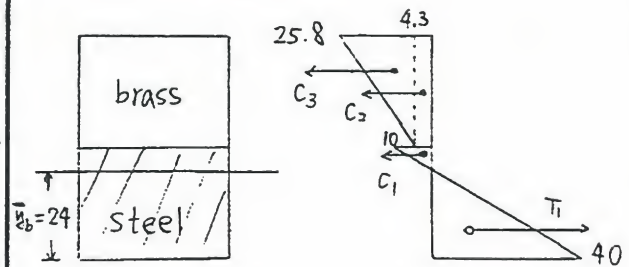
$$\sigma_b = 40 \left( \frac{86}{200} \right) = 17.2 \text{ MPa} < 40 \text{ MPa}$$

The flexural strength

$$M = 2 \left[ \left( \frac{1}{2} \times 30 \times 30 \times 40 \right) + 2 \times \left( \frac{1}{2} \times 10 \times 30 \times 17.2 \right) \right] \times 0.02 \\ = 926.4 \text{ N}\cdot\text{m}$$

8-30

$$\bar{y}_b = \frac{\int_{-30}^{30} (200) y_b dy_b + \int_{30}^{60} (86) y_b dy_b}{\int_{-30}^{30} 200 dy_b + \int_{30}^{60} 86 dy_b} \\ = \frac{200 \times \frac{30^2}{2} + 86 \times \frac{60^2 - 30^2}{2}}{200 \times 30 + 86 \times 30} \\ = 24 \text{ mm}$$



The maximum stress in steel is  $40 \text{ MPa}$

The stress in steel at contact surface

$$\sigma'_s = 40 \left( \frac{6}{24} \right) = 10 \text{ MPa}$$

The maximum stress in brass

$$\sigma'_b = 40 \left( \frac{36}{24} \right) \left( \frac{86}{200} \right) = 25.8 \text{ MPa} < 40 \text{ MPa}$$

The stress in brass at contact surface

$$\sigma_b'' = 25.8 \left( \frac{6}{36} \right) = 4.3 \text{ MPa}$$

Based on these stress quantities

$$T_1 = \frac{1}{2} \times 40 \times 24 \times 40 = 19200 \text{ N}$$

$$C_1 = \frac{1}{2} \times 40 \times 6 \times 10 = 1200 \text{ N}$$

$$C_2 = 40 \times 30 \times 4.3 = 5160 \text{ N}$$

$$C_3 = \frac{1}{2} \times 40 \times 30 \times (25.8 - 4.3) = 12900 \text{ N}$$

The flexural strength

$$M = T_1 \times 0.016 + C_1 \times 0.004 + C_2 \times 0.021 + C_3 \times 0.026 \\ = 755.76 \text{ N}\cdot\text{m}$$



$$M_1 = \frac{\frac{\sigma_{\max}}{n} I}{C_1} = \frac{\frac{40 \times 10^6}{2.3256} (1054440 \times 10^{-12})}{(24 \times 10^{-3})} = 756$$

$$M_2 = \frac{\sigma_{\max} I}{C_2} = \frac{(40 \times 10^6) (1054440 \times 10^{-12})}{(36 \times 10^{-3})} = 1172$$

$$\therefore M_{\max} = M_1 = 756 \text{ N}\cdot\text{m}$$

$$\begin{aligned} 8-31 \quad \bar{y}_b &= \frac{\int_A E_i y_b dA}{\int_A E_i dA} = \frac{15 \times 50 \times 80 (40 + 20) + 40 \times 50 \times 20 \times 10}{15 \times 50 \times 80 + 40 \times 50 \times 20} \\ &= 40 \text{ mm} \end{aligned}$$

$$\begin{aligned} (EI)^* &= \int_A E_i y^2 dA = 15 \times 10^3 \left( \frac{1}{12} \cdot 50 \cdot 80^3 + 50 \cdot 80 \cdot 20^2 \right) \\ &\quad + 40 \times 10^3 \left( \frac{1}{12} \cdot 50 \cdot 20^3 + 50 \cdot 20 \cdot 30^2 \right) \\ &= 9.2 \times 10^{10} \end{aligned}$$

$$\begin{aligned} \sigma_1 &= E_1 \frac{M_2}{(EI)^*} y_1 = 15 \times 10^3 \frac{12 \times 10^6 \times 60}{9.2 \times 10^{10}} \\ &= 117.4 \text{ N/mm}^2 \text{ (MPa)} \end{aligned}$$

$$\begin{aligned} \sigma_2 &= E_2 \frac{M_2}{(EI)^*} y_2 = \frac{12 \times 10^6}{9.2 \times 10^{10}} 40 \times 40 \times 10^3 \\ &= 208.7 \text{ MPa} \end{aligned}$$

$$8-32 \quad \frac{r}{d} = \frac{4}{20} = 0.2 \quad \& \quad \frac{h}{r} = \frac{8}{4} = 2$$

we can find  $K = 1.5$

$$\begin{aligned} \text{at middle spans: } \sigma_{\max} &= \frac{M_{\max}}{S} = \frac{256 \times 10^3}{1568} \\ &= 163 \times 10^6 \text{ N/mm}^2 = 163 \text{ MPa} \end{aligned}$$

$$\begin{aligned} \text{at transition points: } M_{\max} &= \frac{80 \times 160 \times 40}{2} - \\ &\quad \frac{80 \times 40^2}{2} = 192 \times 10^3 \text{ N}\cdot\text{mm} \end{aligned}$$

$$\sigma_{\max} = K \cdot \frac{M}{S} = (1.5) \frac{192 \times 10^3}{800} = 360 \text{ MPa}$$

8-33 from preceding problem

$\sigma_{\max} = 163 \text{ MPa}$  at middle span

at transition points  $K = 1.5$ ,  $S = 800 \text{ mm}^3$

$$\sigma_{\max} = K \cdot \frac{M}{S} = 163 \text{ MPa} \Rightarrow M = 86.9 \text{ N}\cdot\text{m}$$

$$M = \frac{80 \cdot 160 \cdot x}{2} - \frac{80 \cdot x^2}{2} = 86.9 \times 10^3$$

$$\Rightarrow x = 15 \text{ mm}$$

$$8-34 \quad \frac{r}{d} = \frac{2}{10} = 0.2 \quad \frac{h}{r} = \frac{5}{2} = 2.5$$

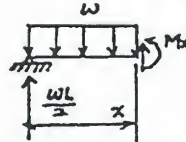
we can find  $K = 1.52$

$$M = \frac{\sigma S}{K} = \frac{(500 \times 10^6) (0.01)^2 / 6}{1.52} = 5482.456$$

$$M = \frac{P}{2} (0.015) \rightarrow P = 730994 \text{ N/m}$$

8-35

$$\sigma_{\max} = \frac{M_{\max} C}{I} = \frac{\frac{WL^2}{8} \left( \frac{h}{2} \right)}{I} = \frac{WL^2 h}{16I}$$



$$M_x = \frac{wx}{2} (L - x)$$

$$U = \int_0^L \frac{M_x^2}{2EI} dx = \frac{1}{2EI} \int_0^L M_x^2 dx$$

$$U = \frac{1}{2EI} \int_0^L \frac{w^2 x^2}{4} (L^2 - 2Lx + x^2) dx$$

$$= \frac{w^2}{8EI} \int_0^L (L^2 x^2 - 2Lx^3 + x^4) dx$$

$$= \frac{w^2 L^5}{8EI} \left( \frac{1}{3} - \frac{1}{2} + \frac{1}{5} \right)$$

$$= \frac{w^2 L^5}{8 \times 30EI}$$

$$= \left( \frac{WL^2 h}{16I} \right)^2 \frac{16^2 \left( \frac{bh^3}{12} \right) L}{h^2 \times 8 \times 30E} = \frac{\sigma_{\max}^2}{2E} \left( \frac{8}{45} AL \right)$$

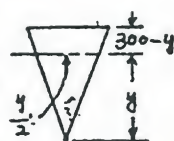
8-36

$$M = -p\pi$$

$$U = \int_0^L \frac{M^2}{2EI} dx = \frac{P^2}{2EI} \int_0^L x^2 dx = \frac{P^2 L^3}{6EI}$$

$$U = \left( \frac{PL \frac{h}{2}}{I} \right)^2 \frac{LI}{\left( \frac{h}{2} \right)^2 6E} = \sigma_{\max}^2 \frac{Ah^2 L}{\left( \frac{h}{2} \right)^2 6E}$$

$$= \frac{\sigma_{\max}^2}{2E} \left( \frac{Vol}{9} \right)$$



$$\frac{1}{2} (y \times \frac{y}{2}) = \frac{1}{2} \left( \frac{y}{2} + 150 \right) (300 - y)$$

$$\frac{y^2}{4} = -\frac{y^2}{4} + 22500$$

$$y = 212 \text{ mm}$$

$$M_u = \sigma_y \left[ \frac{1}{2} (1.212) \left( \frac{-212}{2} \right) \left( \frac{-212}{3} \right) + \left( \frac{-212}{2} \right) (-0.088) \left( \frac{-0.088}{2} \right) + (0.022) (-0.088) \left( \frac{-0.088}{2} \right) \right] = 1.29 \times 10^{-3} \sigma_y$$

$$\frac{M_u}{M_y} = \frac{1.29 \times 10^{-3} \sigma_y}{5.63 \times 10^{-4} \sigma_y} = 2.29$$

8-37

$$M_y = \frac{\sigma_y I}{c} = \frac{\sigma_y \pi r^4}{r/4}$$

$$M_u = \sigma_y \left( \frac{1}{2} \pi r^2 \right) \left( \frac{2 \times 4r}{3\pi} \right)$$

$$\frac{M_u}{M_y} = \frac{(8)(4)}{6\pi} = 1.7$$

8-41

$$\bar{y} = \frac{2\left(\frac{1}{2}\right) + \left(\frac{7}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2} + \frac{7}{2}\right)}{\left(\frac{1}{2}\right)\left(\frac{7}{2}\right) + 2} = 1.185'' \text{ from top}$$

$$I = \frac{1}{12} (4)(.5)^3 + 2(.935)^2 + \frac{1}{12} (.5)(3.5)^3 + \left(\frac{3.5}{2}\right)(1.065)^2 = 5.56 \text{ in}^4$$

$$M_y = \frac{\sigma_y I}{c} = \left( \frac{5.56}{2.815} \right) \sigma_y = 1.98 \sigma_y$$

for equal areas.  $\rightarrow$  NA in flange

$$4x = \frac{1}{2} \left( \frac{7}{2} \right) + \left( \frac{1}{2} - x \right) 4 \rightarrow x = .469'' \text{ from top}$$

$$M_u = \sigma_y \left[ .469(4) \left( \frac{-469}{2} \right) + \left( \frac{3.5}{2} \right) (1.75 + .031) + (.031)(4) \left( \frac{-0.031}{2} \right) \right] = 3.56 \sigma_y$$

$$\frac{M_u}{M_y} = \frac{3.56 \sigma_y}{1.98 \sigma_y} = 1.80$$

8-38

$$M_y = \frac{\sigma_y I}{c} = \left( \frac{\sigma_y}{.15} \right) (2) \left[ \frac{(.2 \times .15)^3}{36} + \frac{(.2 \times .15)}{2} (.05)^2 \right]$$

$$= 7.5 \times 10^{-4} \sigma_y$$

$$M_u = \sigma_y \left( \frac{1}{2} \right) (.2 \times .15) (.1) = 1.5 \times 10^{-3} \sigma_y$$

$$\frac{M_u}{M_y} = \frac{1.5 \times 10^{-3} \sigma_y}{7.5 \times 10^{-4} \sigma_y} = 2.00$$

8-39

W200 x 36

$$Z_x = 380 \times 10^3 \text{ mm}^3, S = 342 \times 10^3 \text{ mm}^3$$

$$\frac{M_p}{M_{yp}} = \frac{380 \times 10^3}{342 \times 10^3} = 1.11$$

8-40

$$M_y = \frac{\sigma_y}{.2} \left( \frac{1}{36} \right) (.15) (.3)^3 = 5.63 \times 10^{-4} \sigma_y$$

at  $M_u$ , stress is constant and  $C=T \rightarrow$   
Area above N.A. = Area below N.A.

8-42

when ultimate moment occurs,

the neutral axis is 130 mm from bottom

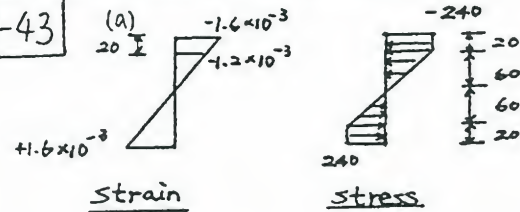
$$M_u = \int \sigma_y \cdot y dA = \sigma_y \int y dA$$

$$= 200 \times 10^3 [0.2 \times 0.05 \times 0.145 + 0.05 \times 0.12 \times 0.06 + 0.05 \times 0.03 \times 0.015 + 0.145 \times 0.1 \times 0.08]$$

$$= 599 \text{ kN}$$



8-43



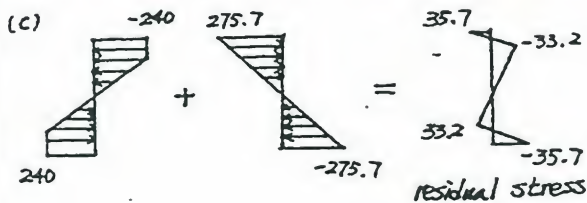
$$M = 2 \left[ 240 \times 20 \times 100 \times (60 + 10) + \frac{1}{2} \times 240 \times 10 \times 60 \times \frac{2}{3} \times 60 \right]$$

$$= 72.96 \times 10^6 \text{ N}\cdot\text{mm} = 72.96 \text{ kN}\cdot\text{m}$$

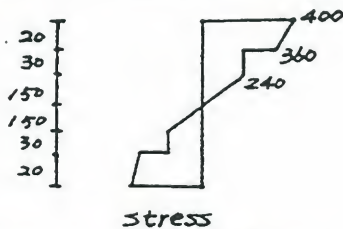
$$(b) \sigma_{\max} = \frac{MC}{I} = \frac{72.96 \times 10^6 \times 80}{21.17 \times 10^6} = 275.7 \text{ N/mm}^2$$

$$\epsilon = \sigma_{\max} / E = \frac{275.7 \times 10^6}{200 \times 10^9} = 1.38 \times 10^{-3}$$

$$\epsilon_{\text{res}} = -1.6 \times 10^{-3} + 1.38 \times 10^{-3} = -0.22 \times 10^{-3}$$



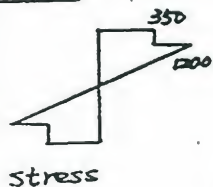
8-44



$$M = 2 \left[ 360 \times 20 \times 240 \times 190 + \frac{40}{2} \times 20 \times 240 \times \left( 180 + \frac{2}{3} \times 20 \right) \right. \\ \left. + 240 \times 60 \times 20 \times \left( 120 + \frac{60}{2} \right) + \frac{240}{2} \times 120 \times 20 \times \frac{2}{3} \times 120 \right]$$

$$= 826 \times 10^6 \text{ N}\cdot\text{mm} = 826 \text{ kN}\cdot\text{m}$$

8-45

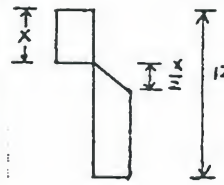


$$M = [350 \times 4 \times 40 \times (40 - 4) + \frac{1}{2} \times 1200 \times 16 \times 40 \times \frac{2}{3} \times 2 \times 16]$$

$$= 102 \times 10^5 \text{ N}\cdot\text{mm}$$

$$M = \frac{P \times 400}{4} \Rightarrow P = 102 \text{ kN}$$

8-46



$$10 \times 2 \times 16 + 10 \times (x - 2) \times 4$$

$$= \frac{10}{2} \times \frac{x}{2} \times 4 + 10 \times \left( 12 - \frac{3}{2}x \right) \times 4$$

$$x = 2.66 \text{ in}$$

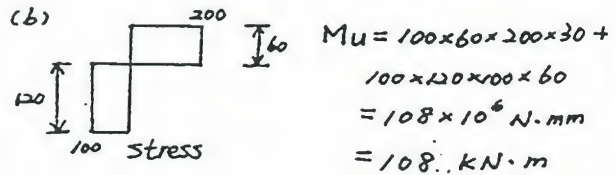
$$M = 10 \times 2 \times 16 \times (2.66 - 1) + 10 \times (2.66 - 2) \times 4 \times \frac{(2.66 - 2)}{2} + \frac{10}{2} \times \frac{2.66}{2} \times 4 \times \frac{2.66}{3} + 10 \times 8.01 \times 4 \times 5.34$$

$$= 2321 \text{ k}\cdot\text{in}$$

8-47

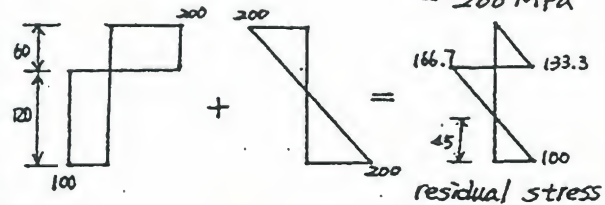
$$(a) M_{yp} = \frac{EI}{C} = \frac{100 \times 48.6 \times 10^6}{90}$$

$$= 54 \times 10^6 \text{ N}\cdot\text{mm} = 54 \text{ kN}\cdot\text{m}$$



$$(c) \sigma_{\max} = \frac{MC}{I} = \frac{108 \times 10^3 \times 90}{48.6 \times 10^6} = 200 \text{ N/mm}^2$$

$$= 200 \text{ MPa}$$



$$C = 100 \times 60 \times \frac{1}{2} \times 133.3 + 100 \times 45 \times \frac{1}{2} \times 100$$

$$= 400 + 225 = 625 \text{ kN}$$

$$T = 100 \times 75 \times \frac{1}{2} \times 166.7 = 625 \text{ kN}$$

$$C = T$$

$$\Sigma M_{\text{bottom}} = 225 \times \frac{45}{3} + 400 \left( \frac{60}{3} + 120 \right) - 625 \times \left( \frac{2}{3} \times 75 + 45 \right) = 0$$

$$\therefore \text{equilibrium}$$



8-48

$$\bar{y}_b = \frac{\int_A E_i y_b dA}{\int_A E_i dA}$$

$$= \frac{70 \times 15 \times 40 \times 20 + 10 \times 40 \times 60 \times 40 + 20 \times 20 \times 30 \times 10}{15 \times 20 \times 40 + 10 \times 40 \times 60 + 20 \times 20 \times 30}$$

$$= 40 \text{ mm}$$

$$(EI)^* = \int_A E_i y^2 dA = 10 \times 10^3 \left( \frac{1}{12} \cdot 60 \cdot 40^3 \right) +$$

$$15 \times 10^3 \left( \frac{1}{12} \cdot 40 \cdot 20^3 + 40 \cdot 20 \cdot 30^2 \right) +$$

$$30 \times 10^3 \left( \frac{1}{12} \cdot 20 \cdot 20^3 + 20 \cdot 20 \cdot 30^2 \right) = 2.56 \times 10^{10}$$

$$\sigma_1 = E_1 \frac{M_2}{(EI)^*} y_1 = 15 \times 10^3 \times \frac{10 \times 10^6}{2.56 \times 10^{10}} \times 40$$

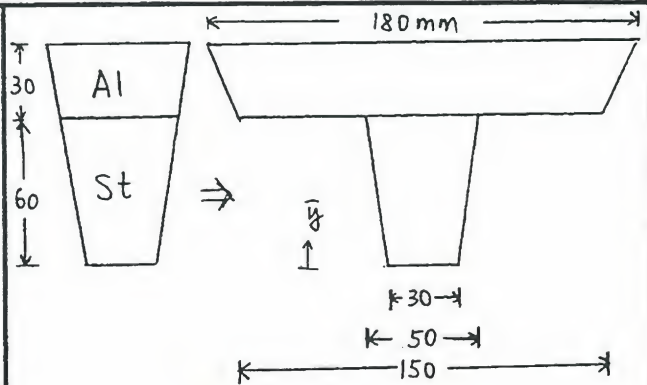
$$= 234.375 \text{ MPa}$$

$$\sigma_2 = E_2 \frac{M_2}{(EI)^*} y_2 = 10 \times 10^3 \times \frac{10 \times 10^6}{2.56 \times 10^{10}} \times 20$$

$$= 78.125 \text{ MPa}$$

$$\sigma_3 = E_3 \frac{M_2}{(EI)^*} y_3 = 30 \times 10^3 \times \frac{10 \times 10^6}{2.56 \times 10^{10}} \times 40$$

$$= 468.75 \text{ MPa}$$



$$\bar{y} = \frac{(30 \times 60 \times 30) + (10 \times 60 \times 40) + (150 \times 30 \times 75) + (15 \times 30 \times 80)}{(30 \times 60) + (10 \times 60) + (150 \times 30) + (15 \times 30)}$$

$$= 61.43 \text{ mm}$$

$$I = \frac{1}{12} (30)(60)^3 + (30 \times 60)(31.43)^2$$

$$+ 2 \left[ \frac{1}{36} (10)(60)^3 + \left( \frac{1}{2} \times 10 \times 60 \right) (21.43)^2 \right]$$

$$+ \frac{1}{12} (150)(30)^3 + (150 \times 30)(13.57)^2$$

$$+ 2 \left[ \frac{1}{36} (15)(30)^3 + \left( \frac{1}{2} \times 15 \times 30 \right) (18.57)^2 \right]$$

$$= 4.06 \times 10^6 \text{ mm}^4$$

$$\sigma_{Al} = \frac{(80 \times 10^6)(28.57)}{(4.06 \times 10^6)} = 563 \text{ MPa}$$

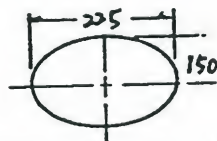
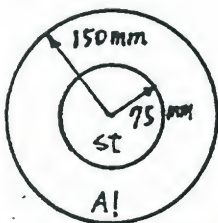
$$\sigma_{st} = \frac{3(80 \times 10^6)(61.43)}{(4.06 \times 10^6)} = 3631 \text{ MPa}$$

8-49

$$\sigma_{AL} = \frac{(80)(10^{-3})(-179)}{(3.01 \times 10^{-4})} = -47.6 \text{ MN/m}^2$$

$$\sigma_{st} = n \frac{MC}{I} = 3 \times \frac{(80)(10^{-3})(-121)}{(3.01 \times 10^{-4})} = 96.5 \text{ MN/m}^2$$

8-50



$$n = \frac{E_{st}}{E_{Al}} = 3$$

$$I_t = \frac{\pi}{4} (225^4 - 150^4) = 4.47 \times 10^{-4} \text{ m}^4$$

$$\sigma_{AL} = \frac{MC}{I_t} = \frac{(80)(10^{-3})(-15)}{(4.47 \times 10^{-4})} = -26.8 \text{ MN/m}^2$$

$$\sigma_{st} = n \frac{MC}{I} = \frac{3(80 \times 10^{-3})(-0.75)}{(4.47 \times 10^{-4})} = -26.8 \text{ MN/m}^2$$

8-51

$$n = \frac{E_{st}}{E_{AL}} = \frac{210}{70} = 3$$

8-52

$$n = \frac{E_{st}}{E_w} = \frac{30 \times 10^6}{1.2 \times 10^6} = 25$$

transformed steel areas are  $\frac{1}{2}$ "  $\times$  50" (top)  
and  $\frac{1}{2}$ "  $\times$  150" (bottom)

$$\bar{y} = \left( \frac{1}{2} \right) (50) \left( \frac{1}{4} \right) + 8(12 \times 65) + \frac{1}{2} (150) (12.75)$$

$$\left( \frac{1}{2} \right) (50) + 8(12) + \frac{1}{2} (150)$$

$$= 8.09" \text{ from top}$$

$$I_t = \frac{(50) \left( \frac{1}{2} \right)^3}{12} + (50 \times \frac{1}{2}) (7.84)^2 + \frac{(8)(12)^3}{12}$$

$$+ (8)(12)(1.6)^2 + \frac{(150) \left( \frac{1}{2} \right)^3}{12} + (150 \times \frac{1}{2}) (4.66)^2$$

$$= 4565 \text{ in}^4$$

$$M_w = \frac{\sigma_w I}{C_w} = \frac{(1.2)(4565)}{7.6} = 721 \text{ k}\cdot\text{in}$$

$$M_{st} = \frac{\sigma_{st} I}{n C_{st}} = \frac{(20)(4565)}{(25)(8.09)} = 451 \text{ k}\cdot\text{in (control)}$$

$$8-53 \quad n = \frac{E_{st}}{E_w} = \frac{30}{1.2} = 25$$

transformed steel - 12.5" x 12"

transformed area - [8 + 2(12.5)] by 12"  
= 33" x 12"

$$I_t = \frac{(33)(12)^3}{12} = 4750 \text{ in}^4$$

$$M_{st} = \frac{20(4750)}{6(25)} = 633 \text{ k}\cdot\text{in (controls)}$$

$$M_w = \frac{1.2(4750)}{6} = 950 \text{ k}\cdot\text{in}$$

$$8-54 \quad \text{steel} - 10 \text{ mm } \phi @ 80 \text{ mm} = 9.82 \times 10^{-4} \text{ m}^2/\text{m}$$

$$\frac{(1) \times 2}{2} = 12(9.82 \times 10^{-4})(.125 - x) \quad x = kd$$

$$d = .125 \text{ m}$$

$$x = 0.44 \text{ m}$$

$$M_t = \bar{\sigma}_{st} A_{st} (d - x) = (150)(10^3)(9.82 \times 10^{-4}) \times (.125 - .044) = 11.9 \text{ kN}\cdot\text{m/m of slab}$$

$$8-55 \quad \text{assume NA is below centroid areas}$$

$$x = kd, \quad d = 28"$$

$$\frac{12x^2}{2} + \frac{6(x-6)^2}{2} = 36(28-x)$$

$$x = 10" \text{ } 76" \rightarrow \text{assumption ok}$$

$$I = \frac{12(10)^3}{3} + \frac{2(3)(4)^3}{3} + 36(18)^2 = 15,800 \text{ in}^4$$

$$M = \frac{\bar{\sigma} I}{n C} = \frac{20(15,800)}{12(18)} = 1460 \text{ k}\cdot\text{in} = 122 \text{ k}\cdot\text{ft}$$

$$8-56 \quad (a) S = \frac{bh^2}{6} = \frac{(0.05)(.1)^2}{6} = 8.33 \times 10^{-5} \text{ m}^4$$

$$\bar{\sigma} = \frac{M}{S} = \frac{(2083)(10^{-6})}{8.33 \times 10^{-5}} = 25.0 \text{ MN/m}^2$$

$$(b) \bar{r} = .25 \text{ m}, \quad r_o = .25 + .05 = .3 \text{ m}, \quad r_i = .2 \text{ m}$$

$$R = \frac{h}{\ln(r_o/r_i)} = \frac{.1}{\ln(.3/.2)} = .24663 \text{ m}$$

$$\bar{\sigma}_i = \frac{M(R - r_i)}{r_i A (\bar{r} - R)} = \frac{(2083)(10^{-6})(.24663 - .2)}{(.2)(.005)(.25 - .24663)} = 28.8 \text{ MN/m}^2$$

$$\bar{\sigma}_o = \frac{M(R - r_o)}{r_o A (\bar{r} - R)} = \frac{(2083)(10^{-6})(.24663 - .3)}{(.3)(.005)(.25 - .24663)} = -22.0 \text{ MN/m}^2$$

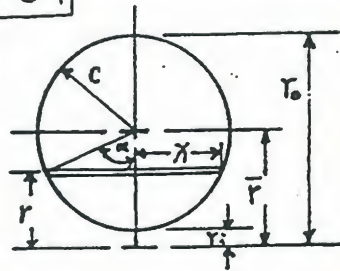
$$(c) \bar{r} = .075 \text{ m}, \quad r_o = .125 \text{ m}, \quad r_i = .025 \text{ m}$$

$$R = \frac{.1}{\ln(.125/.025)} = .06213 \text{ m}$$

$$\bar{\sigma}_i = \frac{(2083)(10^{-6})(.06213 - .025)}{(.025)(.005)(.075 - .06213)} = 48.1 \text{ MN/m}^2$$

$$\bar{\sigma}_o = \frac{(2083)(10^{-6})(.06213 - .125)}{(.125)(.005)(.075 - .06213)} = -16.2 \text{ MN/m}^2$$

8-57



$$R = \frac{A}{\int_A \frac{dA}{r}} \quad A = \pi c^2$$

$$\int_A \frac{dA}{r} = \int \frac{2x dx}{r}$$

$$= \int_0^\pi \frac{2c \sin \alpha}{\bar{r} + c \cos \alpha} c \sin \alpha d\alpha$$

$$= \int_0^\pi \frac{2c^2 (\cos^2 \alpha) d\alpha}{\bar{r} + c \cos \alpha}$$

$$= 2 \int_0^\pi \frac{c^2 - c^2 \cos^2 \alpha + \bar{r}^2 - \bar{r}^2}{\bar{r} + c \cos \alpha} d\alpha$$

$$= 2 \int_0^\pi \left( \frac{c^2 - \bar{r}^2}{\bar{r} + c \cos \alpha} + \bar{r} - c \cos \alpha \right) d\alpha$$

$$= 2\pi (\bar{r} - \sqrt{\bar{r}^2 - c^2})$$

$$R = \frac{\pi c^2}{2\pi (\bar{r} - \sqrt{\bar{r}^2 - c^2})} = \frac{\bar{r} + \sqrt{\bar{r}^2 - c^2}}{2}$$

8-58

$$R = \frac{\bar{r}^2 + \sqrt{\bar{r}^2 - c^2}}{2} = \frac{3 + \sqrt{3^2 - 12}}{2} = 2.915''$$

$$\sigma = \frac{M(R-r)}{rA(\bar{r}-R)} \rightarrow M = \frac{\sigma r A (\bar{r}-R)}{(R-\bar{r})}$$

$$M_o = \frac{12(4)\pi(1)^2(3-2.915)}{2.915-4} = 11.8 \text{ K-in}$$

$$M_i = \frac{12(2)(\pi)(1)^2(3-2.915)}{2.915-2} = 7.00 \text{ K-in}$$



9-1

$$M = \frac{WL}{8} = \frac{4 \times 6}{8} = 3 \text{ kN}\cdot\text{m}$$

$$M_{zz} = M \cos \alpha = 2.819$$

$$M_{yy} = M \sin \alpha = 1.026$$

$$I_{zz} = \frac{1}{12} (150)(200)^3 = 100 \times 10^6 \text{ mm}^4$$

$$I_{yy} = \frac{1}{12} (200)(150)^3 = 56.25 \times 10^6 \text{ mm}^4$$

$$\begin{aligned} \sigma_{\max} &= \frac{M_{zz} C_1}{I_{zz}} + \frac{M_{yy} C_2}{I_{yy}} \\ &= \frac{2.819 \times 100}{100} + \frac{1.026 \times 75}{56.25} \\ &= 4.187 \text{ N/mm}^2 \\ &= 4.187 \text{ MPa} \end{aligned}$$

9-2

$$M = \frac{PL}{4} = \frac{5 \times 6}{4} = 7.5 \text{ kN}\cdot\text{m}$$

$$M_y = 7.5 \times \frac{3}{5} = 4.5 \text{ kN}\cdot\text{m}$$

$$M_z = 7.5 \times \frac{4}{5} = 6 \text{ kN}\cdot\text{m}$$

$$I_z = \frac{1}{12} (0.15)(0.2)^3 = 1 \times 10^{-4} \text{ m}^4$$

$$I_y = \frac{1}{12} (0.2)(0.15)^3 = 5.625 \times 10^{-5} \text{ m}^4$$

$$\begin{aligned} \sigma_{\max} &= \pm \left( \frac{M_z C_1}{I_z} + \frac{M_y C_2}{I_y} \right) \\ &= \pm \left( \frac{6 \times 0.1}{1 \times 10^{-4}} + \frac{4.5 \times 0.075}{5.625 \times 10^{-5}} \right) \\ &= 12 \text{ MPa} \end{aligned}$$

$$\begin{aligned} \tan \beta &= \frac{I_z}{I_y} \tan \alpha \\ &= \frac{1 \times 10^{-4}}{5.625 \times 10^{-5}} \left( \frac{3}{4} \right) \end{aligned}$$

$$\therefore \beta = 53.1^\circ$$

9-3

$$M = PL = 100 \times 0.2 = 20 \text{ N}\cdot\text{m}$$

$$M_y = M \cos 30^\circ = 17.3 \text{ Nm}$$

$$M_z = M \sin 30^\circ = 10.0 \text{ Nm}$$

$$\begin{aligned} I_z &= \frac{1}{12} (0.35)(0.45)^3 - \frac{1}{12} (0.25)(0.35)^3 \\ &= 1.76 \times 10^{-3} \text{ m}^4 \end{aligned}$$

$$\begin{aligned} I_y &= \frac{1}{12} (0.45)(0.35)^3 - \frac{1}{12} (0.35)(0.25)^3 \\ &= 1.15 \times 10^{-3} \text{ m}^4 \end{aligned}$$

$$\begin{aligned} \sigma_{\max} &= \left( \frac{M_z C_1}{I_z} + \frac{M_y C_2}{I_y} \right) \\ &= \frac{10.0 \times 0.6225}{1.76 \times 10^{-3}} + \frac{17.3 \times 0.075}{1.15 \times 10^{-3}} \\ &= 391 \text{ Pa} \end{aligned}$$

9-4

From appendix B:

$$\begin{aligned} I_z &= I_y = 279.3 \text{ in}^4 \\ &= 116 \times 10^6 \text{ mm}^4 \\ &= 1.16 \times 10^{-4} \text{ m}^4 \end{aligned}$$

$$M_1 = P_1 L = 50 \times 0.4 = 20 \text{ N}\cdot\text{m}$$

$$M_2 = P_2 L = 60 \times 0.4 = 24 \text{ N}\cdot\text{m}$$

$$M_z = M_1 + M_2 \cos 20^\circ = 42.6 \text{ N}\cdot\text{m}$$

$$M_x = M_2 \sin 20^\circ = 8.21 \text{ N}\cdot\text{m}$$

$$\begin{aligned} \sigma_{\max} &= \frac{M_z C_1}{I} + \frac{M_x C_2}{I} \\ &= \frac{42.6 \times 0.162}{1.16 \times 10^{-4}} + \frac{8.21 \times 0.162}{1.16 \times 10^{-4}} \\ &= \pm 70.7 \text{ kPa} \end{aligned}$$

$$\begin{aligned} \sigma_{\min} &= \frac{M_z C_1}{I} - \frac{M_x C_2}{I} \\ &= \frac{42.6 \times 0.162}{1.16 \times 10^{-4}} - \frac{8.21 \times 0.162}{1.16 \times 10^{-4}} \\ &= \pm 47.8 \text{ kPa} \end{aligned}$$

9-5

From appendix B.

$$\begin{aligned} I &= 72.99 \text{ in}^4 \\ &= 3.02 \times 10^{-5} \text{ m}^4 \end{aligned}$$

$$M_z = P_1 L = 20 \times 0.55 = 11 \text{ N}\cdot\text{m}$$

$$M_y = P_2 L = 30 \times 0.3 = 9 \text{ N}\cdot\text{m}$$

$$\begin{aligned} \sigma_{\max} &= \frac{M_z C_1}{I} + \frac{M_y C_2}{I} \\ &= \frac{11 \times 0.1095}{3.02 \times 10^{-5}} + \frac{9 \times 0.1095}{3.02 \times 10^{-5}} \\ &= 72.6 \text{ kPa} \end{aligned}$$

9-6

$$M = (3-1)P = 2P$$

$$M_x = M \cos \alpha = 2P \cos \alpha$$

$$M_y = M \sin \alpha = 2P \sin \alpha$$

$$I_x = 127 \times 10^6 \text{ mm}^4$$

$$I_y = 6.53 \times 10^6 \text{ mm}^4$$

$$\sigma_{max} = \frac{M_x C_x}{I_x} + \frac{M_y C_y}{I_y}$$

$$= \frac{2P \cos \alpha \left( \frac{3c5}{2} \right) (1000)}{127 \times 10^6} + \frac{2P \sin \alpha \left( \frac{129}{2} \right) (1000)}{6.53 \times 10^6}$$

$$= P [2.40 \times 10^{-3} \cos \alpha + 2.13 \times 10^{-2} \sin \alpha]$$

$$\alpha = 0^\circ, \sigma_{max} = 2.40P$$

$$\alpha = 1^\circ, \sigma_{max} = 2.77P$$

$$\alpha = 5^\circ, \sigma_{max} = 4.25P$$



9-7

$$I = \frac{1}{12} (0.045) (0.03)^3$$

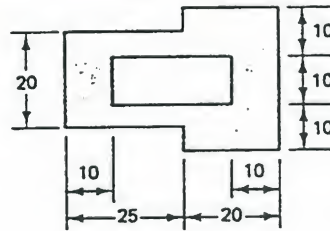
$$- \frac{1}{12} (0.025) (0.01)^3$$

$$- 2 \left[ \frac{1}{12} (0.025) (0.005)^3 + (0.025) (0.005) (0.125)^2 \right]$$

$$= 5.96 \times 10^{-8} \text{ m}^4$$

$$\sigma_{max} = \pm \frac{M C}{I} = \frac{500 \times 0.015}{5.96 \times 10^{-8}}$$

$$= 125.9 \text{ MPa}$$



9-8

$$P_x = 64.3 \text{ N}, P_y = 76.6 \text{ N}$$

$$a. I_{xx} = 114624 \text{ mm}^4$$

$$I_{yy} = 69184 \text{ mm}^4$$

$$\sigma_c = \frac{M_x C_1}{I_x} + \frac{M_y C_2}{I_y}$$

$$= \frac{76.6 \times 150 \times 24}{114624} + \frac{64.3 \times 150 \times 6}{69184} = 3.24 \text{ N/mm}^2$$

$$\sigma_D = \frac{M_x C_2}{I_x} + \frac{M_y C_1}{I_y}$$

$$= \frac{76.6 \times 150 \times 6}{114624}$$

$$+ \frac{64.3 \times 150 \times 24}{69184}$$

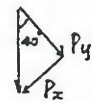
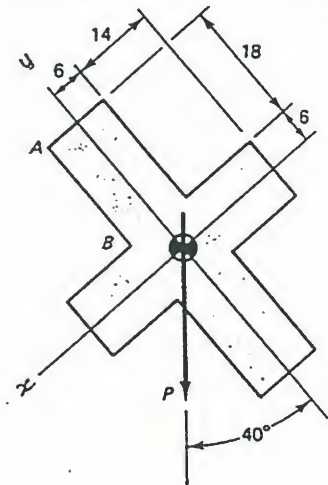
$$= 3.39 \text{ N/mm}^2$$

$$b. \sigma_4 = \frac{76.6 \times 150 \times 24}{114624}$$

$$- \frac{64.3 \times 150 \times 6}{69184}$$

$$= 0$$

$$y = 8.34 \text{ mm}$$



9-9

$$M_y = 15 \times 0.48 = 7.2 \text{ kN-m}$$

$$M_x = 10 \times 0.6 = 6 \text{ kN-m}$$

$$I_x = \frac{1}{12} (0.1) (0.2)^3 = 6.67 \times 10^{-5} \text{ m}^4$$

$$I_y = \frac{1}{12} (0.2) (0.1)^3 = 1.67 \times 10^{-5} \text{ m}^4$$

$$\sigma_A = \frac{6 \times 0.1}{6.67 \times 10^{-5}} + \frac{7.2 \times 0.05}{1.67 \times 10^{-5}}$$

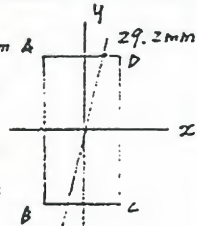
$$= 9 + 21.6 = 30.6 \text{ MPa}$$

$$\sigma_B = -9 + 21.6 = 12.6 \text{ MPa}$$

$$\sigma_C = -9 - 21.6 = -30.6 \text{ MPa}$$

$$\sigma_D = 9 - 21.6 = -12.6 \text{ MPa}$$

$$x = \frac{12.6 \times 100}{12.6 + 30.6} = 29.2 \text{ mm}$$



9-10

$$P = 400 \text{ kN}$$

$$M = Pe = 400 \left( 200 - \frac{257}{2} \right)$$

$$= 30.6 \text{ kN-m}$$

$$\sigma_{max} = \frac{P}{A} + \frac{M}{S} = \frac{400}{0.0066} + \frac{30.6}{0.000572}$$

$$= 117 \text{ MPa}$$

Load is transferred by bearing at the pin.  
Maximum stress will probably occur where the flange ends due to a reduced S.





9-17

$$\sigma_A = -\frac{F_H}{A} + \frac{M_{AB}}{S} = 0$$

$$\sigma_B = -\frac{F_H}{A} - \frac{M_{AB}}{S} = -30$$

$$\sigma_A - \sigma_B = \frac{2M_{AB}}{S} = 30$$

$$M_{AB} = \frac{1}{2} \times 30 \times \frac{1}{6} \times 0.08 \times 1^2 = 2 \text{ kN-m}$$

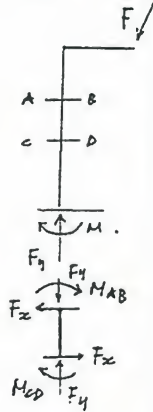
$$\sigma_C = -\frac{F_H}{A} + \frac{M_{CD}}{S} = -24$$

$$\sigma_D = -\frac{F_H}{A} - \frac{M_{CD}}{S} = -6$$

$$\therefore \frac{2M_{CD}}{S} = -18 \rightarrow M_{CD} = \frac{1}{2}(-18) \times \frac{1}{6} \times 0.08 \times 0.1^2 = -1.2 \text{ kN-m}$$

$$\sigma_A = 0.2 F_x - M_{CD}$$

$$\therefore F_x = \frac{M_{AB} + M_{CD}}{0.2} = \frac{2 - 1.2}{0.2} = 4 \text{ kN}$$



9-18

$$I_x = \frac{1}{12} \times 0.2 \times 0.3^3 = 45 \times 10^{-9} \text{ m}^4$$

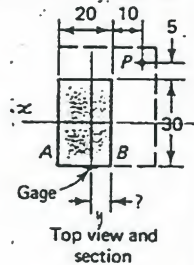
$$I_y = \frac{1}{12} \times 0.3 \times 0.2^3 = 20 \times 10^{-9} \text{ m}^4$$

$$A = 0.03 \times 0.02 = 0.0006 \text{ m}^2$$

$$\sigma_B = \frac{-P}{0.0006} + \frac{P \cdot 0.02 \times 0.015}{45 \times 10^{-9}}$$

$$- \frac{P \cdot 0.02 \times 0.015}{20 \times 10^{-9}} = 0$$

$\therefore x = 5 \text{ mm}$  does not depend on  $P$ .



9-19

$$\sigma_A = E \epsilon_A = 70 \times 10^9 \times 20 \times 10^{-6} = 1.4 \text{ N/mm}^2$$

$$\sigma_A = \frac{P}{N} + \frac{M_x C_x}{I_x} + \frac{M_y C_y}{I_y} = 1.4$$

$$= \frac{\frac{2}{3} F}{1200} + \frac{\frac{F}{2} \times 500 \times 15}{\frac{1}{12} \times 40 \times 30^3} + \frac{\frac{2F}{3} \times 500 \times (-12)}{\frac{1}{12} \times 30 \times 40^3}$$

$$\therefore F = 420 \text{ N}$$

9-20

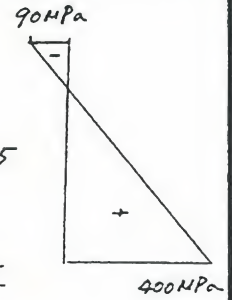
$$a. P = \frac{1}{2} \times 400 \times 10^6 \times 0.002 \times 0.015 - \frac{1}{2} \times 90 \times 10^6 \times 0.02 \times 0.015 = 46.5 \text{ kN}$$

$$b. \epsilon_{mg} = \epsilon_{st} \rightarrow \sigma_{mg} = \frac{E_{mg} \epsilon_{st}}{E_{st}} = 0.225 \sigma_{st}$$

$$P_A = A \times 0.225 \sigma_{st} \times 30 + A \sigma_{st} \times 10 = 16.75 A \sigma_{st}$$

$$P = A \times 0.225 \sigma_{st} + A \sigma_{st} = 1.225 A \sigma_{st}$$

$$a = 13.7 \text{ mm}$$



9-21

$$R = \frac{\bar{r}}{\ln \frac{r_o}{r_i}} = \frac{6.5}{\ln \frac{9.5}{3.5}} = 6.51$$

$$\sigma_z = \frac{P}{A} + \frac{M(R-r_i)}{r_i A (F-R)} = \frac{19}{2 \times 6} + \frac{19 \times 6.5 \times (6.51 - 3.5)}{3.5 \times 2 \times 6 \times (6.5 - 3.5)} = -922 \text{ ksi}$$

$$\sigma_o = \frac{P}{A} + \frac{M(R-r_o)}{r_o A (F-R)} = \frac{19}{12} + \frac{19 \times 6.5 \times (6.51 - 9.5)}{9.5 \times 12 \times (6.5 - 3.5)} = 340 \text{ ksi} < 922 \text{ ksi}$$

9-22

$$M = 10 \times (0.3 - 0.04) = 2.6 \text{ kN-m}$$

$$R = \frac{\bar{r} + \sqrt{r_o^2 - c^2}}{2} = \frac{130 + \sqrt{130^2 - 20^2}}{2} = 129 \text{ mm}$$



$$a. \sigma_i = \frac{P}{A} + \frac{M(R-r_o)}{r_i A (F-R)} = \frac{10}{\pi (0.02)^2} + \frac{2.6 \times (0.129 - 0.11)}{0.11 \pi \times 0.2^2 \times (0.13 - 0.129)} = 475 \text{ MN/m}^2$$

$$\sigma_o = \frac{P}{A} + \frac{M(R-r_o)}{r_o A (F-R)} = \frac{10}{\pi (0.02)^2} + \frac{2.6 \times (0.129 - 0.15)}{0.15 \pi \times 0.2^2 \times (0.13 - 0.129)} = -362 \text{ MN/m}^2 \quad \therefore \sigma_{max} = 475 \text{ MN/m}^2$$

$$b. \frac{\sigma_{max}}{\sigma_{comp}} = \frac{475}{362} = 1.31$$

9-23

$$0 = \frac{-P_1}{\frac{1}{2} \times 0.2^2} + \frac{P_1 e_1 \times 0.0667}{\frac{1}{36} \times 0.2^4}$$

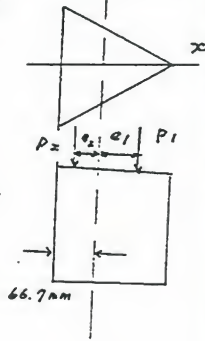
$$\therefore e_1 = 33.3 \text{ mm}$$

$$0 = \frac{-P_2}{\frac{1}{2} \times 0.2^2} + \frac{P_2 e_2 (0.2 - 0.0667)}{\frac{1}{36} \times 0.2^4}$$

$$\therefore e_2 = 16.7 \text{ mm}$$

range, 50 to 100 mm.

from base of the triangular cross-section



9-24

$$\bar{y} = \frac{4 \times 3 \times 2 + \frac{1}{2} \times 6 \times 4 \times 6}{4 \times 3 + \frac{1}{2} \times 6 \times 4}$$

$$= 4 \text{ in}$$

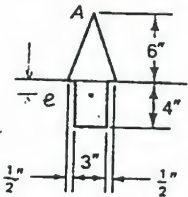
$$I = \frac{1}{12} \times 3 \times 4^3 + 3 \times 4 \times 2^2 + \frac{1}{36} \times 4 \times 6^3 + \frac{1}{2} \times 6 \times 4 \times 2^2$$

$$= 136 \text{ in}^4$$

$$\sigma_A = 0 = -\frac{P}{A} + \frac{Mc}{I} = -\frac{P}{24} + \frac{Pe \times 6}{136}$$

$$e = 0.944 \text{ in}$$

or, 6.944 in from A.



9-25

$$I = \frac{\pi}{4} R^4; A = \pi R^2$$

$$\sigma = -\frac{P}{A} + \frac{Mc}{I} = -\frac{P}{\pi R^2} + \frac{PR}{\frac{\pi R^4}{4}} = 0$$

$$\therefore r = \frac{R}{4}$$

9-26

$$W_1 = 150 \times \frac{1}{2} \times 6 \times 20 = 9000 \text{ lb/ft}$$

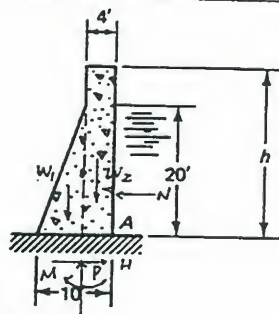
$$W_2 = 150 \times 4 \times h = 600h \text{ lb/ft}$$

$$H = 6 \times 5 \times \frac{1}{2} \times 20^2 = 12500 \text{ lb/ft}$$

$$P = -(W_1 + W_2) = -9000 - 600h$$

$$M = 9000 \times (5-4) + 12500 \times \frac{20}{3} - 600h \times (5-2) = 92333 - 1800h$$

$$\sigma_A = \frac{P}{A} + \frac{M}{S} = \frac{-9000-600h}{10 \times 1} + \frac{92333-1800h}{\frac{1}{6} \times 1 \times 10^2} = 0$$



$$\sigma_A = 4640 - 168h = 0$$

$$\therefore h = 27.6 \text{ ft}$$

9-27

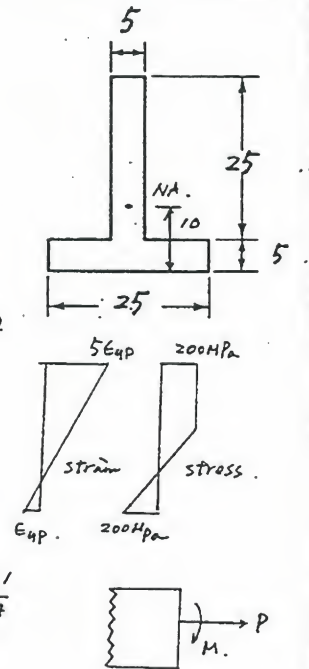
$$\begin{aligned} a. P &= -\frac{1}{2} \times 25 \times 5 \times 200 \\ &\quad + \frac{1}{2} \times 5 \times 5 \times 200 \\ &\quad + 20 \times 5 \times 200 \\ &= -12500 + 2500 + 2000 \\ &= 10 \text{ kN} \end{aligned}$$

$$M = 12500 \times (0.005 + \frac{2}{3} \times 0.005)$$

$$\begin{aligned} &- 2500 \times \frac{1}{3} \times 0.005 \\ &+ 20000 \times 0.01 \\ &= 300 \text{ N-m} \end{aligned}$$

$$\begin{aligned} b. \sigma_{\text{bottom}} &= \frac{-P}{A} + \frac{Mc}{I} \\ &= \frac{10 \times 10^3}{228 \times 10^{-3}} + \frac{300 \times 0.01}{228 \times 10^{-3}} \\ &= 104 \text{ MPa} \end{aligned}$$

$$\begin{aligned} \epsilon_{\text{bottom}} &= \epsilon_{yp} + \frac{\sigma_{\text{bottom}}}{E} \\ &= \frac{-200 + 104}{200 \times 10^3} = -0.00048 \text{ m/m} \end{aligned}$$



9-28

$$a. P = 135 \times 15 \times 20 = 40,500 \text{ N}$$

$$P(22.5 + e)$$

$$= -\frac{1}{2} P \times 5 + \frac{1}{2} P \times 25 + P \times 37.5$$

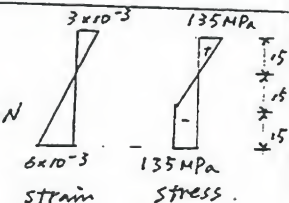
$$\therefore e = 25 \text{ mm}$$

$$\begin{aligned} b. \sigma_{\text{top}} &= -\frac{Pe}{I} + \frac{P}{A} = \frac{-40500 \times 0.025 \times 0.0225}{15,875 \times 10^{-7}} + \frac{40500}{0.0009} \\ &= -105 \text{ MPa} \end{aligned}$$

$$\epsilon_T = 3 \times 10^{-3} - 105 \times 3 \times 10^{-3} / 135 = 6.67 \times 10^{-4}$$

$$\sigma_B = \frac{Pe}{I} + \frac{P}{A} = 195 \text{ MPa}$$

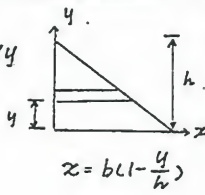
$$\begin{aligned} \epsilon_B &= 195 \times 3 \times 10^{-3} / 135 - 6 \times 10^{-3} \\ &= -1.67 \times 10^{-3} \end{aligned}$$



9-29

$$a. I_{xy} = \int_0^h \frac{b^2}{2} \left(1 - \frac{y}{h}\right)^2 y dy$$

$$= \frac{b^2}{2} \left( \frac{h^2}{2} - \frac{2}{3} h^2 + \frac{1}{4} h^2 \right)$$

$$= -\frac{b^2 h^2}{24}$$


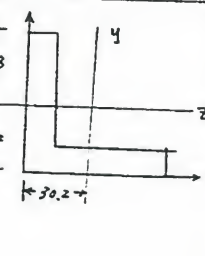
$$x = b \left(1 - \frac{y}{h}\right)$$

$$b. I_{x, y_0} = I_{xy} - A d x d y$$

$$= \frac{b^2 h^2}{24} - \frac{bh}{2} \cdot \frac{b}{3} \cdot \frac{h}{3}$$

$$= -\frac{b^2 h^2}{72}$$

9-30

$$a. \bar{y} = \frac{[102 \times 12.7 \times 51 + (102 - 12.7) \times 12.7 \times \frac{12.7}{2}]}{102 \times 12.7 + (102 - 12.7) \times 12.7}$$


$$= 30.2 \text{ mm from the bottom and left.}$$

$$b. I_y = \frac{1}{12} \times 12.7 \times 89.3^3 + 12.7 \times 89.3 \times (71.8 - 44.65)^2$$

$$+ \frac{1}{12} \times 102 \times 12.7^3 + 102 \times 12.7 \times (-30.2 - 6.35)^2$$

$$= 2.34 \times 10^6 \text{ mm}^4$$

$$I_z = 2.34 \times 10^6 \text{ mm}^4 \text{ (symmetry)}$$

$$I_{yz} = 0 + 12.7 \times 89.3 \times (71.8 - 44.65) \times (30.2 - 6.35)$$

$$+ 0 + 12.7 \times 102 \times (-30.2 + 6.35) \times (-51 + 30.2)$$

$$= 1.38 \times 10^6 \text{ mm}^4$$

$$c. \theta_1 = \frac{1}{2} \tan^{-1} \left[ \frac{2 \times 1.38 \times 10^6}{0} \right] \rightarrow \theta_1 = 90^\circ$$

$$I_{\max/\min} = 2.34 \times 10^6 \pm \sqrt{0 + (1.38 \times 10^6)^2}$$

$$\therefore I_{\max} = 3.72 \times 10^6 \text{ mm}^4$$

$$I_{\min} = 960 \times 10^3 \text{ mm}^4$$

$$d. r_{\min} = 0.782 \text{ in} = 19.8628 \text{ mm}$$

$$I_{\min} = 2429.51 \times 19.8628$$

$$= 959 \times 10^3 \text{ mm}^2$$

$$I_{\max} = I_x + I_y - I_{\min}$$

$$= 3.73 \times 10^6 \text{ mm}^4$$

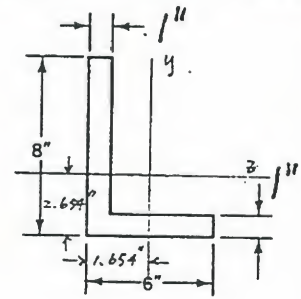
9-31

$$a. \bar{z} = \frac{7 \times 1 \times \frac{1}{2} + 6 \times 1 \times 3}{7 + 6}$$

$$= 1.654 \text{ in.}$$

$$\bar{y} = \frac{8 \times 1 \times 4 + 5 \times 1 \times \frac{1}{2}}{8 + 5}$$

$$= 2.654 \text{ in.}$$



$$I_z = \frac{1}{12} \times 1 \times 7^3 + 7 \times 1.846^2 + \frac{1}{12} \times 6 \times 1^3 + 6 \times 2.154^2$$

$$= 80.8 \text{ in}^4$$

$$I_y = \frac{1}{12} \times 1 \times 7^3 + 7 \times 1.154^2 + \frac{1}{12} \times 1 \times 6^3 + 6 \times 1.346^2$$

$$= 38.8 \text{ in}^4$$

$$I_{zy} = 7 \times 1 \times 1.846 \times (-1.154) + 6 \times 1 \times (-2.154) \times 1.346$$

$$= -32.3 \text{ in}^4$$

$$\theta_1 = \frac{1}{2} \tan^{-1} \left[ \frac{2 \times (-32.3)}{80.8 - 38.8} \right] = -28.5^\circ$$

$$I_{\max/\min} = \frac{80.8 + 38.8}{2} \pm \sqrt{21^2 + (-32.3)^2}$$

$$I_{\max} = 98.3 \text{ in}^4$$

$$I_{\min} = 21.5 \text{ in}^4$$

$$b. r_{\min} = 1.28, I_{\min} = 13 \times 1.28^2 = 21.3 \text{ in}^4$$

$$I_{\max} = I_x + I_y - I_{\min} = 80.8 + 38.8 - 21.3$$

$$= 98.3 \text{ in}^4$$

9-32

$$I_z = \frac{2}{12} \times 30 \times 10^3 + 2 \times 10 \times 30 \times 25^2 + \frac{1}{12} \times 10 \times 60^3$$

$$= 560 \times 10^3 \text{ mm}^4$$

$$I_y = \frac{1}{12} \times 10 \times 40^3 + 10 \times 40 \times 15^2 \times 2 + \frac{1}{12} \times 40 \times 10^3$$

$$= 290 \times 10^3 \text{ mm}^4$$

$$I_{yz} = 2 \times 10 \times 30 \times 25 \times 20 = 300 \times 10^3 \text{ mm}^4$$

$$\theta_1 = \frac{1}{2} \tan^{-1} \left[ \frac{2 \times 300 \times 10^3}{560 \times 10^3 - 290 \times 10^3} \right] = 32.886^\circ$$

$$I_{\max/\min} = \frac{560 + 290}{2} \pm \sqrt{135^2 + 300^2}$$

$$I_{\max} = 753.976 \text{ in}^4$$

$$I_{\min} = 96.0243 \text{ in}^4$$



9-33

$$\begin{aligned}\sigma_B &= \frac{-4 \times 10^6 \times 5.14 \times 10^6}{3.89 \times 22.64 \times 10^{12} - 5.14^2 \times 10^{12}} \times 125.7 \\ &\quad + \frac{4 \times 10^6 \times 22.64 \times 10^6}{3.89 \times 22.64 \times 10^{12} - 5.14^2 \times 10^{12}} \times 4.3 \\ &= -0.3397 \times 125.7 + 1.4969 \times 24.3 \\ &= -36.3 \text{ MPa.}\end{aligned}$$

$$\begin{aligned}\sigma_F &= -0.3397 \times (-74.3) + 1.4969 \times 24.3 \\ &= 61.6 \text{ MPa}\end{aligned}$$

$$\tan \beta = \frac{4 \times 10^6 \times 22.64 \times 10^6}{4 \times 10^6 \times 5.14 \times 10^6} = 4.40$$

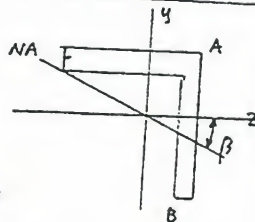
$$\therefore \beta = 77.2^\circ$$

$$\begin{aligned}\sigma_B &= 25.929 \times (-5.346) + 21.88 \times 1154 \\ &= -113.4 \text{ psi}\end{aligned}$$

9-34

$$\tan \beta = \frac{M_z I_{yz}}{M_z I_y}$$

$$= \frac{160 \times 10^3 \times 1.38 \times 10^6}{160 \times 10^3 \times 2.34 \times 10^6}$$



$\beta = 30^\circ$ , by inspection, A+B are the farthest perpendicular points from NA.

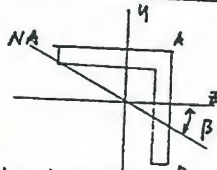
$$\begin{aligned}\sigma_A &= \frac{160 \times 10^3 \times 2.34 \times 10^6}{2.34^2 \times 10^{12} - 1.38^2 \times 10^{12}} \times 30.2 + \frac{160 \times 10^3 \times 1.38 \times 10^6}{2.34^2 \times 10^{12} - 1.38^2 \times 10^{12}} \times (-30.2) \\ &= 1.048 \times 30.2 - 0.6183 \times (-30.2)\end{aligned}$$

$$= 5.03 \text{ MPa}$$

$$\begin{aligned}\sigma_B &= 1.048 \times (-71.8) - 0.6183 \times (-30.2 + 12.7) \\ &= -6.45 \text{ MPa.}\end{aligned}$$

9-35

$$\begin{aligned}M_z &= 200 \times 0.8 \\ &= 160 \text{ N}\cdot\text{m} \\ &= 1416 \text{ lb}\cdot\text{in}\end{aligned}$$



$$\tan \beta = \frac{-32.3}{38.8} \Rightarrow \beta = -39.8^\circ, \text{ by inspection, } B$$

A+B are the farthest perpendicular points from NA.

$$\begin{aligned}\sigma_A &= \frac{1416 \times 38.8}{80.8 \times 38.8 - 32.3^2} \times 2.654 + \frac{-1416 \times (-32.3)}{80.8 \times 38.8 - 32.3^2} \times 1.654 \\ &= 25.929 \times 2.654 + 21.88 \times 1.654 = 105.0 \text{ psi}\end{aligned}$$

10-1  $A_{f_{ghi}} = 50 \times 150 = 7500 \text{ mm}^2$

$\bar{y}_1 = 25 + 50 + 25 = 100 \text{ mm}$

$A_{f_{gpn}} = 4 \times 50 \times 150 = 30000 \text{ mm}^2$

$\bar{y}_2 = 25 \text{ mm}$

$\bar{A}_{f_{ghi}} \bar{y}_1 = 750000 = \bar{A}_{f_{gpn}} \bar{y}_2$

10-2  $V = 250 \text{ lb}$

$I = \frac{1}{12} (6)(8)^3 = 256 \text{ in}^4$

$Q_1 = (6 \times 2)(3) = 36 \text{ in}^3$

$Q_2 = (6 \times 6)(1) = 36 \text{ in}^3$

$q = \frac{VQ}{I} = \frac{(250)(36)}{256} = 35.16 \text{ lb/in}$

$F_{n2,l} = \frac{35.16/2}{6} = 2.93 \text{ lb}$

10-3  $I = \frac{1}{12} (200)(250)^3 - \frac{1}{12} (100)(150)^3$   
 $= 232291667 \text{ mm}^4$

$Q = (100 \times 50)(100) = 500000 \text{ mm}^3$

①  $V = 3 \text{ kN}$

$q = \frac{VQ}{I} = \frac{(3000)(500000)}{(232291667)} = 6.46 \text{ N/mm}$

$S = \frac{2F}{q} = \frac{2(300)}{6.46} = 93 \text{ mm}$

②  $V = 2 \text{ kN}$

$q = \frac{VQ}{I} = \frac{(2000)(500000)}{(232291667)} = 4.3 \text{ N/mm}$

$S = \frac{2F}{q} = \frac{2(300)}{4.3} = 139.5 \text{ mm}$

10-4 (a) Take the design that has the smaller  $Q \rightarrow$  take design (a)

(b)  $Q = (2 \times 6)(4) = 48 \text{ in}^3$

$I = \frac{1}{12} (10)^4 - \frac{1}{12} (6)^4 = 725.33 \text{ in}^4$

$q = \frac{(800)(48)}{(725.33)} = 53 \text{ lb/in}$

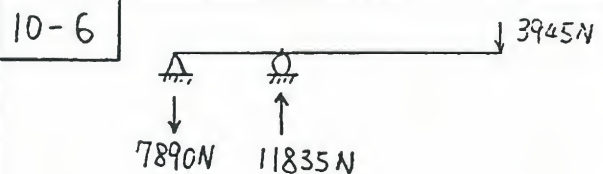
$S = \frac{2(140)}{53} = 5.3 \text{ in}$

10-5  $I = \frac{1}{12} (200)(240)^3 - \frac{1}{12} (188)(208)^3$   
 $+ \frac{2}{12} (240)(20)^3 + 2(240 \times 20)(130)^2$   
 $= 251977045 \text{ mm}^4$

$Q = (240 \times 20)(130) = 624000 \text{ mm}^3$

$q = \frac{VQ}{I} = \frac{(1.5 \times 10^6)(624000)}{(251977045)} = 3715 \text{ N/mm}$

$S = \frac{2(120 \times 10^3)}{3715} = 64.6 \text{ mm}$



(a)  $Q = (250 \times 50)(150) = 1875000 \text{ mm}^3$

$q = \frac{VQ}{I} = \frac{(7890)(1875000)}{(740 \times 10^6)} = 20 \text{ N/mm}$

$S = \frac{2(400)}{20} = 40 \text{ mm}$

(b)  $Q = (150 \times 50)(50) = 375000 \text{ mm}^3$

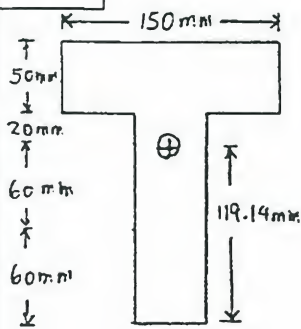
$q = \frac{VQ}{I} = \frac{(7890)(375000)}{(740 \times 10^6)} = 4 \text{ N/mm}$

$S = \frac{2(400)}{4} = 200 \text{ mm}$

<p>10-7 <math>I = 2(34.4 \times 10^6 + 4570 \times 100.5^2)</math>  <math>= 161 \times 10^6 \text{ mm}^4</math></p> <p><math>M = \frac{\sigma I}{y} = \frac{(160 \times 10^3)(161 \times 10^{-6})}{(0.201)} = 128 \text{ kN-m}</math></p> <p><math>q = \frac{2F}{S} = \frac{2(90)}{(0.15)} = 1200</math></p> <p><math>V = \frac{qI}{Q} = \frac{(1200)(161 \times 10^{-6})}{(4570 \times 100.5 \times 10^{-9})} = 420 \text{ kN}</math></p>	<p>10-10 <math>Q = (0.05 \times 0.3)(0.35) + (0.025 \times 0.25)(0.2)</math>  <math>= 0.0065 \text{ m}^3</math></p> <p><math>q = \frac{(450)(0.0065)}{(4300 \times 10^{-6})} = 680.23 \text{ kN/m}</math></p> <p><math>\tau = \frac{qS}{2A} = \frac{(680.23 \times 10^{-3})(0.125)}{2(\pi \times 0.011^2)} = 112 \text{ MN/m}^2</math></p>
<p>10-8 <math>Q_A = (14 \times \frac{1}{2})(20.25) + 2(4.75)(19)</math>  <math>= 322.25 \text{ in}^3</math></p> <p><math>q_A = \frac{(150)(322.25)}{(14560)} = 3.32 \text{ k/in}</math></p> <p><math>S_A = \frac{11.3}{3.32} = 3.40 \text{ in}</math></p> <p><math>Q_B = (14 \times \frac{1}{2})(20.25) = 141.75 \text{ in}^3</math></p> <p><math>q_B = \frac{(150)(141.75)}{(14560)} = 1.46 \text{ k/in}</math></p> <p><math>S = \frac{2(6.63)}{1.46} = 9.08 \text{ in}</math></p>	<p>10-11 <math>q = \frac{(25 \times 10^3)(270 \times 184.8 \times 2)}{(47.6 \times 10^6)} = 52.4</math>  <math>(52.4)(30) = 1572</math></p> <p><math>F.S. = \frac{3560}{1572} = 2.26</math></p>
<p>10-9 <math>\bar{y} = \frac{(3930)(228.5 + 7.16 - 17.1)}{(9460 + 3930)} = 64 \text{ mm}</math></p> <p><math>I = (333 \times 10^6 + 9460 \times (64)^2) + (1.61 \times 10^6 + 3930 \times 232^2)</math>  <math>= 934 \times 10^6 \text{ mm}^4</math></p> <p><math>Q = (3930)(232) = 911760 \text{ mm}^3</math></p> <p><math>q = \frac{(250 \times 10^3)(911760)}{(934 \times 10^6)} = 244 \text{ N/mm}</math></p> <p><math>F = \frac{244/2}{150} = 0.8133 \text{ N}</math></p> <p><math>\tau = \frac{F}{A} = \frac{0.8133}{\frac{\pi}{4}(19 \times 10^{-3})^2} = 2870 \text{ Pa}</math></p>	<p>10-12 <math>A_{\text{rivets}} = \frac{\pi}{4}(4.76)^2 = 17.13 \text{ mm}^2</math></p> <p><math>I = 47.6 \times 10^6 + 2(100 \times 1.3)(200.65)^2 = 58 \times 10^6</math></p> <p><math>Q_A = (270 \times 184.8 \times 2) + (100 \times 1.3 \times 200.65) = 125876.5</math></p> <p><math>q_A = \frac{(25)(125876.5 \times 10^{-9})}{(58 \times 10^{-6})} = 54.3</math></p> <p><math>\tau_A = \frac{(54.3 \times 10^{-3})(30 \times 10^{-3})}{2(17.13 \times 10^{-6})} = 47.5 \text{ MPa}</math></p> <p><math>Q_B = 100 \times 1.3 \times 200.65 = 26084.5</math></p> <p><math>q_B = \frac{(25)(26084.5 \times 10^{-9})}{(58 \times 10^{-6})} = 11.2</math></p> <p><math>\tau_B = \frac{(11.2 \times 10^{-3})(30 \times 10^{-3})}{2(17.13 \times 10^{-6})} = 9.8 \text{ MPa}</math></p>
<p>10-13 <math>\tau_{\max} = \frac{VQ}{It} = \frac{V(\frac{\pi}{2}r^2 \times \frac{4r}{3\pi})}{(\frac{\pi}{4}r^4)(2r)} = \frac{4}{3} \frac{V}{A}</math></p> <p>10-14 <math>\tau_{\max} = \frac{VQ}{It} = \frac{V(\pi r t \times \frac{2r}{\pi})}{(\pi r^3 t)(2t)} = \frac{2V}{2\pi r t} = \frac{2V}{A}</math></p>	



10-15



$$\bar{y}_b = \frac{(50 \times 150 \times 165 + 50 \times 140 \times 70)}{(50 \times 150 + 50 \times 140)} = 119.14 \text{ mm}$$

$$I = \frac{1}{12} \times 150 \times 50^3 + 150 \times 50 \times 45.86^2 + \frac{1}{12} \times 50 \times 140^3 + 50 \times 140 \times 49.14^2 = 45.67 \times 10^6$$

$$\tau_1 = 0$$

$$\tau_2 = \frac{(0.24)(0.15 \times 0.025 \times 0.05836)}{(45.67 \times 10^{-6})(0.15)} = 7.67 \text{ MPa}$$

$$\tau_3 = \frac{(0.24)(0.15 \times 0.05 \times 0.04586)}{(45.67 \times 10^{-6})(0.15 \text{ or } 0.05)} = 12.05 \text{ MPa or } 36.15 \text{ MPa}$$

$$\tau_4 = \frac{(0.24)(0.05 \times 0.12 \times 0.05914)}{(45.67 \times 10^{-6})(0.05)} = 37.29 \text{ MPa}$$

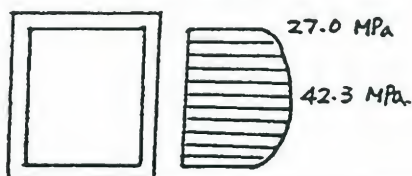
$$\tau_5 = \frac{(0.24)(0.05 \times 0.06 \times 0.08914)}{(45.67 \times 10^{-6})(0.05)} = 28.11 \text{ MPa}$$

$$\tau_6 = 0$$

$$10-16 \quad I = \frac{1}{12} (200 \times 300^3 - 150 \times 250^3) = 254.7 \times 10^6$$

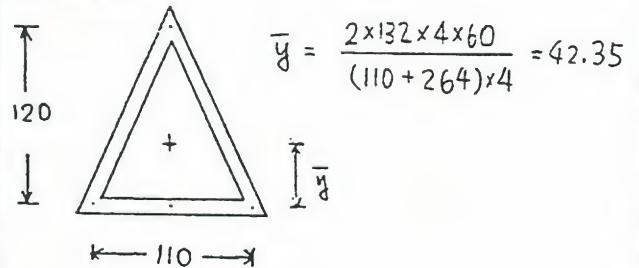
$$\tau_{\max} = \frac{VQ}{It} = \frac{500 \times 10^3 \times (25 \times 200 \times 137.5 + 2 \times 25 \times 250 \times 6.3)}{254.7 \times 10^6 \times 50} = 42.3 \text{ MPa}$$

$$\tau_{\min} = \frac{VQ}{It} = \frac{500 \times 10^3 \times (25 \times 200 \times 137.5)}{254.7 \times 10^6 \times 50} = 27.0 \text{ MPa}$$



10-17

$$L = (55^2 + 120^2)^{1/2} = 132 \text{ mm}$$



$$I = 2 \left( \frac{1}{12} \times 4 \times 132 \times 120^2 + 4 \times 132 \times 17.65^2 \right) + \frac{1}{12} \times 100 \times 4^3 + 100 \times 4 \times 42.35^2 = 2314110 \text{ mm}^4$$

midheight:

$$Q_1 = \left( \frac{132}{2} \times 4 \right) (90 - 42.35) = 12579.6 \text{ mm}^3$$

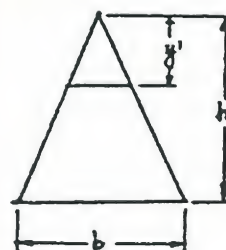
$$\tau_1 = \frac{(100 \times 10^{-3})(12579.6 \times 10^{-9})}{(2314110 \times 10^{-12})(4 \times 10^{-3})} = 136 \text{ MPa}$$

centroidal level:

$$Q_2 = \left[ 132 \times \frac{(120 - 42.35)}{120} \times 4 \right] \frac{(120 - 42.35)}{2} = 13264.95 \text{ mm}^3$$

$$\tau_2 = \frac{(100 \times 10^{-3})(13264.95 \times 10^{-9})}{(2314110 \times 10^{-12})(4 \times 10^{-3})} = 143 \text{ MPa}$$

10-18



$$(a) \tau = \frac{VQ}{It}, \text{ find } \left( \frac{Q}{t} \right)_{\max}$$

$$\frac{Q}{t} = \frac{\frac{1}{2} \left( \frac{1}{2} y' \right) y' \left( \frac{2}{3} h - \frac{2}{3} y' \right)}{\frac{1}{2} y'}$$

$$= \frac{1}{3} (hy' - y'^2)$$

$$\frac{d \left( \frac{Q}{t} \right)}{dy'} = 0 = \frac{1}{3} (h - 2y')$$

$$y' = \frac{1}{2} h$$

$$(b) V = \frac{\tau I t}{Q} = \frac{(100) \left( \frac{1}{36} \times 25 \times 50^3 \times 12.5 \right)}{\frac{1}{2} \times 12.5 \times 25 \times \frac{50}{3}} = 41667 \text{ N}$$



$$10-19 \quad I = 2 \left( \frac{1}{12} \right) (0.16)^4 = 1.092 \times 10^{-4} \text{ m}^4$$

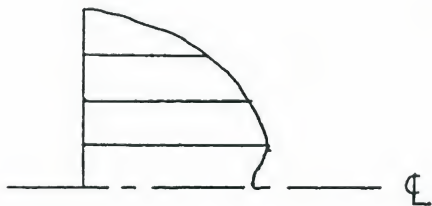
$$\tau_1 = 0$$

$$\tau_2 = \frac{(5) \left( \frac{1}{2} \times 0.04^2 \times \frac{0.4}{3} \right)}{(1.092 \times 10^{-4}) (0.04)} = 120 \text{ kPa}$$

$$\tau_3 = \frac{(5) \left( \frac{1}{2} \times 0.08^2 \times \frac{0.32}{3} \right)}{(1.092 \times 10^{-4}) (0.08)} = 195 \text{ kPa}$$

$$\tau_4 = \frac{(5) \left( \frac{1}{2} \times 0.12^2 \times \frac{0.08}{3} \right)}{(1.092 \times 10^{-4}) (0.12)} = 220 \text{ kPa}$$

$$\tau_5 = \frac{(5) \left( \frac{1}{2} \times 0.16^2 \times \frac{0.16}{3} \right)}{(1.092 \times 10^{-4}) (0.16)} = 195 \text{ kPa}$$



$$10-20 \quad V = \frac{dM}{dx} = \frac{3+2}{10} = 0.5 \text{ k}$$

$$\bar{y}_b = \frac{4 \times 6 \times 5 + 2 \times 8 \times 1}{4 \times 6 + 2 \times 8} = 3.4 \text{ in}$$

$$I = \frac{1}{12} \times 4 \times 6^3 + 4 \times 6 \times 1.6^2$$

$$+ \frac{1}{12} \times 2 \times 8^3 + 8 \times 2 \times 2.4^2$$

$$= 230.9 \text{ in}^4$$

$$(a) Q = 2 \times 6 \times 1.6 = 19.2 \text{ in}^3$$

$$q = \frac{VQ}{I} = \frac{(500)(19.2)}{(230.9)} = 41.58 \text{ lb/in}$$

$$F = (41.58)(12) = 499 \text{ lb}$$

$$(b) Q = 8 \times 2 \times 2.4 = 38.4 \text{ in}^3$$

$$\tau = \frac{VQ}{I t} = \frac{(500)(38.4)}{(230.9)(4)} = 20.79 \text{ lb/in}^2$$

$$10-21 \quad I = \frac{1}{12} (75)(200)^3 - 2 \left[ \frac{\pi}{4} (25)^4 + \pi (25)^2 (40)^2 \right]$$

$$= 43.1 \times 10^6 \text{ mm}^4$$

$$Q = (75 \times 60 \times 70) - \frac{\pi}{2} (25)^2 (40 + \frac{4 \times 25}{3\pi}) = 265313$$

$$V = \frac{\tau I t}{Q} = \frac{(50)(43.1 \times 10^6)(25)}{265313} = 2.03 \times 10^5 \text{ N}$$

$$M = \frac{\sigma I}{c} = \frac{(100)(43.1 \times 10^6)}{(100)(10^3)} = 4.31 \times 10^4 \text{ N-m}$$

$$10-22 \quad \bar{y} = \frac{14 \times 2 \times (17+9) + 2 \times 4 \times (2+2+1)}{14 \times 2 \times 2 + 2 \times 4 \times 3}$$

$$= 9.6 \text{ in}$$

$$\tau = \frac{VQ}{I t} = \frac{(400)(2 \times 4 \times 7.6)}{(2640)(2)} = 4.6 \text{ psi}$$

$$F = \frac{VQs}{I} = \frac{(400)(2 \times 2 \times 4 \times 7.6 + 2 \times 4 \times 8.6)(6)}{2640}$$

$$= 173 \text{ lb}$$

$$10-23$$

$$\bar{y}_b = \frac{25 \times 150 \times 12.5 + 25 \times 200 \times 125 + 25 \times 200 \times 237.5}{25 \times 150 + 25 \times 200 + 25 \times 200}$$

$$= 135.23 \text{ mm}$$



$$\begin{aligned}
 I &= \frac{1}{12} (150)(25)^3 + (150)(25)(122.73)^2 \\
 &\quad + \frac{1}{12} (25)(200)^3 + (25)(200)(15.23)^2 \\
 &\quad + \frac{1}{12} (200)(25)^3 + (200)(25)(102.27)^2 \\
 &= 127 \times 10^6 \text{ mm}^4
 \end{aligned}$$

$$P_1 = \frac{\sigma I}{cL} = \frac{(150 \times 10^{-3})(127 \times 10^6)}{(135.23)(1000)} = 140.87 \text{ kN}$$

$$\begin{aligned}
 P_2 &= \frac{\tau I t}{Q} = \frac{(100 \times 10^{-3})(127 \times 10^6)(25)}{(150 \times 25 \times 122.73) + (25 \times 115.73 \times 57.615)} \\
 &= 507.02 \text{ kN}
 \end{aligned}$$

$$\begin{aligned}
 P_3 &= \frac{q I}{Q} = \frac{(2)(127 \times 10^6)}{(150 \times 25 \times 122.73) + (25 \times 50 \times 85.23)} \\
 &= 448.15 \text{ kN}
 \end{aligned}$$

flexure controls  $\rightarrow P_{max} = 140.87 \text{ kN}$

10-24

$$\begin{aligned}
 \bar{y}_b &= \frac{2 \times 20 \times 400 \times 200 + 100 \times 80 \times 40 + 100 \times 40 \times 380}{2 \times 20 \times 400 + (40 + 80) \times 100} \\
 &= 180
 \end{aligned}$$

$$\begin{aligned}
 I &= \frac{1}{12} (140 \times 400^3 - 100 \times 280^3) + 140 \times 400 \times 20^2 - \\
 &\quad 100 \times 280 \times 40^2 = 541 \times 10^6 \text{ mm}^4
 \end{aligned}$$

$$\begin{aligned}
 (a) \quad V_{PLY} &= \frac{\tau I t}{Q} = \frac{1.5 \times 541 \times 10^6 \times 40}{220 \times 140 \times 110 - 100 \times 180 \times 90} \\
 &= 18360 \text{ N} = 18.36 \text{ kN}
 \end{aligned}$$

$$\begin{aligned}
 (b) \quad V_{u-N} &= \frac{I f}{Q} = \frac{541 \times 10^6}{100 \times 40 \times 200} \quad f = 676.25 \text{ f} \\
 f &= \frac{2F}{S} = \frac{2 \times 500}{50} = 20 \text{ N/mm} \\
 V_{u-N} &= 676.25 \times 20 = 13525 \text{ N} \\
 &= 13.53 \text{ kN}
 \end{aligned}$$

$$(c) \quad f = \frac{2F}{S} = \frac{2 \times 500}{25} = 40 \text{ N/mm}$$

$$\begin{aligned}
 V_{b-N} &= \frac{I f}{Q} = \frac{541 \times 10^6 \times 40}{100 \times 80 \times 140} = 19321 \text{ N} \\
 &= 19.32 \text{ kN}
 \end{aligned}$$

upper nailing governs  $V_{max} = 13.53 \text{ kN}$

$$10-25 \quad I = \frac{1}{12} (80 \times 220^3 - 65 \times 140^3) = 56.12 \times 10^6$$

$$Q = 40 \times 80 \times 90 + 70 \times 15 \times 35 = 324750$$

$$V = \frac{\tau I t}{Q} = \frac{(2 \times 10^{-3})(56.12 \times 10^6)(15)}{(324750)} = 5.2 \text{ kN}$$

$$T = \frac{V Q}{I t} = \frac{(5.2 \times 10^3)(40 \times 80 \times 100 - 15 \times 12 \times 76)}{(56.12 \times 10^6)(2 \times 12 + 15)} = 0.73 \text{ MPa}$$

$$10-26 \quad I = \frac{1}{12} (125 \times 500^3 - 100 \times 300^3) = 1077 \times 10^6$$

$$M = \frac{\sigma I}{c} = \frac{(10^4)(1077 \times 10^6)}{(250 \times 10^{-3})} = 43.1 \text{ kN}\cdot\text{m}$$

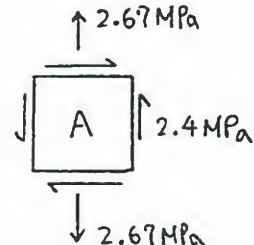
$$V_{glue} = \frac{\tau I b}{Q} = \frac{(300)(1077 \times 10^6)(0.1)}{(0.05 \times 0.1 \times 0.2)} = 32.3 \text{ kN}$$

$$V_{wood} = \frac{(600)(1077 \times 10^6)(0.025)}{(0.125 \times 0.1 \times 0.2 + 0.025 \times 0.15 \times 0.075)} = 5.8 \text{ kN}$$

$\Rightarrow V_{allowable} = 5.8 \text{ kN}$

$$10-27 \quad \sigma = \frac{M y}{I} = \frac{(10^4 \times 200)(15)}{\frac{1}{12} \times 40 \times 150^3} = 2.67 \text{ MPa}$$

$$T = \frac{V Q}{I t} = \frac{(10^4)(60 \times 40 \times 45)}{\frac{1}{12} \times 40 \times 150^3 \times 40} = 2.4 \text{ MPa}$$





10-28 W 14x90  $M_A = M_B = \frac{1}{2} \times 4 \times 20 \times 5 -$

$$\frac{1}{2} \times 4 \times 5^2 = 150 \text{ K} \cdot \text{ft}$$

$$V_A = V_B = \frac{1}{2} \times 4 \times 20 - 5 \times 4 = 20 \text{ K}$$

$$\sigma_A = \frac{My_1}{I} = \frac{(150 \times 12) \times 0.01}{999} = 0.0018 \text{ Ksi (tensile)}$$

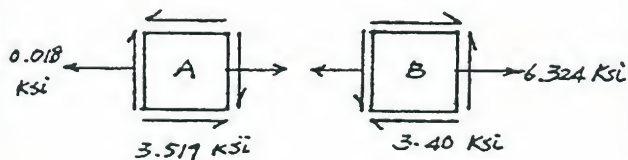
$$\sigma_B = \frac{My_2}{I} = \frac{(150 \times 12) \times 3.51}{999} = 6.324 \text{ Ksi (tensile)}$$

$$\tau_B = \frac{VQ}{It} = \frac{20}{999 \times 0.44} \left[ 14.52 \times 0.71 \times (7.01 - 0.355) + 2.79 \times 0.44 \times (3.51 + \frac{2.79}{2}) \right]$$

$$= 3.40 \text{ Ksi}$$

$$\tau_A = \frac{VQ}{It} = \frac{20}{999 \times 0.44} \left[ 14.52 \times 0.71 \times (7.01 - 0.355) + 6.29 \times 0.44 \times 3.155 \right]$$

$$= 3.519 \text{ Ksi}$$



10-29  $M_L = \frac{2}{5} \times 100 \times 1.2 = 48 \text{ K} \cdot \text{m}$

$$M_R = \frac{2}{5} \times 100 \times 1.3 = 52 \text{ K} \cdot \text{m}$$

$$I = \frac{1}{12} \times 0.2 \times 0.3^3 = 4.5 \times 10^{-4} \text{ m}^4$$

$$(\sigma_{\text{upper}})_L = (\sigma_{\text{upper}})_R = 0$$

$$(\sigma_{\text{bottom}})_L = \frac{My}{I} = \frac{(48 \times 10^{-3})(0.15)}{(4.5 \times 10^{-4})} = 16 \text{ MPa}$$

$$(\sigma_{\text{bottom}})_R = \frac{(52 \times 10^{-3})(0.15)}{(4.5 \times 10^{-4})} = 17.33 \text{ MPa}$$

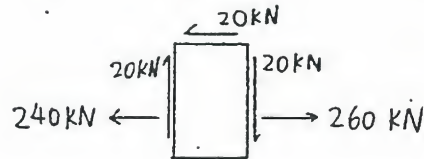
$$\tau_{\text{upper}} = \frac{VQ}{It} = \frac{(\frac{2}{5} \times 0.1)(0.2 \times 0.15 \times 0.075)}{(4.5 \times 10^{-4})(0.2)} = 1 \text{ MPa}$$

$$V_1 = \tau_{\text{upper}} \times 100 \times 200 = 20 \text{ K}$$

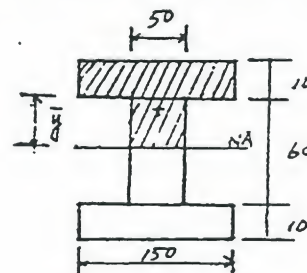
$$F_L = \frac{1}{2} \times (\sigma_{\text{bottom}})_L \times 150 \times 200 = 240 \text{ K}$$

$$F_R = \frac{1}{2} \times (\sigma_{\text{bottom}})_R \times 150 \times 200 = 260 \text{ K}$$

$$V_2 = V_3 = \frac{(F_R - F_L) \times 100}{100} = 20 \text{ K}$$



10-30



transformed section

$$I = \frac{1}{12} (150 \times 80^3 - 100 \times 60^3) = 4.6 \times 10^6 \text{ mm}^4$$

$$\tau = \frac{VQ}{It} = \frac{5 \times 10^3 \times (150 \times 10 \times 35 + 50 \times 30 \times 15)}{4.6 \times 10^6 \times 50}$$

$$= 1.63 \text{ MPa}$$

10-31

$$n = E_w / E_f = 2$$

$$t_w = 12 \times 2 = 24$$

$$\bar{y}_b = \frac{(50 \times 324 \times 475 + 48 \times 450 \times 225 + 50 \times 100 \times 25)}{50 \times 324 + 48 \times 450 + 50 \times 100}$$

$$= 296.26 \text{ mm}$$

$$I = \frac{1}{12} (324 \times 500^3 - 100 \times 400^3 - 176 \times 450^3)$$

$$+ (324 \times 500 \times 46.26^2 - 100 \times 400 \times 46.26^2 - 176 \times 450 \times 71.26^2)$$

$$= 1364.07 \times 10^6 \text{ m}^4$$

$$(a) (\tau_w)_{\text{max}} = \frac{(20 \times 10^6)(50 \times 324 \times 178.74 + 48 \times 153.74 \times 78.87)}{(1364.07 \times 10^6)(48)}$$

$$= 1.06 \text{ MPa}$$

$$1b) \tau_{max} = \frac{(20 \times 10^3)(176 \times 50 \times 178.74)}{(1364.07 \times 10^6)(50+50)}$$

$$= 0.23 \text{ MPa}$$

$$10-32 \quad I = \frac{1}{12} \times 56 \times 40^3 - 2 \left( \frac{1}{12} \times 10 \times 36^3 + 10 \times 36 \times 2^2 \right)$$

$$- \left( \frac{1}{12} \times 20 \times 36^3 + 20 \times 36 \times 2^2 \right)$$

$$= 137387 \text{ mm}^4$$

$$\tau_1 = \frac{(10^4)(4 \times 20 \times 10)}{(137387)(4)} = 14.56 \text{ MPa}$$

$$\tau_2 = \frac{(10^4)(4 \times 20 \times 10 + 4 \times 10 \times 18)}{(137387)(4)} = 27.66 \text{ MPa}$$

$$10-33(a) \quad I = 2 \times \frac{1}{12} (3 \times 100 \times 80^3) + 2 \times \frac{1}{12} (3 \times 80^3) + 2 \times \frac{1}{12} \times 130 \times 3^3 + 2 \times 130 \times 3 \times 40^2$$

$$= 1.82 \times 10^6 \text{ mm}^4$$

$$(b) \quad \tau_{aa} = \frac{VQ}{It} = \frac{10 \times 10^3 \times (50 \times 3 \times 20)}{1.82 \times 10^6 \times 3}$$

$$= 5.49 \text{ MPa}$$

$$\tau_{bb} = \frac{10 \times 10^3 \times (35 \times 3 \times 40)}{1.82 \times 10^6 \times 3} = 7.69 \text{ MPa}$$

$$\tau_{cc} = \frac{10 \times 10^3 \times (65 \times 3 \times 40 + 40 \times 3 \times 20)}{1.82 \times 10^6 \times 3}$$

$$= 18.68 \text{ MPa}$$

10-34

$$\tau_{aa} = \frac{VQ}{It} = 0.006 \times \frac{1}{20} \times (200 \times 20 \times 62.1)$$

$$= 74.52 \text{ MPa}$$

$$\tau_{bb} = \frac{VQ}{It} = 0.006 \times \frac{1}{25} \times (400 \times 20 \times 62.1 + 62.1 \sqrt{2} \times 25 \times \frac{62.1}{2})$$

$$= 135.6 \text{ MPa}$$

10-35

angles  $\tau_{aa} = \frac{VQ}{It} = 0.01 \times \frac{10 \times 4 \times 25}{4}$

$$= 2.5 \text{ MPa}$$

plate  $\tau_{aa} = 0.01 \times \frac{40 \times 4 \times 48 + 2 \times 30 \times 4 \times 35 + 20 \times 4 \times 40}{4}$

$$= 48.2 \text{ MPa}$$

10-36

Pipe:  $\frac{A}{2} = 1025 \text{ mm}^2$ ,  $t = 6.02 \text{ mm}$

$$\bar{R} = \frac{1}{4} (114 + 102) = 54, \quad \bar{y} = 100 + \frac{2\bar{R}}{\pi} = 134.38 \text{ mm}$$

$$I = \frac{1}{12} \times 10 \times 260^3 + 2(2.98 \times 10^6 + 2050 \times 100^2)$$

$$= 61.6 \times 10^6$$

$$\tau_{\text{pipe}} = \frac{(200 \times 10^3)(1025 \times 134.38)}{(61.6 \times 10^6)(2 \times 6.02)} = 37 \text{ MPa}$$

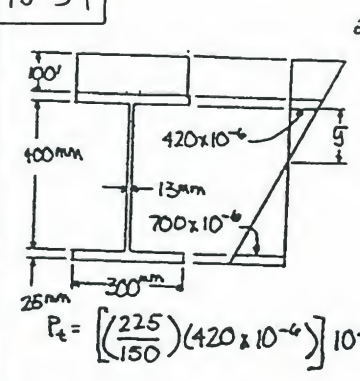
$$\tau_{\text{web}} = \frac{(200 \times 10^3)(10 \times 30 \times 115)}{(61.6 \times 10^6)(10)} = 11 \text{ MPa}$$

10-37

a)  $\bar{y} = \frac{400}{420 + 700}$

$$\bar{y} = 150 \text{ mm}$$

$P_c = \sigma_{AVE} A_c$   
 $\sigma_{AVE} = \epsilon_{AVE} E_c$



$$P_t = \left[ \left( \frac{225}{150} \right) (420 \times 10^{-6}) \right] 10^4 (1)(1.3) = 1.89 \text{ MN}$$

$$= 1890 \text{ kN}$$

b) Assume each bolt carries equal force

$$\# \text{ of rows} = \frac{5}{.6} = 8.33 = 8 \quad \# \text{ of bolts} = 16$$

$$\text{Force/bolt} = \frac{1890}{16} = 118 \text{ kN/bolt}$$

10-38

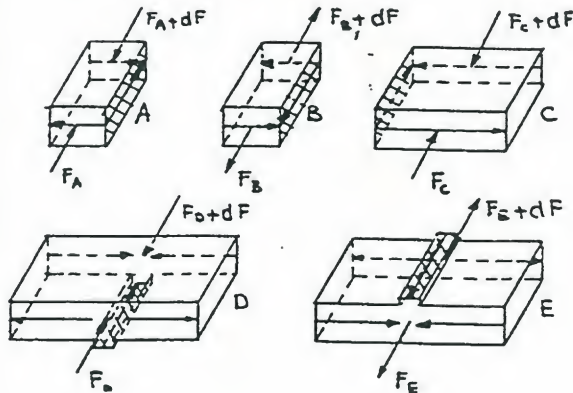
$$I = \frac{1}{12} [ .15(.24)^3 + .14(.2)^3 ] = 7.95 \times 10^{-5} \text{ m}^4$$

$$\tau_A = \tau_B = \frac{VQ}{I} = \frac{100(.03)(.02)(.11)}{7.95 \times 10^{-5}} = 83.0 \text{ kN/m}$$



$$f_c = \frac{100(.07)(.02)(.11)}{7.95 \times 10^{-5}} = 194 \text{ KN/m}$$

$$f_D = f_E = \frac{100(.15)(.02)(.11)}{7.95 \times 10^{-5}} = 415 \text{ KN/m}$$

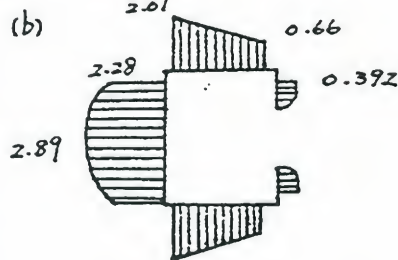


10-39 (a)  $\bar{I}_{a-a} = \frac{VQ}{It} = \frac{7(1)(\frac{1}{2} \times 2)}{35.7(\frac{1}{2})} = 0.392 \text{ ksi}$

$$\bar{I}_{bb} = \frac{7[1(\frac{1}{2} \times 2) + 3(\frac{1}{2})(2.75)]}{35.7(\frac{1}{2})} = 2.01 \text{ ksi}$$

$$\bar{I}_{c-c} = \frac{7[1(\frac{1}{2})(2) + 3(\frac{1}{2})(2.75) + 3(\frac{1}{2})(1.5)]}{35.7(\frac{1}{2})}$$

$$= 2.89 \text{ ksi}$$



10-40  $I = \frac{1}{12}(.002)(.05)^3 + 2(.002)(.026 \times .025)^2$   
 $= 8.58 \times 10^{-8} \text{ m}^4$

$$V_e = F_A h_A - F_L h_L, h_A = h_L, \bar{y}_1 = \bar{y}_2 = .025 \text{ m}$$

$$F_A = \frac{V A_1 \bar{y}_1}{2 I t} A_1 = \frac{V(.002)(.02)^2(.025)}{2(8.58 \times 10^{-8})}$$

$$F_L = \frac{V(.002)(.006)^2(.025)}{2(8.58 \times 10^{-8})}$$

$$e = \frac{.002(.025)(.05)}{2(8.58 \times 10^{-8})} (.02^2 - .006^2)$$

$$= .00530 \text{ m} = 5.30 \text{ mm from } \phi \text{ vert. member}$$

10-41

$$I = \frac{1}{12}(.002)(.1)^3 + 2(.002 \times .04)(.04^2 + .05^2)$$

$$= 8.23 \times 10^{-7} \text{ m}^4$$

$$V_e = h_1 F_1 + h_2 F_2, A_1 = t b = A_2$$

$$F_1 = \frac{V A_1 \bar{y}_1}{2 I t} A_1, F_2 = \frac{V A_2 \bar{y}_2}{2 I t} A_2$$

$$e = \frac{t b^2}{4 I} (h_1^2 + h_2^2) = \frac{.002(.04)^2}{4(8.23 \times 10^{-7})} (.08^2 + .1^2)$$

$$= .0159 \text{ m} = 15.9 \text{ mm}$$

(from  $\phi$  of vert. member)

10-42 from problem 10-41,  $I = 8.23 \times 10^{-7}$

$$e = \frac{(0.002)(0.04)^2}{4(8.23 \times 10^{-7})} (0.1^2 - 0.08^2)$$

$$= 0.0035 \text{ m} = 3.5 \text{ mm}$$

(from  $\phi$  of vertical member)

10-40

$$I = \frac{1}{12}(.002)(.05)^3 + 2(.002)(.026 \times .025)^2$$

$$= 8.58 \times 10^{-8} \text{ m}^4$$

$$V_e = F_A h_A - F_L h_L, h_A = h_L, \bar{y}_1 = \bar{y}_2 = .025 \text{ m}$$

10-43

$$Q_z = \int_{-\alpha}^{\alpha} R \sin \theta dA$$

$$= \int_{-\alpha}^{\alpha} R \sin \theta R t d\theta$$

$$= R^2 t (\cos \theta - \cos \alpha)$$

$$I = \int_{-\alpha}^{\alpha} R^2 \sin^2 \theta R t d\theta = R^3 t \left[ \alpha - \frac{\sin 2\alpha}{2} \right]$$



$$\tau = \frac{VQ}{It} = \frac{VR^3x(\cos\theta - \cos\alpha)}{R^3x^2[\alpha - \sin\frac{2\alpha}{2}]}$$

$$= \frac{V(\cos\theta - \cos\alpha)}{Rx[\alpha - \sin\frac{2\alpha}{2}]}$$

$$\text{Torque about } O = T = \int_{-\alpha}^{\alpha} \tau R dA$$

$$= \int_{-\alpha}^{\alpha} RV \frac{\cos\theta - \cos\alpha}{[\alpha - \frac{\sin 2\alpha}{2}]} d\theta$$

$$= \frac{2R(\sin\alpha - \alpha\cos\alpha)}{\alpha - \sin\alpha\cos\alpha}$$

$$e = \frac{T}{V} = \frac{2R(\sin\alpha - \alpha\cos\alpha)}{\alpha - \sin\alpha\cos\alpha}$$

10-44

$$I = \left[ \frac{1}{12} \times 4 \times 40^3 + 4 \times 40 \times 80^2 + \frac{1}{12} \times 4^3 \times 100 + 4 \times 100 \times 100^2 + \frac{1}{12} \times 4 \times 125 \times 100^2 + 125 \times 4 \times 50^2 \right] \times 2$$

$$= 13.43 \times 10^6 \text{ mm}^4$$

$$\tau_1 = \frac{V}{4I} Q = \frac{V(4 \times a \times (60 + \frac{a}{2}))}{4I}$$

$$= (60a + \frac{a^2}{2}) \frac{V}{I}$$

$$F_1 = \int \tau_1 dA = \int_0^{40} (60a + \frac{a^2}{2}) \frac{V}{I} \times 4 \times da$$

$$= 0.0103 V$$

$$\tau_2 = \frac{V}{4I} (40 \times 4 \times 80 + 4 \times a \times 100)$$

$$= (3200 + 100a) \frac{V}{I}$$

$$F_2 = \int \tau_2 dA = \int_0^{100} (3200 + 100a) \frac{V}{I} \times 4 \times da$$

$$= 0.2442 V$$

$$T = 2F_1 \times 175 + 2F_2 \times 100$$

$$= 2 \times 0.0103 V \times 175 + 2 \times 0.2442 V \times 100$$

$$= 52.45 V$$

$$e = \frac{T}{V} = 52.45 \text{ mm from left corner}$$

10-45  $\sigma_x = \frac{M_z(I_y z - I_{yz} y)}{I_y I_z - I_{yz}^2}$

$$\tau = \frac{dF}{x dx} = \frac{d(\int \sigma_x dA)}{x dx}$$

$$= \frac{dM_z}{dx} \frac{1}{x(I_y I_z - I_{yz}^2)} (I_{yz} \int z dA - I_y \int y dA)$$

$$= \frac{V_y}{x(I_y I_z - I_{yz}^2)} [I_{yz} \int z dA - I_y \int y dA]$$

take the absolute value

$$\tau = \frac{V_y (I_y Q_z - I_{yz} Q_y)}{x (I_y I_z - I_{yz}^2)} = \frac{10(0.533 Q_z - 0.8 Q_y)}{0.1(0.533 \times 2.133 - 0.8^2)}$$

$$= 107.3 Q_z - 161 Q_y \text{ psi}$$

$$\text{Flanges: } Q_y = xt(b - \frac{S}{2}) = 0.25 - 0.05 S^2$$

$$Q_z = xt \frac{h}{2} = 0.2 S$$

$$\tau_f = 107.3(0.25) - 161(0.25 - 0.05 S^2)$$

$$= 8.05 S^2 - 10.74 S$$

$$F_f = \int \tau_f dA$$

$$= \int_0^2 (8.05 S^2 - 10.74 S) \times 0.1 \times ds$$

$$= 0$$

$$\text{Web: } Q_y = t \times b \times \frac{b}{2} = 0.2$$

$$Q_z = t \times b \times \frac{b}{2} + t \times S \times (\frac{h}{2} - \frac{S}{2})$$

$$= -0.05 S^2 + 0.2 S + 0.4$$

$$\tau_b = 107.3(-0.05 S^2 + 0.2 S + 0.4)$$

$$- 161(0.2)$$

$$= -5.365 S^2 + 21.46 S + 10.72$$

$$F_b = \int \tau_b dA$$

$$= \int_0^2 (-5.365 S^2 + 21.46 S + 10.72) \times 0.1 \times ds$$

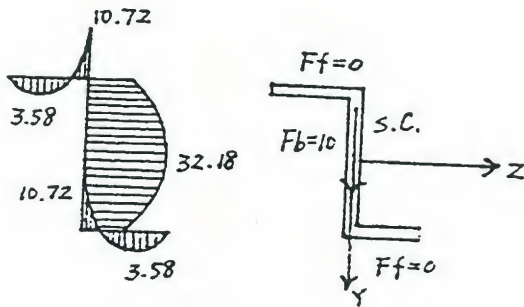
$$= 5.0 \text{ lb}$$

$$F_b = 10 \text{ lb}$$

Torque about centroid  $T = 0 \times 10 = 0$

$$\therefore e = 0$$

Shear center passes the centroid.



$$Z_b = 429.3 \times 0.2 -$$

$$161(-0.05S^2 + 0.2S + 0.4) \\ = 8.05S^2 - 32.2S + 21.46$$

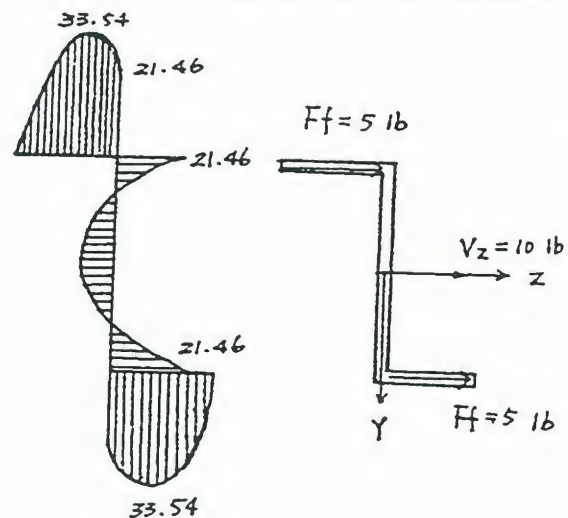
$$F_b = \int Z_b dA$$

$$= \int_0^2 (8.05S^2 - 32.2S + 21.46) \times 0.1 dS \\ = 0$$

Torque about centroid

$$eV = F_f \cdot \frac{h}{2} - F_f \cdot \frac{h}{2} = 0 \quad e = 0$$

Shear center passes the centroid



10-46 
$$\sigma_y = \frac{M_y (I_z \cdot z - I_{yz} y)}{I_y I_z - I_{yz}^2}$$

the same as Prob. 7-41

$$Z = \frac{V_z (I_z Q_y - I_{yz} Q_z)}{(I_y I_z - I_{yz}^2) t} \\ = \frac{10 (2.133 Q_y - 0.8 Q_z)}{0.1 (0.533 \times 2.133 - 0.8)^2} \\ = 429.3 Q_y - 161 Q_z \text{ psi}$$

Flanges: from Prob. 7-41

$$Q_y = 0.25 - 0.05 S^2$$

$$Q_z = 0.2 S$$

$$\therefore Z_f = 429.3 (0.05 S^2 - 0.25) - \\ 161 (-0.2) S \\ = 53.66 S - 21.46 S^2$$

$$F_f = \int Z_f dA \\ = \int_0^2 (53.66 S - 21.46 S^2) \times 0.1 dS \\ = 5.0 \text{ lb.}$$

Web:  $Q_y = 0.2$

$$Q_z = -0.05 S^2 + 0.25 + 0.4$$

10-47 
$$I = 2A (a \sin \alpha)^2$$

$$q = \frac{VQ}{I} = \frac{VA (a \sin \alpha)}{2A (a \sin \alpha)^2} = \frac{V}{2a \sin \alpha}$$

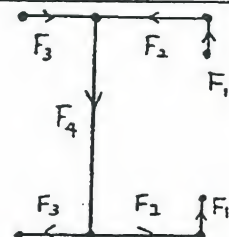
$$P_e = V_e = \frac{2\pi R (2\alpha)}{2\pi} a q = \frac{VaR}{\sin \alpha}$$

$$e = \frac{d}{\sin \alpha} R \text{ from } 0$$

10-48

$$I = (3A \times 75^2 + A \times 50^2) \times 2$$

$$= 38750A$$



$$F_1 = \frac{VQ}{I} \times S = \frac{V(50A)}{(38750A)} (25) = \frac{2V}{62}$$

$$F_2 = \frac{V(50A+75A)}{(38750A)} (75) = \frac{15V}{62}$$

$$F_3 = \frac{V(75A)}{(38750A)} (50) = \frac{6V}{62}$$

$$eV = 2F_1 \times 75 + F_2 \times 150 - F_3 \times 150$$

$$e = \frac{1}{62} (300 + 2250 - 900) = 26.61 \text{ mm}$$

(left from  $\bar{C}$  of vertical member)

$$\tau_B = 0$$

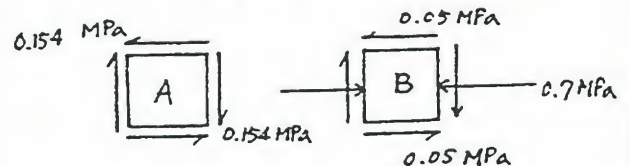
due to torsion

$$A_m = (20-2)(30-2) = 504 \text{ mm}^2$$

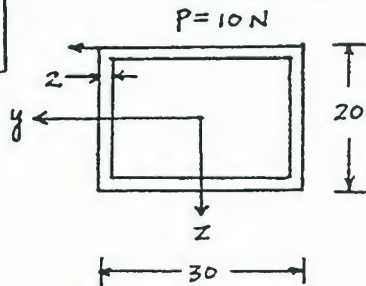
$$\tau_A = \tau_B = \frac{T}{2A_m t} = \frac{100}{2 \times 504 \times 2} = 0.05 \text{ MPa}$$

$$\text{total } \tau_A = 0.05 + 0.104 = 0.154 \text{ MPa}$$

$$\tau_B = 0.05$$



10-49



$$V = 10 \text{ N}, M = 10 \times 100 = 1000 \text{ N}\cdot\text{mm}$$

$$T = 10 \times 10 = 100 \text{ N}\cdot\text{mm}$$

$$I_z = \frac{1}{12} (20 \times 30^3 - 16 \times 26^3) = 21565 \text{ in}^4$$

$$I_y = \frac{1}{12} (30 \times 20^3 - 26 \times 16^3) = 11125 \text{ in}^4$$

$$J = I_z + I_y = 32690 \text{ in}^4$$

due to bending

$$\sigma_A = \frac{My}{I_z} = 0$$

$$\sigma_B = \frac{My}{I_z} = \frac{1000 \times 15}{21565} = 0.70 \text{ MPa}$$

due to shear

$$\tau_A = \frac{VQ}{I_z t} = \frac{10(20 \times 15 \times 7.5 - 16 \times 13 \times 6.5)}{21565 \times 4}$$

$$= 0.104$$

10-50

$$M = 20 \times 100 = 2000 \text{ N}\cdot\text{mm}$$

$$T = 20 \times 13 = 260 \text{ N}\cdot\text{mm}$$

$$V = 20 \text{ N}$$

$$I = \frac{\pi}{4} (13^4 - 9^4) = 16500 \text{ mm}^4$$

$$J = 2I = 33000 \text{ mm}^4$$

due to bending

$$\sigma_A = \frac{My}{I} = 0$$

$$\sigma_B = \frac{2000 \times 13}{16500} = 1.58 \text{ MPa}$$

due to torsion

$$\tau_A = \tau_B = \frac{T}{2A_m t} = \frac{260}{2 \times \pi \times 11^2 \times 4} = 0.09 \text{ MPa}$$

due to shear

$$\tau_B = 0$$

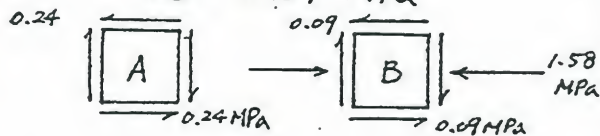
$$\tau_A = \frac{VQ}{I t} = \frac{20(\pi \times 13^2 \times \frac{2 \times 13}{3\pi} - \pi \times 9^2 \times \frac{2 \times 9}{3\pi})}{16500 \times 8}$$

$$= 0.15 \text{ MPa}$$



$$\text{total } \tau_A = 0.09 + 0.15 = 0.24 \text{ MPa}$$

$$\tau_B = 0.09 \text{ MPa}$$



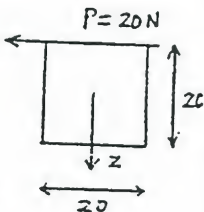
10-51

$$M = 20 \times 100 = 2000 \text{ N}\cdot\text{mm}$$

$$V = 20 \text{ N}$$

$$T = 20 \times 10 = 200 \text{ N}\cdot\text{mm}$$

$$I_z = \frac{1}{12} 20^4 = 13333 \text{ mm}^4$$



$$\alpha = 0.208 \text{ from Eq. 4-30 \& Table}$$

due to bending

$$\sigma_A = 0$$

$$\sigma_B = \frac{2000 \times 10}{13333} = 1.5 \text{ MPa}$$

due to torsion

$$\tau_A = \tau_B = \frac{T}{\alpha A^3} = \frac{200}{0.208 \times 20^3} = 0.12 \text{ MPa}$$

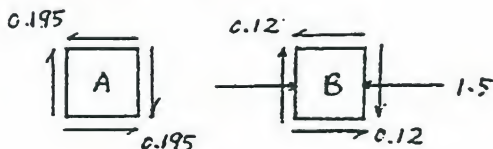
due to shear

$$\tau_A = \frac{VQ}{It} = \frac{20 \times 10 \times 20 \times 5}{13333 \times 20} = 0.075 \text{ MPa}$$

$$\tau_B = 0$$

$$\text{total } \tau_A = 0.12 + 0.075 = 0.195 \text{ MPa}$$

$$\tau_B = 0.12 \text{ MPa}$$



10-52

$$m = \frac{2\bar{r}}{d} = \frac{2[\frac{1}{2}(2) - \frac{1}{2}(\frac{1}{4})]}{\frac{1}{4}} = 7 \rightarrow K = 1.20$$

$$\Delta = \frac{64 F \bar{r}^3 N}{G d^4} = \frac{64(50)(.875)^3(8)}{11.6 \times 10^6 (.25)^4}$$

$$= .378 \text{ in.}$$

$$\tau_{\max} = K \frac{1.6 F \bar{r}}{\pi d^3} = \frac{1.2(16)(70)(.875)}{\pi (.25)^3}$$

$$= 24,000 \text{ psi}$$

10-53

$$K_1 = \frac{G d^4}{64 \bar{r}^3 N} = \frac{(82 \times 10^6)(0.006)^4}{64(0.012)^3(16)} = 60$$

$$K_2 = \frac{(82 \times 10^6)(0.008)^4}{64(0.016)^3(18)} = 71$$

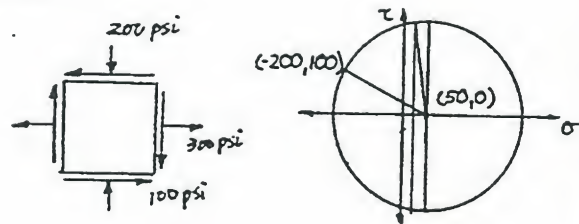
$$K = \frac{K_1 K_2}{K_1 + K_2} = \frac{60 \times 71}{60 + 71} = 32.52 \text{ kN/m}$$

$$m_1 = 2\bar{r}_1/d_1 = 2(12)/6 = 4 \rightarrow K = 1.37$$

$$m_2 = 2\bar{r}_2/d_2 = 2(16)/8 = 4 \rightarrow K = 1.37$$

$$\therefore F_{\max} = \frac{T_{\max} \pi d^3}{16 K \bar{r}} = \frac{(480 \times 10^3) \pi (0.006)^3}{16(1.37)(0.012)} = 1.24 \text{ kN}$$

10-54



$$\text{center} = \frac{1}{2}(300 - 200) = 50$$

$$\text{radius} = \sqrt{(300 - 50)^2 + 100^2} = 269$$

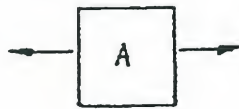
$$\theta = \tan^{-1} \frac{100}{200} + 2(30) = 86.6^\circ$$

$$\tau' = 269 \sin 86.6^\circ = 269 \text{ psi}$$

$$\tau_{\text{all}} = 150 \text{ psi} < 269 \text{ psi}$$

not permissible

11-1.



$$\sigma_A = \frac{P_1}{A}$$



$$\sigma_B = \frac{P_1}{A} + \frac{My}{I}$$



$$\sigma_C = \frac{P_1}{A}$$

$$\tau_C = \frac{VQ}{It}$$



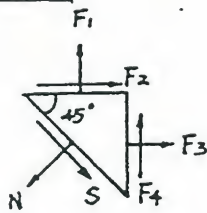
$$\sigma_D = \frac{My}{I}$$

$$\tau_D = \frac{T_r}{J}$$



$$\tau_E = \frac{T_r}{J} - \frac{P_1 \alpha}{I \pi}$$

11-2



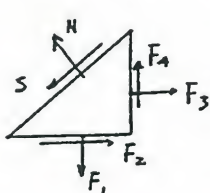
$$F_1 = 30 \times \frac{1}{\sqrt{2}}, F_2 = \frac{20}{\sqrt{2}}$$

$$F_3 = \frac{20}{\sqrt{2}}, F_4 = \frac{20}{\sqrt{2}}$$

$$N = \frac{1}{\sqrt{2}} \left( \frac{30}{\sqrt{2}} + \frac{20}{\sqrt{2}} + \frac{20}{\sqrt{2}} + \frac{20}{\sqrt{2}} \right) = 45$$

$$S = \frac{1}{\sqrt{2}} \left( \frac{30}{\sqrt{2}} + \frac{20}{\sqrt{2}} - \frac{20}{\sqrt{2}} - \frac{20}{\sqrt{2}} \right) = 5$$

$$\sigma = 45 \text{ MPa}, \tau = 5 \text{ MPa}$$



$$F_1 = \frac{30}{\sqrt{2}}$$

$$F_2 = \frac{20}{\sqrt{2}}$$

$$F_3 = \frac{20}{\sqrt{2}}$$

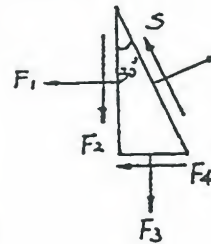
$$F_4 = \frac{20}{\sqrt{2}}$$

$$N = \frac{1}{\sqrt{2}} \left( \frac{30}{\sqrt{2}} - \frac{20}{\sqrt{2}} + \frac{20}{\sqrt{2}} - \frac{20}{\sqrt{2}} \right) = 5$$

$$S = \frac{1}{\sqrt{2}} \left( -\frac{30}{\sqrt{2}} - \frac{20}{\sqrt{2}} + \frac{20}{\sqrt{2}} + \frac{20}{\sqrt{2}} \right) = -5$$

$$\sigma = 5 \text{ MPa}, \tau = -5 \text{ MPa}$$

11-3



$$F_1 = \frac{\sqrt{3}}{2} \times 20 = 10\sqrt{3}$$

$$F_2 = \frac{\sqrt{3}}{2} \times 20 = 10\sqrt{3}$$

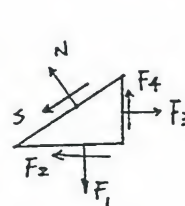
$$F_3 = \frac{1}{2} \times 30 = 15$$

$$F_4 = \frac{1}{2} \times 20 = 10$$

$$N = \frac{\sqrt{3}}{2} (10\sqrt{3} + 10) + \frac{1}{2} (10\sqrt{3} + 15) = 39.8$$

$$S = \frac{\sqrt{3}}{2} (10\sqrt{3} + 15) - \frac{1}{2} (10\sqrt{3} + 10) = 14.3$$

$$\sigma = 39.8 \text{ MPa}, \tau = 14.3 \text{ MPa}$$



$$F_1 = 30 \times \frac{\sqrt{3}}{2} = 15\sqrt{3}$$

$$F_2 = 20 \times \frac{\sqrt{3}}{2} = 10\sqrt{3}$$

$$F_3 = 20 \times \frac{1}{2} = 10$$

$$F_4 = 20 \times \frac{1}{2} = 10$$

$$N = \frac{\sqrt{3}}{2} (15\sqrt{3} - 10) + \frac{1}{2} (-10\sqrt{3} + 10)$$

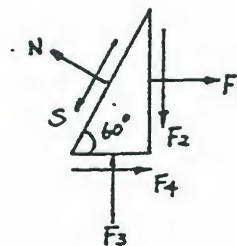
$$= 10.18$$

$$S = \frac{1}{2} (-15\sqrt{3} + 10) + \frac{\sqrt{3}}{2} (-10\sqrt{3} + 10)$$

$$= -14.33$$

$$\sigma = 10.18 \text{ MPa}, \tau = -14.33 \text{ MPa}$$

11-4



$$F_1 = \frac{\sqrt{3}}{2} \times 20 = 10\sqrt{3}$$

$$F_2 = \frac{\sqrt{3}}{2} \times 12 = 6\sqrt{3}$$

$$F_3 = \frac{1}{2} \times 10 = 5$$

$$F_4 = \frac{1}{2} \times 12 = 6$$

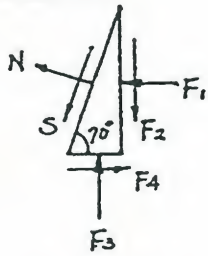
$$N = \frac{\sqrt{3}}{2} (10\sqrt{3} + 6) + \frac{1}{2} (6\sqrt{3} - 5) = 22.9$$

$$S = \frac{1}{2} (10\sqrt{3} + 6) + \frac{\sqrt{3}}{2} (5 - 6\sqrt{3}) = 6.99$$

$$\sigma = 22.9 \text{ ksi}, \tau = 6.99 \text{ ksi}$$



11-5



$$F_1 = 10 \times 0.940 = 9.40$$

$$F_2 = 6 \times 0.940 = 5.64$$

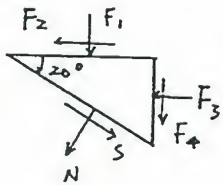
$$F_3 = 8 \times 0.342 = 2.74$$

$$F_4 = 6 \times 0.342 = 2.05$$

$$N = 0.94(-9.4 + 2.05) + 0.342(5.64 - 2.74) = -5.92$$

$$S = 0.342(-9.4 + 2.05) + 0.94(5.64 - 2.74) = -5.24$$

$$\bar{\sigma}_v = -5.92 \text{ ksi}, \tau = -5.24 \text{ ksi}$$



$$F_1 = 8 \times 0.940 = 7.52$$

$$F_2 = 6 \times 0.940 = 5.64$$

$$F_3 = 10 \times 0.342 = 3.42$$

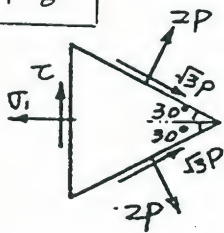
$$F_4 = 6 \times 0.342 = 2.052$$

$$N = 0.94(-7.52 - 2.052) + 0.342(-5.64 - 3.42) = -12.09$$

$$S = 0.342(-7.52 - 2.052) + 0.94(5.64 + 3.42) = 5.24$$

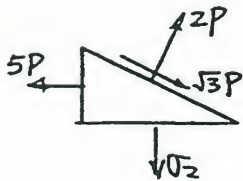
$$\sigma = -12.09 \text{ ksi}, \tau = 5.24 \text{ ksi}$$

11-6



$$\sigma_1 = 2 \times 2P \times 0.5 + 2 \times \sqrt{3}P \times \frac{1}{\sqrt{3}} = 5P$$

$$\tau = 0$$

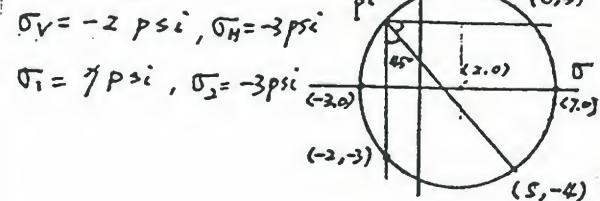


$$\sigma_2 = 2P - \sqrt{3}P \cdot \frac{1}{\sqrt{3}} = P$$

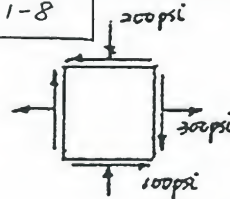
11-7

assume area of vertical plane  $A_v = 1$   
 then area of inclined plane  $A_i = \sqrt{2}$   
 $N = (\frac{5}{\sqrt{2}} - \frac{4}{\sqrt{2}}) \times \sqrt{2} - 3 = -2$   
 $S = (\frac{4}{\sqrt{2}} + \frac{5}{\sqrt{2}}) \times \sqrt{2} - 6 = 3$   
 $\sigma_v = -2/1 = -2 \text{ psi}, \tau_v = -3 \text{ psi}$

check by Mohr's circle



11-8



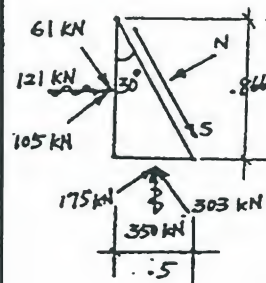
$$\theta = -60^\circ$$

$$\tau' = \frac{300 - (-200)}{2} \sin(-120^\circ) + (-100) \cos(-120^\circ) = 266.5 \text{ psi}$$

$$\tau_{allowable} = 150 \text{ psi} < 266.5 \text{ psi}$$

not permissible.

11-9



$$N = 105 + 175 = 280 \text{ kN}$$

$$S = 303 - 61 = 242 \text{ kN}$$

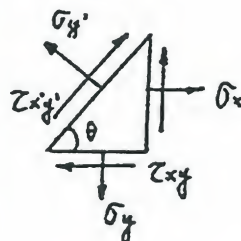
$$\therefore \tau = 242 \text{ kN/m}^2$$

$$\tau_{all} = 280(0.5) + 85 = 225 \text{ kN/m}^2$$

$$< \tau$$

$\therefore$  state of stress NOT permissible.

11-10

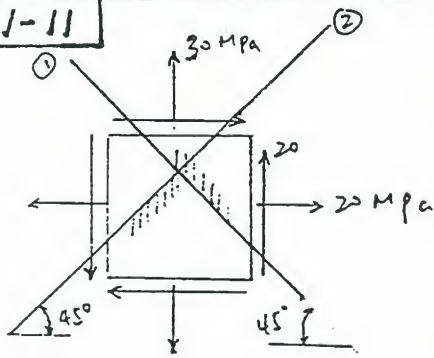


$$\Sigma F_{x'} = 0$$

$$\tau_{x'y'} = -(\sigma_x \cos \theta + \tau_{xy} \sin \theta) \sin \theta + (\sigma_y \sin \theta + \tau_{xy} \cos \theta) \cos \theta = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$



11-11

For plane ①  $\theta = -135^\circ$ 

$$\sigma_{x'} = \frac{30+20}{2} + \frac{20-30}{2} \cos(-270^\circ) + 20 \sin(-270^\circ)$$

$$= 45 \text{ MPa}$$

$$\tau_{x'y'} = -\frac{20-30}{2} \sin(-270^\circ) + 20 \cos(-270^\circ)$$

$$= 5 \text{ MPa}$$

For plane ②,  $\theta = 135^\circ$ 

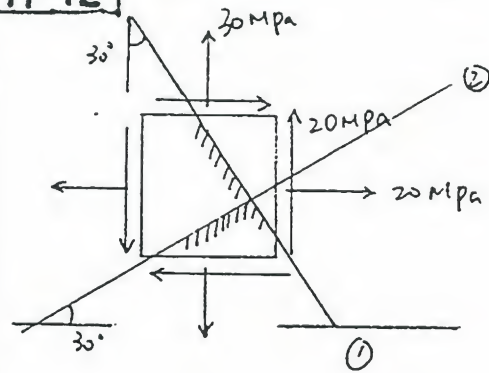
$$\sigma_{x'} = \frac{30+20}{2} + \frac{20-30}{2} \cos(270^\circ) + 20 \sin(270^\circ)$$

$$= 5 \text{ MPa}$$

$$\tau_{x'y'} = -\frac{20-30}{2} \sin(270^\circ) + 20 \cos(270^\circ)$$

$$= -5 \text{ MPa}$$

11-12

For plane ①,  $\theta = 30^\circ$ 

$$\sigma_{x'} = \frac{30+20}{2} + \frac{20-30}{2} \cos 60^\circ + 20 \sin 60^\circ$$

$$= 39.8 \text{ MPa}$$

$$\tau_{x'y'} = -\frac{20-30}{2} \sin 60^\circ + 20 \cos 60^\circ$$

$$= 14.3 \text{ MPa}$$

For plane ②,  $\theta = 120^\circ$ 

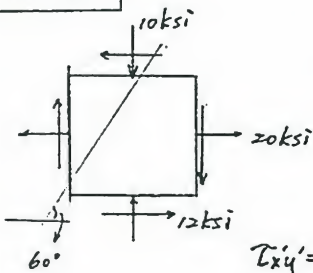
$$\sigma_{x'} = \frac{20+30}{2} + \frac{20-30}{2} \cos 240^\circ + 20 \sin 240^\circ$$

$$= 10.18 \text{ MPa}$$

$$\tau_{x'y'} = -\frac{20-30}{2} \sin 240^\circ + 20 \cos 240^\circ$$

$$= -14.33 \text{ MPa}$$

11-13



$$\theta = 150^\circ$$

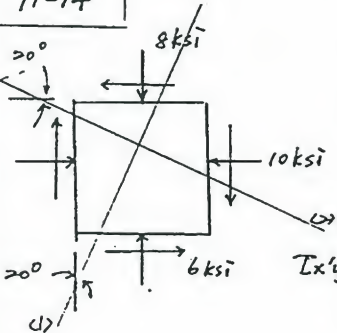
$$\sigma_x' = \frac{20-10}{2} + \frac{20-(-10)}{2} \cos 300^\circ - 12 \sin 300^\circ$$

$$= 22.89 \text{ ksi}$$

$$\tau_{xy}' = -\frac{20-(-10)}{2} \sin 300^\circ - 12 \cos 300^\circ$$

$$= 6.99 \text{ ksi}$$

11-14



$$\text{For plane (1), } \theta = 160^\circ$$

$$\sigma_x' = \frac{-10-8}{2} + \frac{(-10)-(-8)}{2} \cos 320^\circ - 6 \sin 320^\circ$$

$$= -5.91 \text{ ksi}$$

$$\tau_{xy}' = -\frac{(-10)-(-8)}{2} \sin 320^\circ - 6 \cos 320^\circ$$

$$= -5.24 \text{ ksi}$$

$$\text{For plane (2), } \theta = -110^\circ$$

$$\sigma_x' = \frac{-10-8}{2} + \frac{(-10)-(-8)}{2} \cos (-220^\circ) - 6 \sin (-220^\circ)$$

$$= -12.09 \text{ ksi}$$

$$\tau_{xy}' = -\frac{(-10)-(-8)}{2} \sin (-220^\circ) - 6 \cos (-220^\circ)$$

$$= 5.24 \text{ ksi}$$

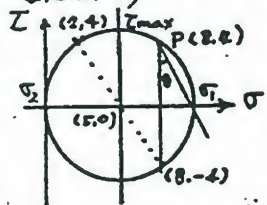
11-15

Using Eq. 11-7 and 11-8.

$$\sigma_1 = \frac{8+2}{2} + \sqrt{\left(\frac{8-2}{2}\right)^2 + 4^2} = 10 \text{ ksi}$$

$$\sigma_2 = 5-5 = 0, \theta = \frac{1}{2} \tan^{-1} \frac{4}{(8-2)/2} = 26.6^\circ$$

check by Mohr's circle



$$\sigma_1 = 10 \text{ ksi}$$

$$\sigma_2 = 0$$

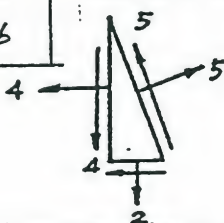
$$\theta = 26.6^\circ$$

$$\tau_{max} = 5 \text{ ksi}$$

$$\sigma = 5 \text{ ksi}$$



11-16



$$\tau_{max} = \left[ \left( \frac{8-2}{2} \right)^2 + 4^2 \right]^{1/2}$$

$$= 5 \text{ ksi}$$

$$\sigma = (8+2)/2$$

$$= 5 \text{ ksi}$$

11-17

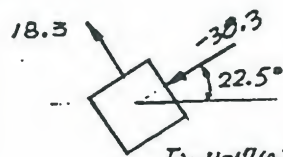


Fig. 11-17(a)

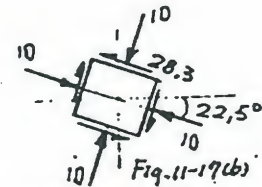


Fig. 11-17(b)

$$(a) \sigma_1 = \frac{-20}{2} + \left[ \left( \frac{40}{2} \right)^2 + 20^2 \right]^{1/2} = 18.3 \text{ ksi}$$

$$\sigma_2 = -10 - 28.3 = -38.3 \text{ ksi}$$

$$(b) \tau_{max} = 28.3 \text{ ksi}, \sigma = -10 \text{ ksi}$$

$$(c) \sigma_1 + \sigma_2 = -20 \text{ ksi} = \sigma_x + \sigma_y \quad \text{O.K.}$$

11-18

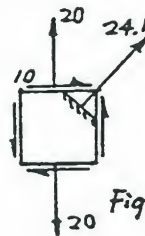


Fig. 11-18(a)

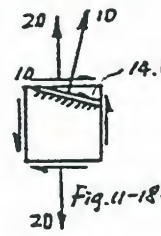


Fig. 11-18(b)

$$(a) \sigma_1 = \frac{20}{2} + \left( \frac{10^2 + 10^2}{2} \right)^{1/2} = 24.1 \text{ ksi}$$

$$\sigma_2 = 10 - 14.1 = -4.1 \text{ ksi}$$

$$(b) \tau_{max} = 14.1 \text{ ksi}, \sigma = 10 \text{ ksi}$$

$$(c) \sigma_1 + \sigma_2 = 20 \text{ ksi} = \sigma_x + \sigma_y$$

11-19

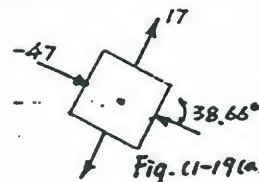


Fig. 11-19(a)

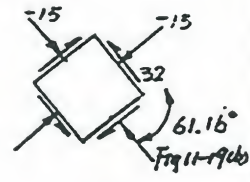


Fig. 11-19(b)

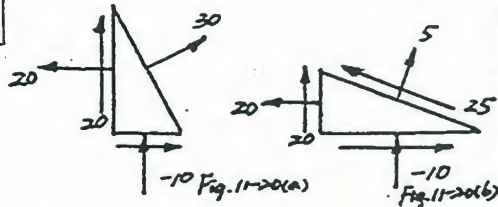
$$(a) \sigma_1 = -15 + \left( 25^2 + 32^2 \right)^{1/2} = 17 \text{ MPa}$$

$$\sigma_2 = -15 - 32 = -47.0 \text{ MPa}$$

$$(b) \tau_{max} = 32.0 \text{ MPa}, \sigma = -15 \text{ MPa}$$

$$(c) \sigma_1 + \sigma_2 = -30 = \sigma_x + \sigma_y$$

11-20



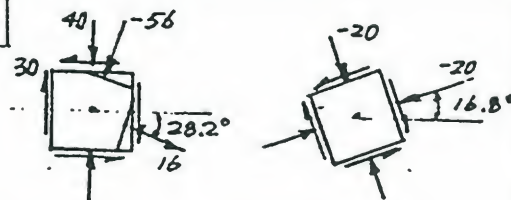
$$(a) \sigma_1 = 5 + (15^2 + 20^2)^{\frac{1}{2}} = 30 \text{ MPa}$$

$$\sigma_2 = 5 - 25 = -20 \text{ MPa}$$

$$(b) \tau_{\max} = 25 \text{ MPa}, \sigma = 5 \text{ MPa}$$

$$(c) \sigma_1 + \sigma_2 = 10 = \sigma_x + \sigma_y$$

11-21



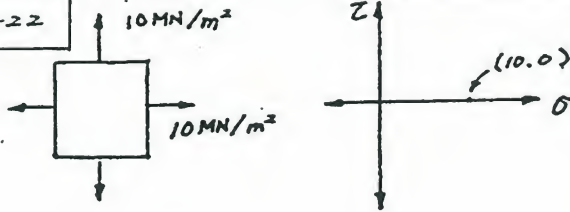
$$(a) \sigma_1 = -20 + (20^2 + 30^2)^{\frac{1}{2}} = 16.1 \text{ ksi}$$

$$\sigma_2 = -20 - 36.1 = -56.1 \text{ ksi}$$

$$(b) \tau_{\max} = 36.1 \text{ ksi}, \sigma = -20 \text{ ksi}$$

$$(c) \sigma_1 + \sigma_2 = -40 \text{ ksi} = \sigma_x + \sigma_y$$

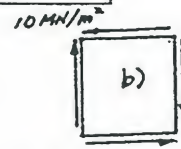
11-22



(a) Already in principal state  
Mohr's circle is a single point.

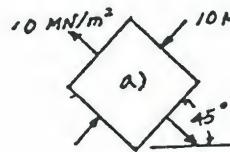
(b) No shear.

11-23



$$(a) \sigma_1 = 10 \text{ MN/m}^2, \sigma_2 = -10 \text{ MN/m}^2$$

$$\theta = 45^\circ$$



check

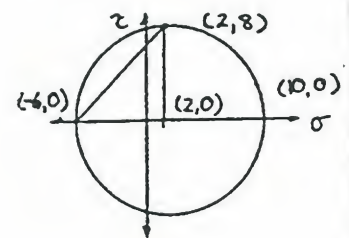
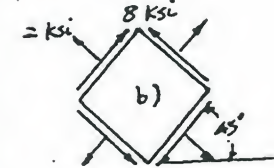
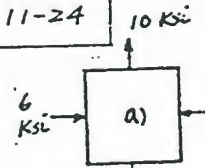
$$\sigma_1 + \sigma_2 = 0 = \sigma_x + \sigma_y$$

O.K.

b) already in state of principal shear.

$$\tau = 10 \text{ MN/m}^2$$

11-24



a) already in principal state  
 $\sigma_1 = 10, \sigma_2 = -6$

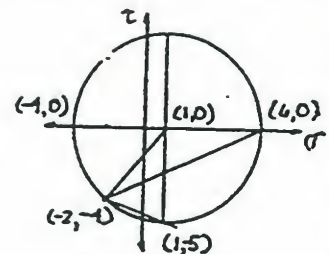
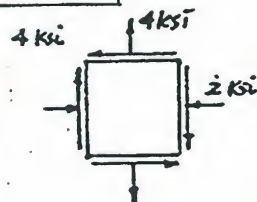
$$b) \text{ center} = \frac{10 - 6}{2} = 2$$

$$\text{radius} = 10 - 2 = 8$$

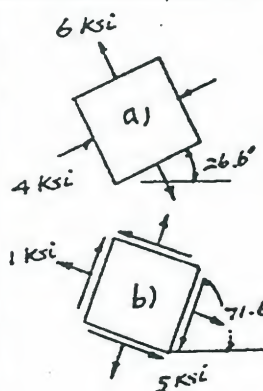
$$\theta_2 = \tan^{-1} \frac{8}{8} = 45^\circ$$

$$\tau = 8 \text{ ksi}$$

11-25





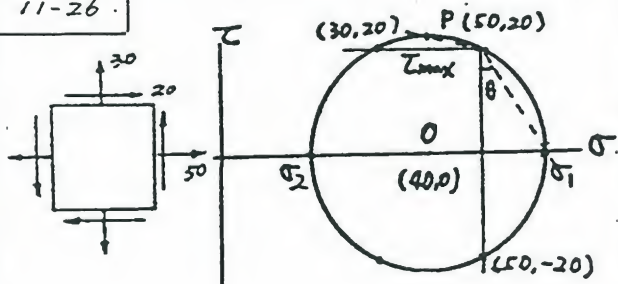


a) center =  $\frac{4+6}{2} = 5$   
 radius =  $\sqrt{(1-5)^2 + (5-1)^2} = 5$   
 $\theta_1 = \tan^{-1} \frac{4}{5} = 26.6^\circ$   
 $\sigma_1 = 1+5 = 6 \text{ ksi}$   
 $\sigma_2 = 1-5 = -4 \text{ ksi}$

b)  $\tau = 5 \text{ ksi}$   
 $\theta_2 = \tan^{-1} \frac{5}{1} = 71.6^\circ$   
 $\sigma = 1 \text{ ksi}$

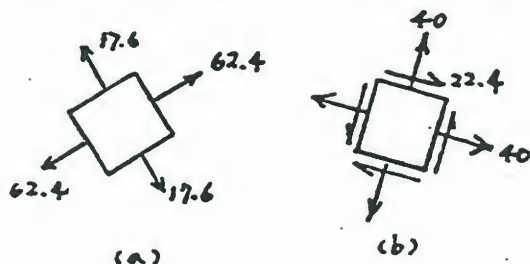
check  $\sigma_1 + \sigma_2 = 2 = \sigma_x + \sigma_y$  O.K.

11-26.

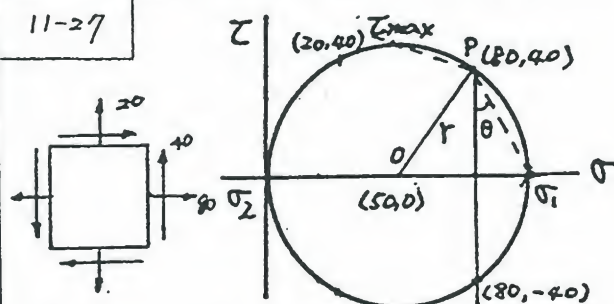


(a) radius =  $(10^2 + 20^2)^{\frac{1}{2}} = 10\sqrt{5}$   
 $\sigma_1 = 40 + 10\sqrt{5} = 62.4 \text{ MPa}$   
 $\sigma_2 = 40 - 10\sqrt{5} = 17.6 \text{ MPa}$   
 $\sigma_1 + \sigma_2 = 80 = \sigma_x + \sigma_y$  O.K.  
 $\theta = \tan^{-1} \left( \frac{10\sqrt{5}-10}{20} \right) = 31.7^\circ$

(b)  $\tau_{max} = 10\sqrt{5} = 22.4$ ,  $\sigma = 40 \text{ MPa}$

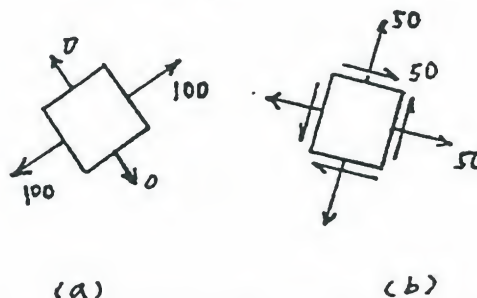


11-27

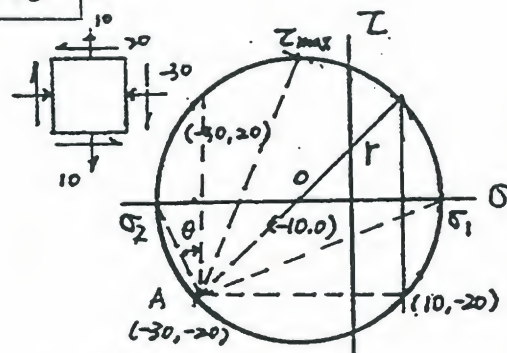


(a)  $r = (30^2 + 40^2)^{\frac{1}{2}} = 50$   
 $\sigma_1 = 50 + 50 = 100 \text{ psi}$   
 $\sigma_2 = 50 - 50 = 0 \text{ psi}$   
 $\sigma_1 + \sigma_2 = 100 = \sigma_x + \sigma_y$  O.K.  
 $\theta = \tan^{-1} \left( \frac{20}{40} \right) = 26.6^\circ$

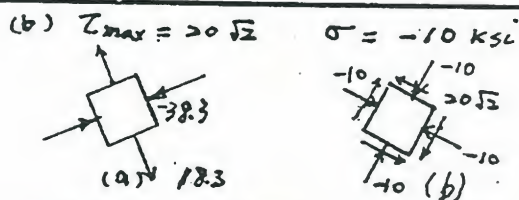
(b)  $\tau_{max} = 50 \text{ psi}$ ,  $\sigma = 50 \text{ psi}$



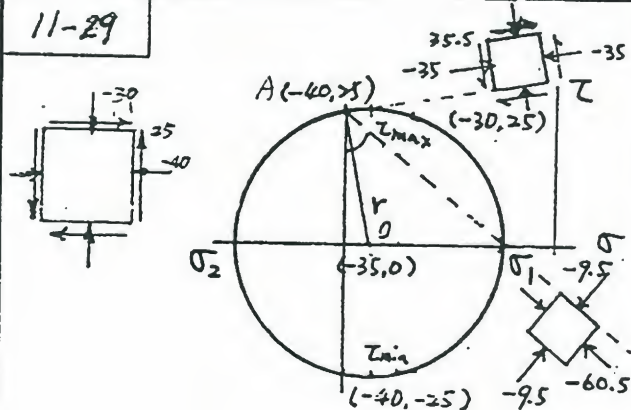
11-28



(a)  $r = (20^2 + 20^2)^{\frac{1}{2}} = 20\sqrt{2}$   
 $\sigma_1 = -10 + 20\sqrt{2} = 18.3 \text{ ksi}$   
 $\sigma_2 = -10 - 20\sqrt{2} = -38.3 \text{ ksi}$   
 $\sigma_1 + \sigma_2 = -20 = \sigma_x + \sigma_y$  O.K.  
 $\theta = \tan^{-1} \left( \frac{20\sqrt{2}-20}{20} \right) = 22.5^\circ$



11-29



(a)  $r = (5^2 + 25^2)^{1/2} = 25.5 \text{ MPa}$

$\sigma_1 = -35 + 25.5 = -9.5 \text{ MPa}$

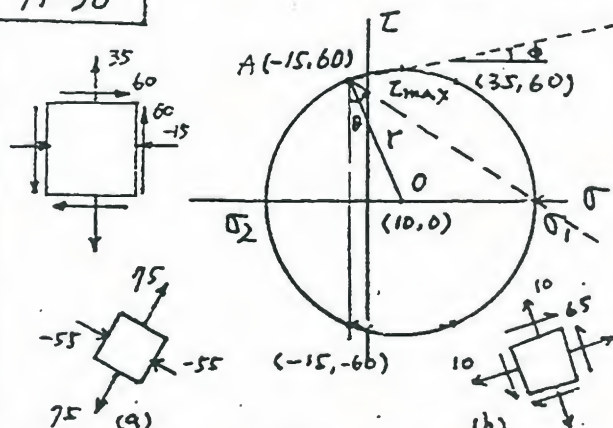
$\sigma_2 = -35 - 25.5 = -60.5 \text{ MPa}$

$\sigma_1 + \sigma_2 = -70 = \sigma_x + \sigma_y \text{ O.K.}$

$\theta = \tan^{-1} \left( \frac{30.5}{25} \right) = 50.7^\circ$

(b)  $\tau_{max} = 25.5 \text{ MPa}$ ,  $\sigma = -35 \text{ MPa}$

11-30



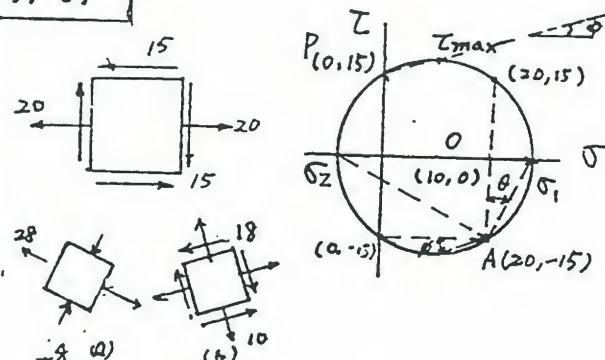
(a)  $r = (25^2 + 60^2)^{1/2} = 65$

$\sigma_1 = 75 \text{ MPa}$ ,  $\sigma_2 = -55 \text{ MPa}$

$\sigma_1 + \sigma_2 = 20 = \sigma_x + \sigma_y \text{ O.K.}$

(b)  $\phi = \tan^{-1} \left( \frac{5}{25} \right) = 11.3^\circ$   
 $\tau_{max} = 65 \text{ MPa}$ ,  $\sigma = 10 \text{ MPa}$

11-31



(a)  $r = (10^2 + 15^2)^{1/2} = 18.0$

$\sigma_1 = 10 + 18 = 28.0 \text{ Ksi}$

$\sigma_2 = 10 - 18 = -8.0 \text{ Ksi}$

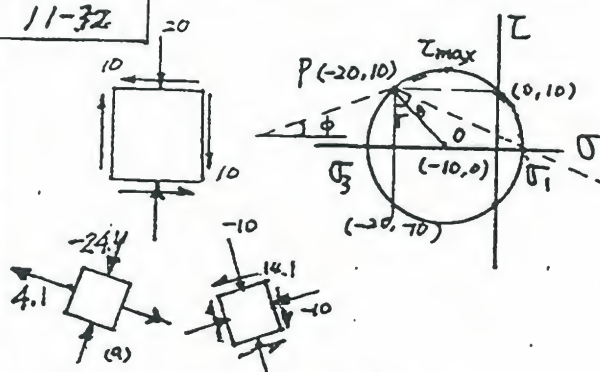
$\sigma_1 + \sigma_2 = 20 = \sigma_x + \sigma_y \text{ O.K.}$

$\theta = \tan^{-1} \left( \frac{8}{15} \right) = 28.07^\circ$

(b)  $\phi = \tan^{-1} \left( \frac{3}{10} \right) = 16.7^\circ$

$\tau_{max} = 18.0 \text{ Ksi}$ ,  $\sigma = 10 \text{ Ksi}$

11-32



(a)  $r = (10^2 + 10^2)^{1/2} = 14.1$

$\sigma_1 = -10 + 14.1 = 4.1 \text{ Ksi}$

$\sigma_2 = -10 - 14.1 = -24.1 \text{ Ksi}$

$\sigma_1 + \sigma_2 = -20 \text{ Ksi} = \sigma_x + \sigma_y \text{ O.K.}$

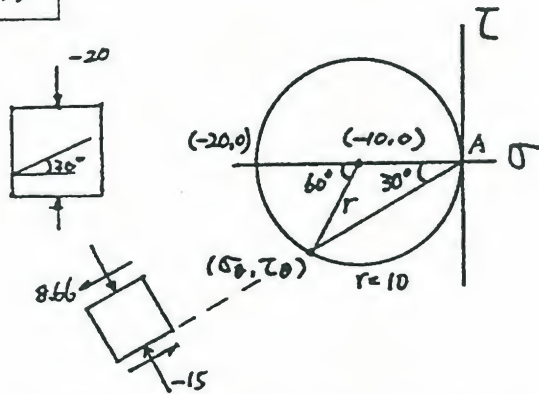
$\theta = \tan^{-1} \left( \frac{4.1}{10} \right) = 22.3^\circ$

(b)  $\phi = \tan^{-1} \left( \frac{4.1}{10} \right) = 22.3^\circ$

$\tau_{max} = 14.1 \text{ Ksi}$ ,  $\sigma = -10 \text{ Ksi}$



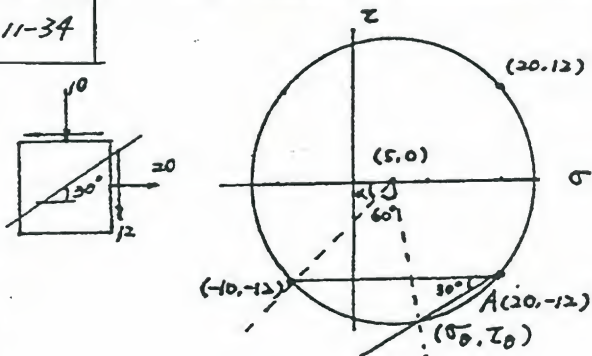
11-33



$$\sigma_{\theta} = -10 - 10 \cos 60^{\circ} = -15 \text{ Ksi}$$

$$\tau_{\theta} = -10 \sin 60^{\circ} = 8.66 \text{ Ksi}$$

11-34



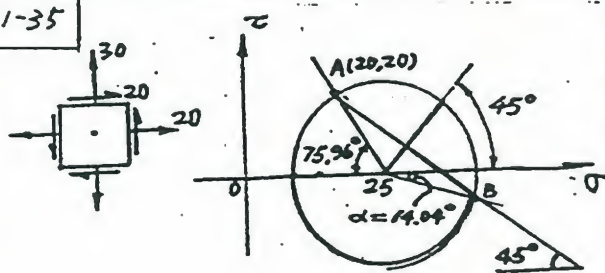
$$r = (15^2 + 12^2)^{1/2} = 19.2$$

$$\alpha = \tan^{-1} \left( \frac{12}{15} \right) = 38.7^{\circ}$$

$$\sigma_{\theta} = 5 + 19.2 \cos (180^{\circ} - 38.7^{\circ} - 60^{\circ}) = 7.9 \text{ Ksi}$$

$$\tau_{\theta} = -19.2 \sin (180^{\circ} - 38.7^{\circ} - 60^{\circ}) = -19.0 \text{ Ksi}$$

11-35



$$R = \sqrt{20^2 + 5^2} = 20.62$$

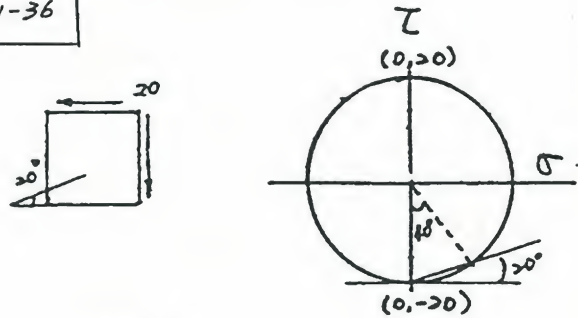
$$\sigma_x = 25 + 20.62 \cos \alpha = 45 \text{ MPa}$$

$$\tau = -20.62 \sin \alpha = -5.00 \text{ MPa}$$

$$\text{here, } \tan \frac{20}{5} = 75.96^{\circ}$$

$$\alpha = [180 - (75.96 + 45)] - 45 = 14.04^{\circ}$$

11-36

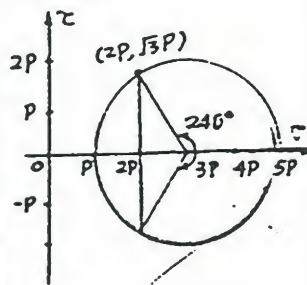


$$r = 20$$

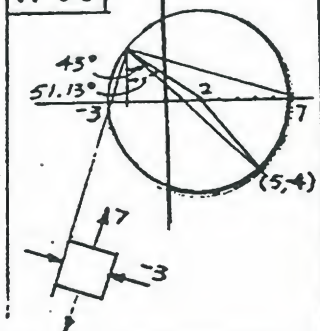
$$\sigma_{\theta} = +20 \sin 40^{\circ} = +12.9 \text{ Ksi}$$

$$\tau_{\theta} = -20 \cos 40^{\circ} = -15.3 \text{ Ksi}$$

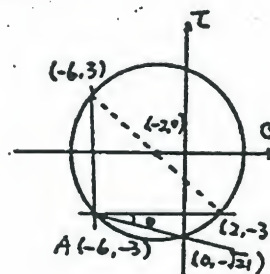
11-37



11-38



11-39



$$r = (4^2 + 3^2)^{1/2} = 5$$

$$\tau = -\sqrt{5^2 - 4^2} = -\sqrt{9} = -3$$

$$\theta = \tan^{-1} \frac{5-3}{4} = 14.78^{\circ}$$





check by wedge method

assume  $A_v = 1$ ,  $A_v = \sin \theta$ ,  $A_H = \cos \theta$

$$N = (2 \cos \theta - 3 \sin \theta) \cos \theta$$

$$S = (2 \sin \theta + 3 \cos \theta) \cos \theta + (-3 \sin \theta + 6 \cos \theta) \sin \theta$$

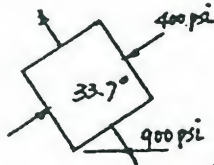
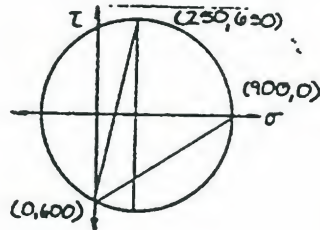
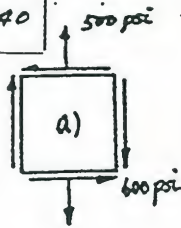
$$= 2.29 + 1.29 = 4.58$$

$$\sigma_v = N/A_v = 0$$

$$\tau_v = S/A_v = 4.58 \text{ ksi O.K.}$$



11-40



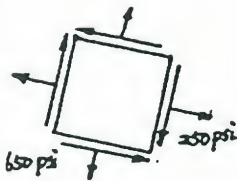
$$\text{center} = \frac{1}{2}(500) = 250 \text{ psi}$$

$$\text{radius} = \sqrt{250^2 + 600^2} = 650$$

$$a) \sigma_1 = 250 + 650 = 900 \text{ psi}$$

$$\sigma_2 = 250 - 650 = -400 \text{ psi}$$

$$\theta_1 = \tan^{-1} \frac{600}{900} = 33.7^\circ$$



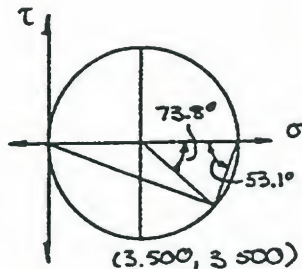
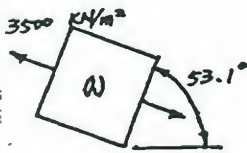
$$b) \tau = 650 \text{ psi}$$

$$\sigma' = 250 \text{ psi}$$

$$\theta_2 = \tan^{-1} \frac{600 + 650}{250}$$

$$= 28.70$$

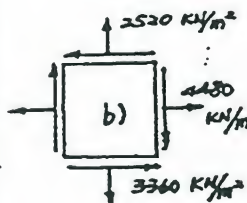
11-41



$$a) \text{radius} = \tau_{\max} = 3500$$

$$\sigma_1 = 7000 \text{ KN/m}^2$$

$$\sigma_2 = 0$$



$$b) \theta' = 180 - 2(53.1) = 73.8^\circ$$

$$\sigma_x = 3500 + 3500 \cos 73.8^\circ$$

$$= 4480 \text{ KN/m}^2$$

$$\sigma_y = 3500 - 3500 \cos 73.8^\circ$$

$$= 2520 \text{ KN/m}^2$$

$$\tau = -3500 \sin 73.8^\circ$$

$$= -3360 \text{ KN/m}^2$$

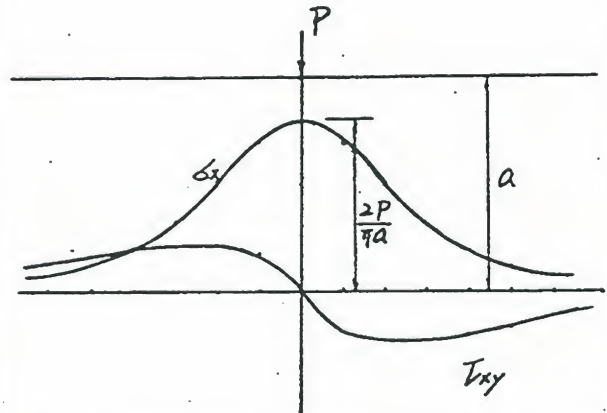
11-42

$$\sigma_x = \sigma_r \cos^2 \theta = -\frac{2P \cos^4 \theta}{\pi a}$$

$$\sigma_y = \sigma_r \sin^2 \theta = -\frac{2P}{\pi a} \sin^2 \theta \cos^2 \theta$$

$$\tau_{xy} = \sigma_r \sin \theta \cos \theta = -\frac{2P \sin \theta \cos^3 \theta}{\pi r}$$

$$\tau_{xy} = -\frac{2P}{\pi a} \sin \theta \cos^3 \theta$$



11-43

$$\sigma_x = \sigma_r \cos^2 \theta = \frac{-P \cos \theta}{r(\alpha + \frac{1}{2} \sin 2\alpha)} \cos^2 \theta$$

$$\sigma_x = \frac{-P \cos^4 \theta}{a(\alpha + \frac{1}{2} \sin 2\alpha)}$$

$$(a) \alpha = 10^\circ$$

$$\sigma_{x \max} = -\frac{P}{a(0.175 + \frac{1}{2} 0.341)} = -\frac{P}{0.346a}$$

$$\sigma_x = \frac{-P}{2a \tan 10^\circ} = \frac{-P}{0.352a}$$

$$(b) \alpha = 45^\circ$$

$$\sigma_{x \max} = \frac{-P}{a(0.707 + 0.5)} = \frac{-P}{1.207a}$$

$$\sigma_x = \frac{-P}{2a \tan 45^\circ} = \frac{-P}{2a}$$

11-44

$$\delta_x = \delta_r \sin^2 \theta = \frac{-P \cos \theta}{r(\alpha - \frac{1}{2} \sin 2\alpha)} \sin^2 \theta$$

$$\delta_x = \frac{-Py \sin^4 \theta}{x^2(\alpha - \frac{1}{2} \sin 2\alpha)}$$

$$\tau_{xy} = \delta_r \sin \theta \cos \theta = \frac{-P \cos^2 \theta \sin \theta}{r(\alpha - \frac{1}{2} \sin 2\alpha)}$$

$$\tau_{xy} = \frac{-Py^2 \sin^4 \theta}{x^3(\alpha - \frac{1}{2} \sin 2\alpha)}$$

Elementary solution

$$\delta_x = \frac{My}{I} = -\frac{Pxy}{I}$$

$$\tau_{xy} = \frac{VQ}{It} = \frac{-PQ}{I}$$

$$\text{If } \alpha = 30^\circ \quad \alpha - \frac{1}{2} \sin 2\alpha = 0.09$$

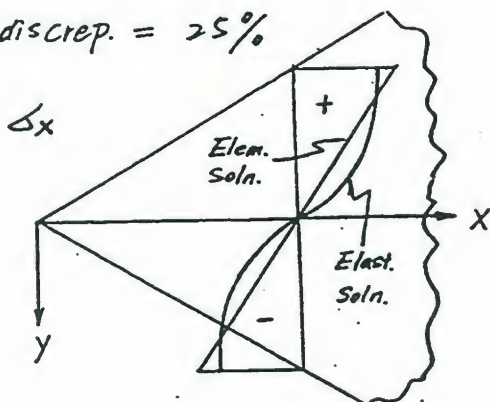
$$\text{Elast. Soln. } \delta_x = -11.1 \frac{Py}{x^2} \sin^4 \theta$$

$$\delta_{x, \theta=60^\circ} = \delta_{x, \max} = -3.61 \frac{P}{x}$$

$$\text{Elem Soln } \delta_{x, \max} = -\frac{Pxy}{\frac{1}{12}(20)^3}$$

$$\delta_{x, \max} = -\frac{3Pxc}{2c^3} = -4.5 \frac{P}{x} \quad (c = x \tan 30^\circ)$$

$$\text{discrep.} = 25\%$$



$$\tau_{xy, \text{Elast}} = -11.1 \frac{Py^2}{x^3} \sin^4 \theta \quad \tau_{xy, \theta=60^\circ, \text{Elast}} = -2.06 \frac{P}{x}$$

$$\tau_{xy, \max, \text{Elem.}} = \frac{-PQ}{I} = \frac{-P \frac{c^2}{12}}{\frac{1}{12}(2c)^3} = -1.30 \frac{P}{x}$$

11-45 from eq. (11-23)

$$(2 - \sigma_n)(5 - \sigma_n) - 4 = 0$$

$$\sigma_1 = 6, \quad \sigma_3 = 1, \quad \sigma_2 = 3$$

$$\text{for } n_i^{(1)}, \quad n_1^{(1)} = 0, \quad \text{let } n_2^{(1)} = 1$$

$$(2-6)n_2^{(1)} + 2n_3^{(1)} = 0, \quad n_3^{(1)} = 2$$

$$\text{normalize} \Rightarrow n^{(1)} = (0, \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}})$$

$$\text{for } n_i^{(2)} \Rightarrow n^{(2)} = (1, 0, 0)$$

$$\text{for } n_i^{(3)}, \quad n_1^{(3)} = 0, \quad \text{Let } n_2^{(3)} = 1$$

$$(2-1)n_2^{(3)} + 2n_3^{(3)} = 0, \quad n_3^{(3)} = -\frac{1}{2}$$

$$\text{normalize} \Rightarrow n^{(3)} = (0, \frac{2}{\sqrt{5}}, \frac{-1}{\sqrt{5}})$$

11-46

from eq. (11-23)

$$\sigma_n^3 - 18\sigma_n^2 - 120\sigma_n + 1872 = 0$$

$$(a) \quad I\sigma = 18, \quad II\sigma = -120, \quad III\sigma = -1872$$

$$(b) \quad \sigma_1 = 19.1646, \quad \sigma_2 = 9.3181$$

$$\sigma_3 = -10.4827$$

$$(c) \quad (10 - 19.1646)n_1 + 4n_2 - 6n_3 = 0$$

$$4n_1 - (6 + 19.1646)n_2 + 8n_3 = 0$$

$$n_1 = -0.55n_3, \quad n_2 = 0.23n_3$$

$$n_1^2 + n_2^2 + n_3^2 = 1, \quad n_3 = 0.86$$

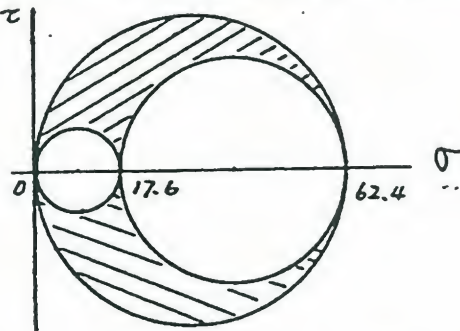
$$n_1 = -0.47, \quad n_2 = 0.20$$



11-47

from solution of Prob. 11-26

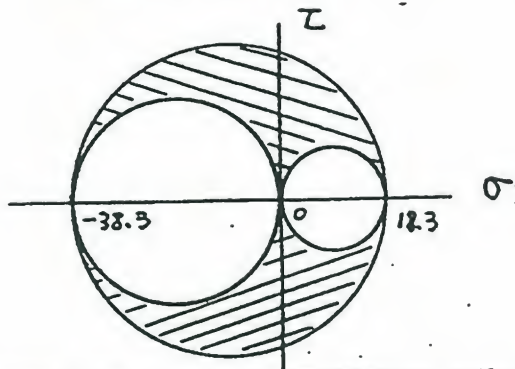
$$\sigma_1 = 62.4, \sigma_2 = 17.6, \sigma_3 = 0$$



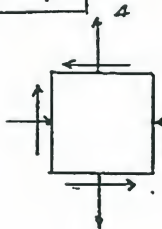
11-48

from solution of Prob. 11-28

$$\sigma_1 = 18.3, \sigma_2 = 0, \sigma_3 = -38.3$$



11-49



$$\sigma_x = -2, \sigma_y = 4, \tau_{xy} = -4$$

$$\sigma_o = \frac{1}{3}(\sigma_x + \sigma_y + \sigma_z) = \frac{2}{3}$$

$$S_x = -\frac{8}{3}, S_y = \frac{10}{3}, S_z = -\frac{2}{3}$$

$$J_2 = -\frac{80}{9} - \frac{20}{9} + \frac{16}{9} - 16 = -\frac{228}{9}$$

$$J_3 = -\left(\frac{160}{27} + \frac{2}{3} \times 16\right) = -\frac{448}{27}$$

$$\theta = \frac{1}{3} \arccos \left[ \frac{448/27}{2 \left( \frac{228}{9} \right)^{3/2}} \right] = 24.31^\circ$$

$$\sigma_1 = \frac{2}{3} + 2 \left[ \frac{228}{27} \right]^{1/2} (0.9176) = 6.0$$

$$\sigma_2 = \frac{2}{3} - 2 \left[ \frac{228}{27} \right]^{1/2} (0.11476) = 0$$

$$\sigma_3 = \frac{2}{3} - 2 \left[ \frac{228}{27} \right]^{1/2} (0.80292) = -4$$

using eq. 11-33, 11-34. for  $\sigma_1$ 

$$(\sigma_x - \sigma_1)l_1 + \tau_{xy}m_1 = 0$$

$$0x l_1 + 0y m_1 + (0 - \sigma_1)n_1 = 0$$

$$l_1^2 + m_1^2 + n_1^2 = 1$$

$$\therefore l_1 = 1/\sqrt{5}, m_1 = -2/\sqrt{5}, n_1 = 0$$

$$\text{for } \sigma_2: (\sigma_x - \sigma_2)l_2 + \tau_{xy}m_2 = 0$$

$$\tau_{xy}l_2 + (\sigma_y - \sigma_2)m_2 = 0$$

$$l_2^2 + m_2^2 + n_2^2 = 1$$

$$\therefore l_2 = m_2 = 0, n_2 = \pm 1$$

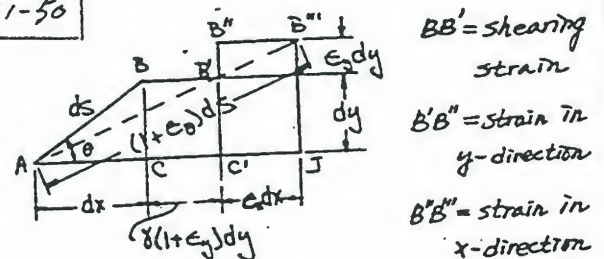
$$\text{for } \sigma_3: (\sigma_x - \sigma_3)l_3 + \tau_{xy}m_3 = 0$$

$$0x l_3 + 0y m_3 + (0 - \sigma_3)n_3 = 0$$

$$l_3^2 + m_3^2 + n_3^2 = 1$$

$$\therefore l_3 = 2/\sqrt{5}, m_3 = 1/\sqrt{5}, n_3 = 0$$

11-50



$$AB''' = (1 + \epsilon_o) ds$$

$$\begin{aligned} (AB''')^2 &= [(1 + \epsilon_o) ds]^2 \\ &= (AJ)^2 + (JB)^2 \\ &= [(1 + \epsilon_y) dy + (1 + \epsilon_x) dx]^2 \\ &\quad + [(1 + \epsilon_y) dy]^2 \\ &= [(1 + \epsilon_o) ds]^2 / ds^2 \end{aligned}$$

$$\begin{aligned} (1 + \epsilon_o)^2 &= [(1 + \epsilon_x) \frac{dx}{ds} + (1 + \epsilon_y) \frac{dy}{ds}]^2 \\ &\quad + [(1 + \epsilon_y) \frac{dy}{ds}]^2 \end{aligned}$$

$$\text{but } \frac{dx}{ds} = \cos \theta \text{ and } \frac{dy}{ds} = \sin \theta$$

$$\therefore \epsilon_o = \epsilon_x \cos^2 \theta + \epsilon_y \sin^2 \theta + \dots \sin \theta \cos \theta$$



11-51 see Fig. 11-20

$$\beta \approx \tan \beta = \frac{-BB' \cos \theta + BB'' \sin \theta + BB''' \cos \theta}{dy'}$$

$$= \frac{-\epsilon_x dx \cos \theta + \epsilon_y dy \sin \theta + \gamma_{xy} dy \cos \theta}{dy'}$$

but  $\frac{dx}{dy'} = \sin \theta$  and  $\frac{dy}{dy'} = \cos \theta$

$$\therefore \beta = \frac{-\epsilon_x \sin \theta \cos \theta + \epsilon_y \sin \theta \cos \theta + \gamma_{xy} \cos^2 \theta}{1}$$

$$= -(\epsilon_x - \epsilon_y) \sin \theta \cos \theta + \gamma_{xy} \cos^2 \theta$$

11-52  $\epsilon_1 = \frac{1000}{2} + (620^2 + 100^2)^{\frac{1}{2}} = 1128 \text{ Mm/m}$

$\epsilon_2 = \frac{1000}{2} - (620^2 + 100^2)^{\frac{1}{2}} = -128 \text{ Mm/m}$

$\tan 2\theta_1 = \frac{-200}{-1240}, \theta_1 = 4.58^\circ$

11-53  $\epsilon_1 = \frac{-1000}{2} + (300^2 + 400^2)^{\frac{1}{2}} = 0 \text{ Mm/m}$

$\epsilon_2 = \frac{-1000}{2} - (300^2 + 400^2)^{\frac{1}{2}} = -1000 \text{ Mm/m}$

$\tan 2\theta_1 = \frac{800}{-600}, \theta_1 = -26.6^\circ$

11-54  $\epsilon_3 = -20 \text{ Mm/m}$

$(70 - \lambda)(50 - \lambda) - 30 = 0$

$\lambda = 60 \pm 20$

$\epsilon_1 = 80 \text{ Mm/m}, \epsilon_2 = 40 \text{ Mm/m}$

for  $\epsilon_1$ , Let  $n_1^{(1)} = 1$

$$\begin{bmatrix} -10 & -\sqrt{3} \\ -\sqrt{3} & -30 \end{bmatrix} \begin{Bmatrix} n_1^{(1)} \\ n_2^{(1)} \end{Bmatrix} = 0$$

$n_2^{(1)} = -\frac{1}{\sqrt{3}}$

normalize  $n_1^{(1)} = \frac{\sqrt{3}}{2}, n_2^{(1)} = -\frac{1}{2}$

$n_3^{(1)} = 0$ ,

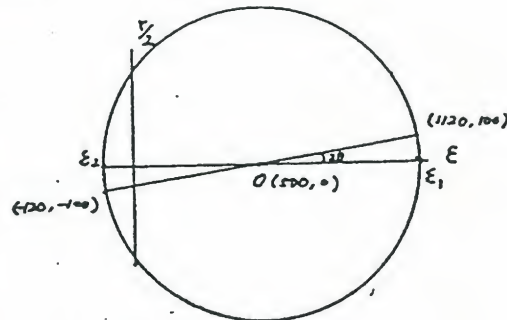
for  $\epsilon_2$ ,  $n_1^{(2)} = -\sqrt{3} n_2^{(2)}$

$(n_1^{(2)})^2 + (n_2^{(2)})^2 = 1, n_3^{(2)} = 0$

$n_2^{(2)} = \frac{1}{2}, n_1^{(2)} = -\frac{\sqrt{3}}{2}$

for  $\epsilon_3$ ,  $n^{(3)} = (0, 0, 1)$

11-55



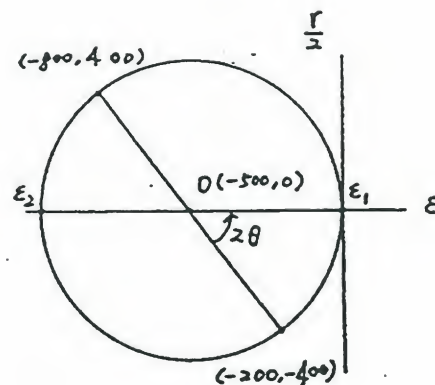
$r = [(1120 - 500)^2 + 100^2]^{\frac{1}{2}} = 628$

$\epsilon_1 = 500 + 628 = 1128 \text{ Mm/m}$

$\epsilon_2 = 500 - 628 = -128 \text{ Mm/m}$

$\tan 2\theta = \frac{100}{1120 - 500}, \theta = 4.58^\circ$

11-56



$r = [(-500 + 200)^2 + 400^2]^{\frac{1}{2}} = 500$

$\epsilon_1 = -500 + 500 = 0 \text{ Mm/m}$

$\epsilon_2 = -500 - 500 = -1000 \text{ Mm/m}$

$-\tan 2\theta = \frac{400}{300}, \theta = -26.6^\circ$

11-57

$$\gamma = 2(-.00012) - (-.00022 + .00022) = .00024$$

$$\epsilon_{1,2} = \frac{-.00022 + .00022}{2} \pm \sqrt{\left(\frac{-.00022 - .00022}{2}\right)^2 + \left(\frac{.00024}{2}\right)^2}$$

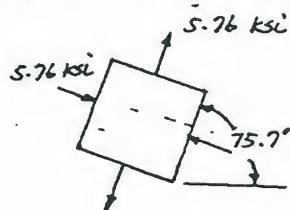
$$= \pm .000251$$

$$\sigma_1 = \frac{30 \times 10^6}{1-.3^2} [.000251 - .3(.000251)]$$

$$= 5.76 \text{ ksi}$$

$$\sigma_2 = -5.76 \text{ ksi}$$

$$\theta_1 = \frac{1}{2} \tan^{-1} \frac{24}{-2(22)} = 75.7^\circ$$



11-58

$$\epsilon_x = \epsilon_0 = .00040$$

$$\epsilon_y = \frac{1}{3} (2\epsilon_{60^\circ} + 2\epsilon_{120^\circ} - \epsilon_0)$$

$$= \frac{1}{3} (.00080 - .00120 - .00040) = -.000267$$

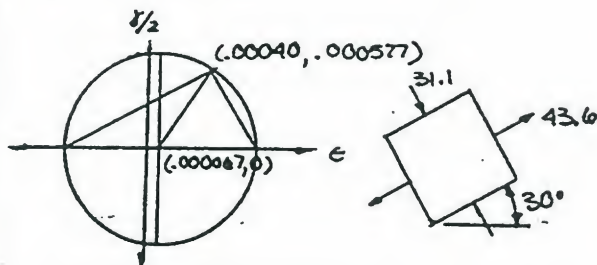
$$\gamma_{xy} = \frac{2}{\sqrt{3}} (\epsilon_{60^\circ} - \epsilon_{120^\circ}) = \frac{2}{\sqrt{3}} (.00040 + .00060)$$

$$= .001155$$

$$\text{center} = \frac{\epsilon_x + \epsilon_y}{2} = \frac{.00040 - .000267}{2}$$

$$= .000067$$

$$\frac{\gamma}{2} = .000577$$



$$\text{radius} = \sqrt{(.00040 - .000067)^2 + (.000577)^2}$$

$$= .000677$$

$$\epsilon_1 = .000067 + .000677 = .000734$$

$$\epsilon_2 = .000067 - .000677 = -.000600$$

$$\sigma_1 = \frac{70000}{1-.25^2} [.000734 - .25(-.000600)]$$

$$= 43.6 \text{ MPa}$$

$$\sigma_2 = \frac{70000}{1-.25^2} [-.000600 + .25(.000734)]$$

$$= -31.1 \text{ MPa}$$

11-59

Using Eq. 11-50 and plug in

$$\theta = 135^\circ$$

$$\epsilon_{135^\circ} = \frac{\epsilon_x}{2} + \frac{\epsilon_y}{2} - \frac{\gamma_{xy}}{2}$$

$$\therefore \gamma_{xy} = \epsilon_x + \epsilon_y - 2\epsilon_{135^\circ}$$

Using  $\epsilon_{0^\circ}$ ,  $\epsilon_{90^\circ}$ ,  $\epsilon_{45^\circ}$  to compute

$$\gamma_{xy} = 2\epsilon_{45^\circ} - (\epsilon_{0^\circ} + \epsilon_{90^\circ})$$

$$= 2 \times 400 - (-120 + 1120)$$

$$= -200 \mu\text{m}/\text{m}$$

Using  $\epsilon_{0^\circ}$ ,  $\epsilon_{90^\circ}$ ,  $\epsilon_{135^\circ}$  to compute

$$\gamma_{xy} = \epsilon_{0^\circ} + \epsilon_{90^\circ} - 2\epsilon_{135^\circ}$$

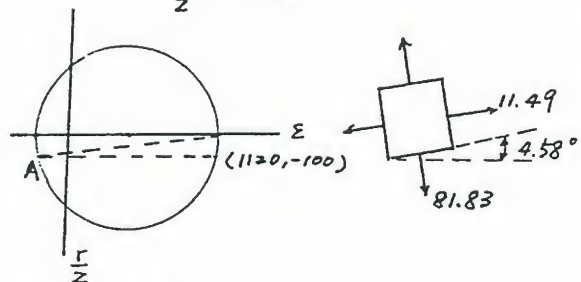
$$= (-120 + 1120) - 2 \times 600$$

$$= -200 \mu\text{m}/\text{m}$$

 $\therefore$  Consistent.

$$\text{center} = \frac{1}{2} (\epsilon_x + \epsilon_y) = 500 \mu\text{m}/\text{m}$$

$$\frac{\gamma_{xy}}{2} = -100$$



$$\text{radius} = \sqrt{(-120 - 500)^2 + (-100)^2} = 628$$

$$\epsilon_1 = 500 + 628 = 1128 \mu\text{m}/\text{m}$$

$$\epsilon_2 = 500 - 628 = -128 \mu\text{m}/\text{m}$$

$$\therefore \sigma_1 = \frac{70,000}{1-0.75^2} [1128 - 0.75 \times 128] / 10^6$$

$$= 81.83 \text{ MPa}$$

$$\sigma_2 = \frac{70,000}{1-0.75^2} [-128 + 0.75 \times 1128] / 10^6$$

$$= 11.49 \text{ MPa}$$

$$\theta = \arctan\left(\frac{100}{2 \times 628 - 8}\right) = 4.58^\circ$$

11-60

$$\sigma_x + \sigma_y = 27.5 \text{ MPa}$$

$$\therefore \epsilon_x + \epsilon_y = \frac{(\sigma_x + \sigma_y)(1-\nu)}{E} \times 10^6$$

$$= 103.125 \mu\text{m}/\text{m}$$

$$\text{radius} = \frac{500}{2} = 250$$

$$\therefore \epsilon_1 = \frac{\epsilon_x + \epsilon_y}{2} + 250 = 301.56$$

$$\epsilon_2 = \frac{\epsilon_x + \epsilon_y}{2} - 250 = -198.43$$

$$\therefore \sigma_1 = \frac{200,000}{1-0.75^2} [301.56 - 0.75 \times 198.43] / 10^6$$

$$= 53.75 \text{ MPa}$$

$$\sigma_2 = \frac{200,000}{1-0.75^2} [-198.43 + 0.75 \times 301.56] / 10^6$$

$$= -26.75 \text{ MPa}$$

$$\sigma_1 + \sigma_2 = 27.5 \text{ MPa} \text{ . o.k.}$$



12-1  $\bar{\sigma} = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3} = \frac{(10) + (-6) + (14)}{3} = 6$

$$\begin{pmatrix} 10 & 4 & -6 \\ 4 & -6 & 8 \\ -6 & 8 & 14 \end{pmatrix} = \underbrace{\begin{pmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{pmatrix}}_{\text{spherical}} + \underbrace{\begin{pmatrix} 4 & 4 & -6 \\ 4 & -12 & 8 \\ -6 & 8 & 8 \end{pmatrix}}_{\text{deviatoric}}$$

12-2  $\bar{\sigma} = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3} = \frac{40 + 50 + 60}{3} = 50$

$$\begin{pmatrix} 40 & -30 & 20 \\ -30 & 50 & 10 \\ 20 & 10 & 60 \end{pmatrix} = \underbrace{\begin{pmatrix} 50 & 0 & 0 \\ 0 & 50 & 0 \\ 0 & 0 & 50 \end{pmatrix}}_{\text{spherical}} + \underbrace{\begin{pmatrix} -10 & -30 & 20 \\ -30 & 0 & 10 \\ 20 & 10 & 10 \end{pmatrix}}_{\text{deviatoric}}$$

12-3  $\bar{\sigma} = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3} = \frac{50 + 70 + 60}{3} = 60$

$$\begin{pmatrix} 50 & -30 & 0 \\ -30 & 70 & -20 \\ 0 & -20 & 60 \end{pmatrix} = \underbrace{\begin{pmatrix} 60 & 0 & 0 \\ 0 & 60 & 0 \\ 0 & 0 & 60 \end{pmatrix}}_{\text{spherical}} + \underbrace{\begin{pmatrix} -10 & -30 & 0 \\ -30 & 10 & -20 \\ 0 & -20 & 0 \end{pmatrix}}_{\text{deviatoric}}$$

12-4 (a)  $\sigma_y = 300 \text{ MPa}$ ,  $\sigma_x = \tau_{xy} = 0$

The three criteria have the same  $\sigma_{yp}$

(b)  $\sigma_x = \sigma_y = 0$ ,  $\tau_{xy} = -150 \text{ MPa}$

$\Rightarrow \sigma_1 = 150 \text{ MPa}$ ,  $\sigma_2 = -150 \text{ MPa}$

maximum normal stress  $\sigma_{yp} = 150 \text{ MPa}$

distortion - energy  $\sigma_{yp}/\sqrt{3} = 150 \text{ MPa}$

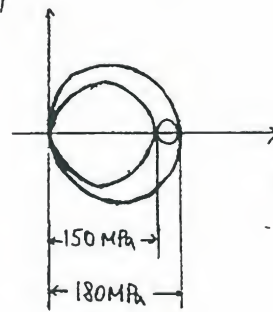
$\sigma_{yp} = 259.8 \text{ MPa}$

maximum shear stress  $\sigma_{yp}/2 = 150 \text{ MPa}$

$\sigma_{yp} = 300 \text{ MPa}$

$\Rightarrow$  maximum shear stress theory control!

(c)



$\sigma_x = 150$ ,  $\sigma_y = 180$

$\tau_{xy} = 0$

$\Rightarrow \sigma_1 = 180$ ,  $\sigma_2 = 150$

$\sigma_3 = 0$

maximum normal stress  $\sigma_{yp} = 180 \text{ MPa}$

distortion - energy  $(30)^2 + (180)^2 + (150)^2 = 2 \sigma_{yp}^2$

$\sigma_{yp} = 167 \text{ MPa}$

maximum shear stress  $\sigma_{yp} = 90 \text{ MPa}$

$\Rightarrow$  maximum normal stress theory control!

12-5 (a)  $\sigma_x = -150 \text{ MPa}$ ,  $\sigma_y = \tau_{xy} = 0$

The three criteria have the same  $\sigma_{yp}$

(b)  $\sigma_x = 0$ ,  $\sigma_y = 60 \text{ MPa}$ ,  $\tau_{xy} = 75 \text{ MPa}$



$\Rightarrow \sigma_1 = 110.78 \text{ MPa}$

$\sigma_2 = 0 \text{ MPa}$

$\sigma_3 = -50.78 \text{ MPa}$

maximum normal stress  $\sigma_{YP} = 110.78 \text{ MPa}$

distortion - energy

$$(161.56)^2 + (110.78)^2 + (50.78)^2 = 2 \sigma_{YP}^2$$

$$\sigma_{YP} = 143.1 \text{ MPa}$$

$$\text{maximum shear stress } \sigma_{YP} = \frac{\sigma_1 - \sigma_3}{2} = 80.78 \text{ MPa}$$

$\Rightarrow$  distortion-energy theory control!

$$(c) \sigma_x = 75 \text{ MPa}, \sigma_y = -100 \text{ MPa}, \tau_{xy} = 0$$

$$\Rightarrow \sigma_1 = 75 \text{ MPa}, \sigma_2 = 0, \sigma_3 = -100 \text{ MPa}$$

maximum normal stress = 100 MPa

$$\text{distortion-energy } (75)^2 + (75)^2 + (100)^2 = 2 \sigma_{YP}^2$$

$$\sigma_{YP} = 152.07 \text{ MPa}$$

$$\text{maximum shear stress} = \frac{\sigma_1 - \sigma_3}{2} = 87.5 \text{ MPa}$$

$\Rightarrow$  distortion-energy theory control!

12-6

from von Mises yield criterion

$$(60\alpha - 50\alpha)^2 + (50\alpha - 70\alpha)^2 + (70\alpha - 60\alpha)^2 = 2(250)^2$$

$$600\alpha^2 = 125000$$

$$\alpha = 14.43$$

12-7

Tresca:

$$\tau_{max} = \frac{\sigma_{YP}}{2} = \left[ \left( \frac{\sigma_x}{2} \right)^2 + \tau_{xy}^2 \right]^{\frac{1}{2}}$$

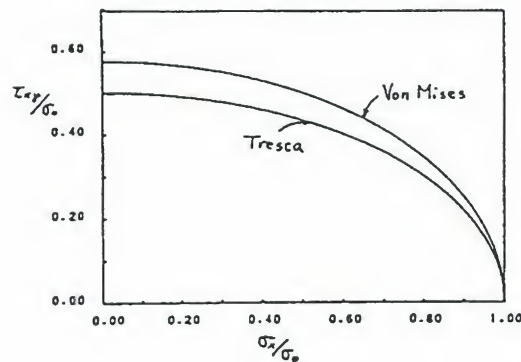
$$\left( \frac{\sigma_x}{\sigma_{YP}} \right)^2 + 4 + \left( \frac{\tau_{xy}}{\sigma_{YP}} \right)^2 = 1$$

Von Mises:

$$\sigma_1^2 - \sigma_1 \sigma_2 + \sigma_2^2 = \sigma_{YP}^2$$

$$\sigma_1 \text{ or } \sigma_2 = \frac{\sigma_x}{2} \pm \left[ \left( \frac{\sigma_x}{2} \right)^2 + \tau_{xy}^2 \right]^{\frac{1}{2}}$$

$$\left( \frac{\sigma_x}{\sigma_{YP}} \right)^2 + 3 \left( \frac{\tau_{xy}}{\sigma_{YP}} \right)^2 = 1$$



12-8

In pure shear  $\sigma_1 = \tau_0, \sigma_2 = 0,$

$$\sigma_3 = -\tau_0$$

$$J_2 = \frac{1}{6} [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2] = \tau_0^2$$

for simple tension  $\sigma_2 = \sigma_3 = 0$

$$\frac{2\sigma_1^2}{6} = \tau_0^2 \Rightarrow \sigma_1 = \sqrt{3} \tau_0 \text{ in Von Mises}$$

$$\tau_{max} = \frac{\sigma_1}{2} = \tau_0 \Rightarrow \sigma_1 = 2\tau_0 \text{ in Tresca}$$

for  $\sigma_1 = \sigma_2$

$$\frac{2\sigma_1^2}{6} = \tau_0^2 \Rightarrow \sigma_1 = \sqrt{3} \tau_0 \text{ in Von Mises}$$

$$\tau_{max} = \frac{\sigma_1}{2} = \tau_0 \Rightarrow \sigma_1 = 2\tau_0 \text{ in Tresca}$$

$$12-9 \quad (a) \quad \frac{5P+P}{2} = \frac{60}{2} \Rightarrow P = 10$$

$$\sigma_1 = 50 \text{ ksi}, \quad \sigma_2 = 20 \text{ ksi}, \quad \sigma_3 = -10 \text{ ksi}$$

$$(b) \quad [(5-2)^2 + (2+1)^2 + (5+1)^2] P^2 = 2 \times 60^2$$

$$P = 11.547$$

$$\sigma_1 = 57.7 \text{ ksi}, \quad \sigma_2 = 23.1 \text{ ksi}, \quad \sigma_3 = -11.5 \text{ ksi}$$

$$12-10 \quad \epsilon_3 = \frac{1}{E} (\sigma_3 - \nu \sigma_1 - \nu \sigma_2) = 0$$

$$\therefore \sigma_2 = 0 \quad \therefore \sigma_3 = \nu \sigma_1$$

$$\bar{J}_2 = \frac{1}{3} \sigma_{yp}^2 \Rightarrow \sigma_1 = \frac{\sigma_{yp}}{(1-\nu+\nu^2)^{\frac{1}{2}}}$$

Use Tresca :

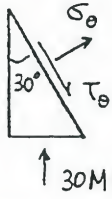
$$\frac{\sigma_1 - \sigma_3}{2} = \frac{\sigma_{yp}}{2}, \quad \sigma_3 = \nu \sigma_1$$

$$\sigma_1 = \frac{\sigma_{yp}}{1-\nu}$$



13-1

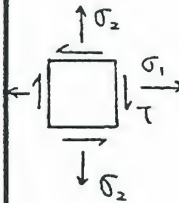
$$\sigma_\theta = \frac{-30}{2} + \frac{30}{2} \cos 60^\circ = -\frac{15}{2} \text{ MPa}$$



$$\tau_\theta = -\frac{30}{2} \sin 60^\circ = -\frac{15\sqrt{3}}{2} \text{ MPa}$$

13-2

$$\tau = \frac{T\rho}{I_p} = \frac{(200 \times 10^4)(50)}{\frac{\pi}{2}(50^4 - 46^4)} = 35.92 \text{ MPa}$$



$$\sigma_1 = \frac{Pr}{t} = \frac{(1.5)(50)}{(4)} = 18.75 \text{ MPa}$$

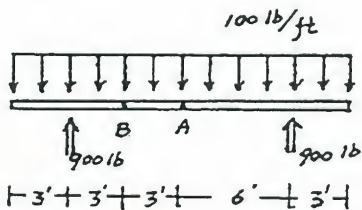
$$\sigma_2 = \frac{Pr}{2t} = \frac{(1.5)(50)}{2(4)} = 9.38 \text{ MPa}$$

$$R = \sqrt{[(18.75 - 9.38)/2]^2 + (35.92)^2} = 36.22 \text{ MPa}$$

$$\sigma_{\max} = \frac{18.75 + 9.38}{2} + 36.22 = 50.29 \text{ MPa}$$

$$\sigma_{\min} = \frac{18.75 + 9.38}{2} - 36.22 = -22.16 \text{ MPa}$$

13-3



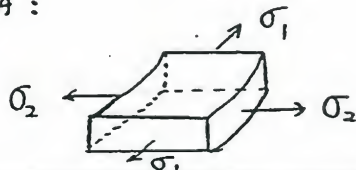
$$V_A = 0$$

$$M_A = (900 \times 6 - \frac{1}{2} \times 100 \times 9^2) \times 12 = 16200 \text{ lb-in}$$

$$V_B = 300 \text{ lb}$$

$$M_B = (900 \times 3 - \frac{1}{2} \times 100 \times 6^2) \times 12 = 10800 \text{ lb-in}$$

element A:



$$\sigma_1 = \frac{Pr}{t} = \frac{100 \times 20}{0.2} = 10000 \text{ psi}$$

$$\sigma_2 = \frac{Pr}{2t} + \frac{M_A r}{I}$$

$$= \frac{100 \times 20}{2 \times 0.2} + \frac{16200 \times 20}{\pi \times 20^3 \times 0.2}$$

$$= 5000 + 64.46$$

$$= 5064.46 \text{ psi}$$

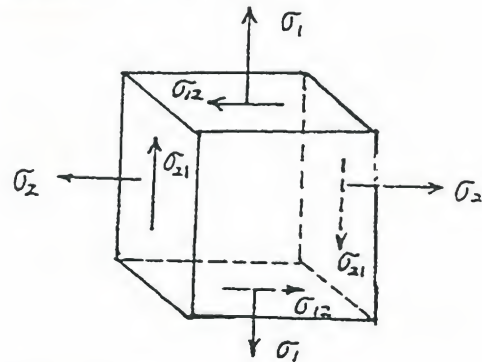
$$\frac{M_A r}{I} = 64.46 \text{ psi} \ll \frac{Pr}{2t} = 5000 \text{ psi}$$

(due to dead weight)

$$\sigma_3 \approx 0$$

$$\sigma_{13} = 0 \quad (\because V_A = 0)$$

element B:



$$\sigma_1 = \frac{Pr}{t} = \frac{100 \times 20}{0.2} = 10000 \text{ psi}$$

$$\sigma_2 = \frac{Pr}{2t} + \frac{M_B \cdot 0}{I} = 5000 \text{ psi}$$

$$\sigma_3 \approx 0$$

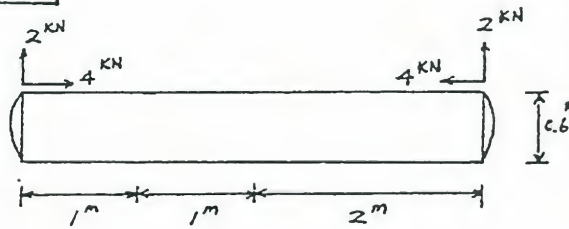
$$\sigma_{12} = \frac{V_B \cdot Q_B}{I \cdot 2t} = \frac{V_B \cdot 2tr^2}{\pi r^4 \cdot 2t} = \frac{V_B}{\pi r t}$$

$$= \frac{300}{\pi \times 10 \times 0.2} = 47.74 \text{ psi}$$

$$\frac{V_B Q_B}{I \cdot 2t} = 47.74 \text{ psi} \ll \frac{Pr}{2t} = 5000 \text{ psi}$$

$\therefore$  the dead weight has little influence on the total stress.

13-4



$$W = 102 \times 9.81 = 1000 \text{ N/m} = 1 \text{ kN/m}$$

$$V_A = 0, \quad M_A = \frac{1}{8} W L^2 = \frac{1}{8} \times 1 \times 4^2 = 2 \text{ kN-m}$$

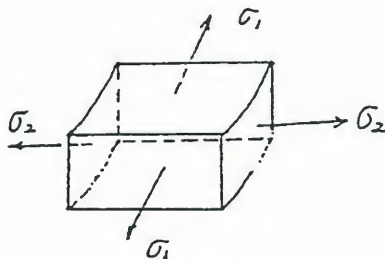
$$V_B = 2 - 1 \times 1 = 1 \text{ kN}$$

$$M_B = 2 \times 1 - \frac{1}{2} \times 1 \times 1^2 = 1.5 \text{ kN-m}$$

$$P_A = P_B = -4 \text{ kN}, \quad P_i = 500 \text{ kPa}$$

$$M = 4 \times 0.3 = 1.2 \text{ kN-m}$$

element A:

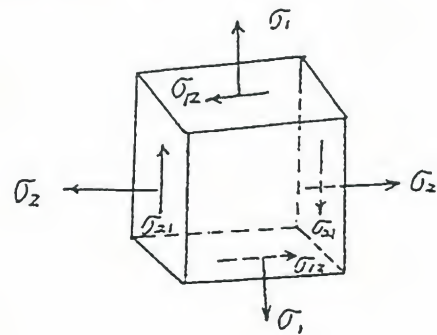


$$\sigma_1 = \frac{P_i r}{t} = \frac{500 \times 300}{6} = 25 \text{ MPa}$$

$$\begin{aligned} \sigma_2 &= \frac{P_i r}{2t} + \frac{P_A}{A} + \frac{(M_A + M)r}{I} \\ &= 12.5 - \frac{4 \times 10^{-3}}{\pi \times 0.6 \times 6 \times 10^{-3}} + \frac{3.2 \times 10^{-3} \times 0.3}{\pi \times (0.3)^2 \times 6 \times 10^{-3}} \\ &= 12.5 - 0.35 + 1.89 \\ &= 14.04 \text{ MPa} \end{aligned}$$

$$\sigma_3 \approx 0.5 \text{ MPa} \approx 0$$

element B:



$$\sigma_1 = \frac{P_i r}{t} = 25 \text{ MPa}$$

$$\sigma_2 = \frac{P_i r}{2t} - \frac{P_B}{A} = 12.5 - 0.35 = 12.15 \text{ MPa}$$

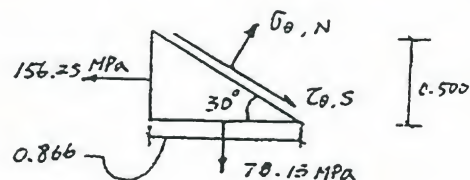
$$\sigma_3 \approx 0$$

$$\begin{aligned} \sigma_{12} &= \frac{V_B \theta_B}{I \cdot 2t} = \frac{V_B}{\pi \cdot r t} = \frac{1 \times 10^{-3}}{\pi \times 0.3 \times 6 \times 10^{-3}} \\ &= 0.178 \text{ MPa} \approx 0 \end{aligned}$$

13-5

$$\sigma_1 = \frac{P_i r_i}{t} = \frac{1.5 \times (2.5/2)}{0.012} = 156.25 \text{ MPa}$$

$$\sigma_2 = \frac{1}{2} \sigma_1 = 78.13 \text{ MPa}$$



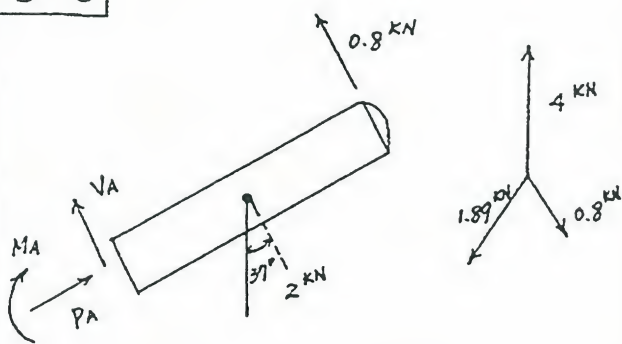
$$\begin{aligned} N &= 156.25 (0.5 \times 0.5) + 78.13 (0.866) (0.866) \\ &= 97.66 \text{ MN} \end{aligned}$$

$$\sigma_\theta = 97.66 \text{ MPa}$$

$$\begin{aligned} S &= 156.25 (0.5) (0.866) - 78.13 (0.866) (0.5) \\ &= 33.83 \text{ MN} \end{aligned}$$

$$\tau = 33.83 \text{ MPa}$$

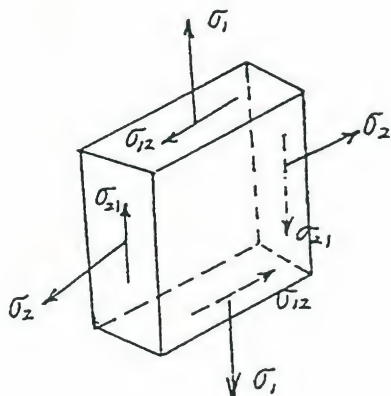
13-6



$$V_A = 2 \times \frac{4}{5} - 0.8 = 0.8 \text{ kN}$$

$$P_A = -2 \times \frac{3}{5} = -1.2 \text{ kN}$$

$$P_i = 0.5 \text{ MPa}$$



$$\sigma_1 = \frac{P_i r}{t} = \frac{500 \times 300}{6} = 25 \text{ MPa}$$

$$\sigma_2 = \frac{P_i r}{2t} + \frac{P_A}{A} = 12.5 - \frac{1.2 \times 10^{-3}}{\pi \times 0.6 \times 6 \times 10^{-3}}$$

$$= 12.4 \text{ MPa}$$

$$\sigma_3 \approx 0$$

$$\sigma_{12} = \frac{V_A Q_A}{I \cdot 2t} = \frac{V_A}{\pi r t} = \frac{0.8 \times 10^{-3}}{\pi \times 0.3 \times 6 \times 10^{-3}}$$

$$= 0.14 \text{ MPa} \approx 0$$

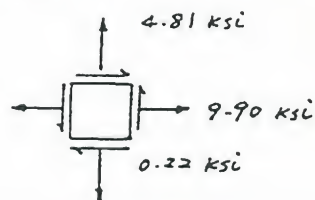
13-7

$$\sigma_1 = \frac{P r_{\text{mean}}}{t} = \frac{600 \times 6.19}{0.375 \times (10)^3} = 9.90 \text{ ksi}$$

$$\sigma_2 = \sigma_1 / 2 = 4.95 \text{ ksi}$$

$$\sigma_w = \frac{W}{A} = - \frac{49.56 \times 40}{14.58 \times (10)^3} = -0.14 \text{ ksi}$$

$$\tau = \frac{VQ}{It} = \frac{(40 \times 40) \times \frac{14.58 \times 2}{2} \times 6.19}{279.3 \times 0.75 \times (10)^3} = 0.22 \text{ ksi}$$



13-8

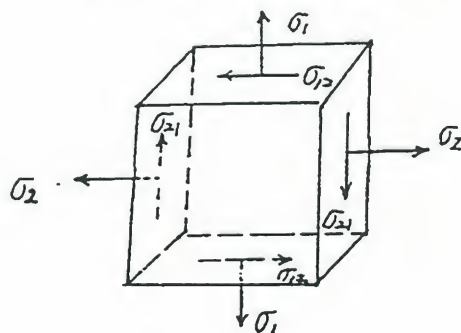
$$V_A = P = 31.4 \text{ K}$$

$$T = 31.4 \times 10 = 314 \text{ K-in}$$

$$M_A = 31.4 \times 20 = 628 \text{ K-in}$$

$$P_i = 250 \text{ psi}$$

$$r = 10 \text{ in}, t = 0.25 \text{ in}$$



$$(a) \sigma_1 = \frac{P_i r}{t} = 250 \times \frac{10}{0.25} = 10000 \text{ psi} = 10 \text{ ksi}$$

$$\sigma_2 = \frac{P_i r}{2t} = 5 \text{ ksi}$$

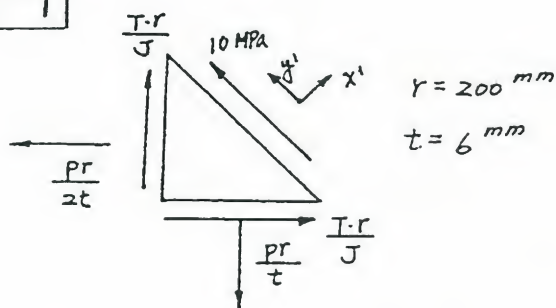
$$\sigma_3 \approx 0 = \sigma_3'$$



$$\begin{aligned}\sigma_{12} &= \frac{V_A Q_A}{I \cdot 2t} + \frac{T r}{J} \\ &= \frac{V_A}{\pi r t} + \frac{T}{2\pi r^2 t} \\ &= \frac{31.4}{\pi \times 10 \times 0.25} + \frac{314}{2\pi \times 10^3 \times 0.25} \\ &= 4 + 2 = 6 \text{ ksi}\end{aligned}$$

$$\begin{aligned}(b) \quad \sigma_1^P &= \frac{10+5}{2} + \sqrt{\left(\frac{10-5}{2}\right)^2 + 6^2} = 14 \text{ ksi} \\ \sigma_2^P &= \frac{10+5}{2} - \sqrt{\left(\frac{10-5}{2}\right)^2 + 6^2} = 1 \text{ ksi} \\ \tau_{max} &= \frac{\sigma_1^P - \sigma_2^P}{2} = \frac{14}{2} = 7 \text{ ksi}\end{aligned}$$

13-9



$$\sum F_{x'} = 0$$

$$2 \times \frac{1}{\sqrt{2}} \cdot \frac{T \cdot r}{J} - \frac{pr}{2t} \cdot \frac{1}{\sqrt{2}} - \frac{pr}{t} \cdot \frac{1}{\sqrt{2}} = 0$$

$$\sum F_{y'} = 0$$

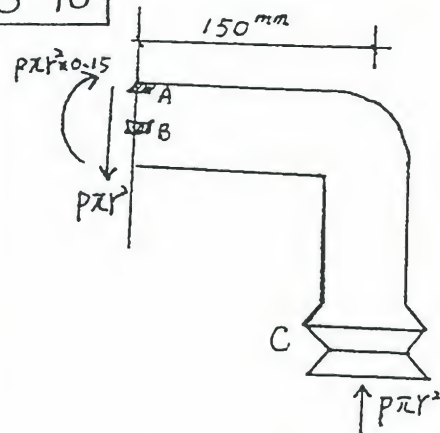
$$\frac{pr}{2t} \cdot \frac{1}{\sqrt{2}} + 10\sqrt{2} - \frac{pr}{t} \cdot \frac{1}{\sqrt{2}} = 0$$

$$\Rightarrow \frac{p \cdot r}{t} = 40 \text{ MPa}, \quad \frac{T r}{J} = 10 \text{ MPa}$$

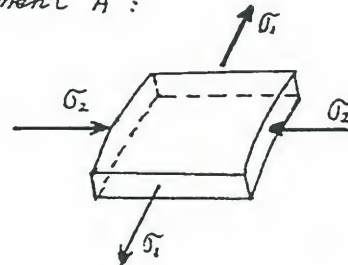
$$\Rightarrow p = 40 \times \frac{6}{200} = 1.2 \text{ MPa}$$

$$T = 2\pi r^2 t \times 10 = 15.08 \text{ kN-m}$$

13-10



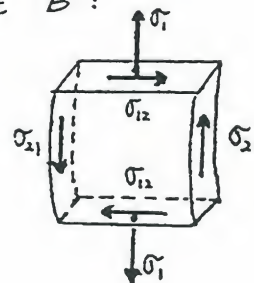
element A :



$$\sigma_1 = \frac{p \cdot r}{t} = \frac{2 \times 30}{2} = 30 \text{ MPa}$$

$$\begin{aligned}\sigma_2 &= \frac{M r}{I} = \frac{p \pi r^2 \cdot 0.15 \times r}{\pi r^3 t} = \frac{2 \times 0.15}{2 \times 10^{-3}} \\ &= -150 \text{ MPa}\end{aligned}$$

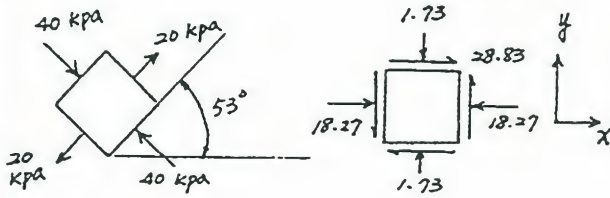
element B :



$$\sigma_1 = \frac{p \cdot r}{t} = \frac{2 \times 30}{2} = 30 \text{ MPa}$$

$$\begin{aligned}\sigma_{12} &= \frac{V Q}{I \cdot 2t} = \frac{V}{\pi \cdot r \cdot t} = \frac{p \pi \cdot r^2}{\pi \cdot r \cdot t} \\ &= \frac{pr}{t} = 30 \text{ MPa}\end{aligned}$$

13-11



$$\sigma_x = \frac{20 + (-40)}{2} + \frac{20 - (-40)}{2} \cos(2 \times 53^\circ)$$

$$= -18.27 \text{ kPa}$$

$$\sigma_y = \frac{20 + (-40)}{2} + \frac{20 - (-40)}{2} \cos(2 \times 37^\circ)$$

$$= -1.73 \text{ kPa}$$

$$\tau_{xy} = -\frac{20 - (-40)}{2} \sin(2 \times 53^\circ)$$

$$= 28.83 \text{ kPa}$$

$$Q_A = 2 \times 2 \times \frac{1}{4} \times 8 = 8 \text{ in}^3$$

$$V_A = 12 \text{ K}, P_A = 24.$$

$$M_A = 12 \times 36 - 24(2 \times 12 - 9) = 72 \text{ K-in}$$

$$\sigma = \frac{M_A \cdot Y_A}{I} - \frac{P_A}{A} = \frac{-72 \times 7}{165} - \frac{24}{15}$$

$$= -4.65 \text{ ksi}$$

$$\tau = \frac{V_A \cdot Q_A}{I t} = \frac{12 \times 8}{165 \times \frac{7}{4}} = 1.16 \text{ ksi}$$

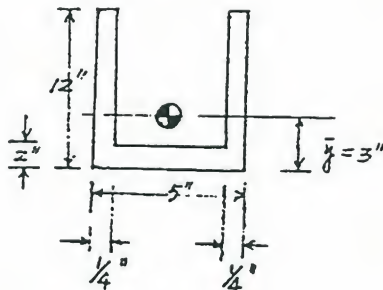
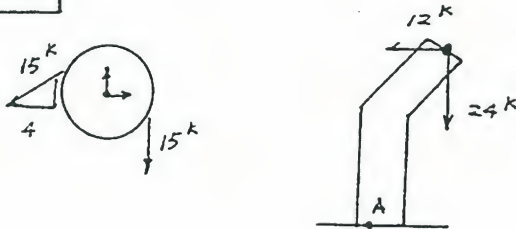
$$\sigma_{\max} = \frac{-4.65}{2} + \sqrt{\left(\frac{4.65}{2}\right)^2 + (1.16)^2}$$

$$= 0.272 \text{ ksi}$$

$$\sigma_{\min} = \frac{-4.65}{2} - \sqrt{\left(\frac{4.65}{2}\right)^2 + (1.16)^2}$$

$$= -4.93 \text{ ksi}$$

13-12

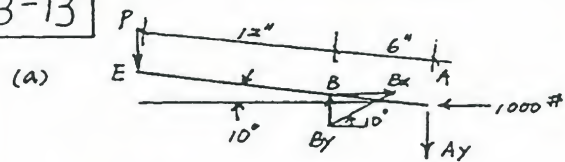


$$\bar{y} = \frac{12 \times 5 \times 6 - 10 \times 4.5 \times 7}{12 \times 5 - 10 \times 4.5} = \frac{45}{15} = 3''$$

$$A = 15 \text{ in}^2$$

$$I = 2 \times \left( \frac{1}{3} \times \frac{1}{4} \times 9^3 + \frac{1}{3} \times \frac{1}{4} \times 3^3 \right) + \frac{1}{12} \times 4.5 \times 2^3 + 4.5 \times 2 \times 2^2 = 165 \text{ in}^4$$

13-13



$$\sum F_x = 0$$

$$B_x = 1000 \text{ lb} \rightarrow$$

$$B_y = B_x \tan 10^\circ$$

$$= 1000 (0.176)$$

$$= 176.3 \text{ lb}$$

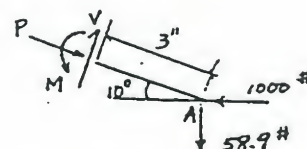
$$\sum M_A = 0 \quad (+)$$

$$18 \cos 10^\circ P - 176 (6 \cos 10^\circ) - 1000 (6 \sin 10^\circ) = 0$$

$$P = 117.4 \text{ lb}$$

$$\sum F_y = 0, A_y = 176.3 - 117.4 = 58.9 \text{ lb}$$

(b)



$$P = 1000 \cos 10^\circ - 58.9 \sin 10^\circ$$

$$= 974.6 \text{ lb} (-)$$

$$V = 1000 \sin 10^\circ + 58.9 \cos 10^\circ$$

$$= 231.6 \text{ lb}$$

$$M = 1000 (3 \sin 10^\circ) + 58.9 (3 \cos 10^\circ)$$

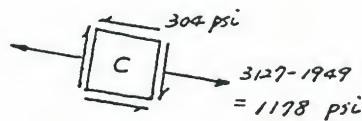
$$= 694.9 \text{ lb}$$

$$\sigma_P = \frac{P}{A} = \frac{974.6}{\frac{1}{4} \times 2} = 1949.2 \text{ psi} (-)$$

$$\sigma_M = \frac{My}{I} = \frac{694.9 \times (\frac{3}{4})}{\frac{\pi}{4} (2)^3 \times \frac{1}{2}} = 3127.1 \text{ psi} (+)$$

$$\tau = \frac{VQ}{It} = \frac{231.6 (\frac{1}{4} \times \frac{1}{4} \times \frac{7}{8})}{\frac{\pi}{4} (2)^3 \times \frac{1}{2} (\frac{1}{4})} = 304.0 \text{ psi}$$

(c)



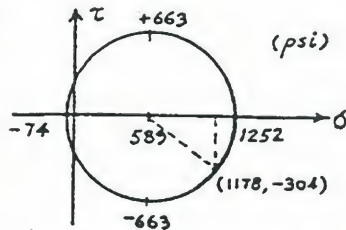
(d) center :  $\frac{1178}{2} = 589 \text{ psi}$

$$r = \sqrt{(589)^2 + (304)^2} = 663 \text{ psi}$$

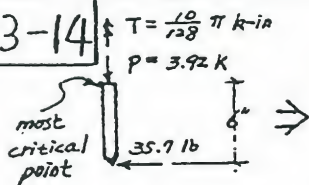
$$\sigma_1 = 1252 \text{ psi}$$

$$\sigma_2 = -74 \text{ psi}$$

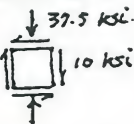
$$\tau_{max} = 663 \text{ psi}$$



13-14



At the critical point



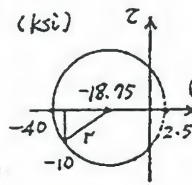
$$\sigma = \frac{P}{A} + \frac{MC}{I} = \frac{3.92}{0.196} + \frac{(0.0357 \times 6) 0.25}{0.00306}$$

$$= 57.5 \text{ ksi} (-)$$

$$\tau = \frac{TC}{J} = \frac{10 \pi (0.25)}{128 (0.00612)} = 10.0 \text{ ksi}$$

$$\text{center} = \frac{-37.5}{2} = -18.75$$

$$r = \sqrt{(18.75)^2 + (10)^2} = 21.25$$

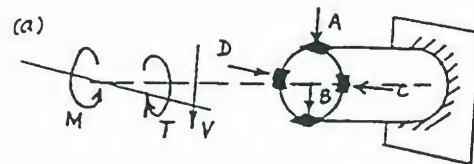


$$\sigma_1 = -18.75 + 21.25 = 2.5 \text{ ksi}$$

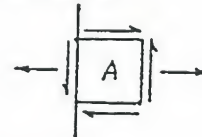
$$\sigma_2 = -18.75 - 21.25 = -40.0 \text{ ksi}$$

$$\therefore \sigma_{max} = -40.0 \text{ ksi}$$

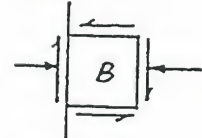
13-15



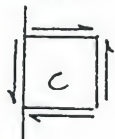
Point A



Point B



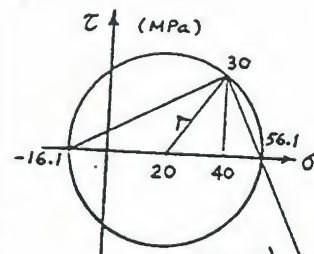
Point C



Point D



(b)



Point A

$$\text{center} = \frac{40}{2} = 20$$

$$r = \sqrt{(20)^2 + (30)^2} = 36.1$$

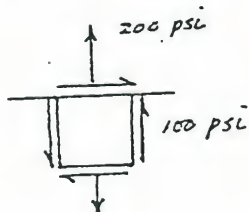




13-16

Point A

(a)

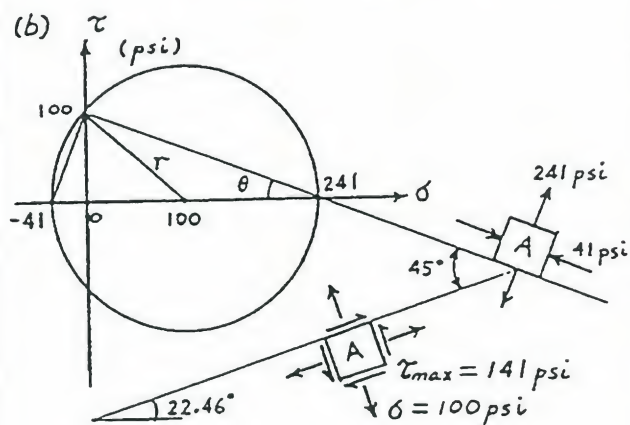


$$\text{center} = \frac{200}{2} = 100$$

$$r = \sqrt{(100)^2 + (100)^2} = 141 \text{ psi}$$

$$\theta = \tan^{-1} \left( \frac{100}{241} \right) = 22.54^\circ$$

(b)



13-17

(a)



$$F_x = 2000 \text{ N}$$

$$F_y = -2000 \text{ N}$$

$$F_z = -1000 \text{ N}$$

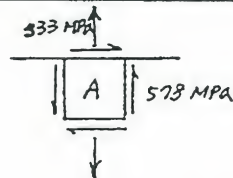
$$A = 12 \times 12 = 144 \text{ mm}^2$$

$$I = \frac{12(12)^3}{12} = 1728 \text{ mm}^4$$

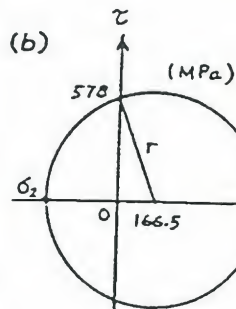
Point A

$$\sigma_A = \frac{F_y}{A} + \frac{M_{xx} \cdot c}{I} = -\frac{2000}{144} - \frac{(1000 \times 100) \times 6}{1728} = 333 \text{ MPa}$$

$$\tau_A = \frac{3V}{2A} + \frac{T}{2bc^3} = \frac{3}{2} \left( \frac{2000}{144} \right) + \frac{1000 \times 200}{0.208 \times 12 \times 12^3} = 578 \text{ MPa}$$



(b)



$$\text{center} = \frac{333}{2} = 166.5$$

$$r = \sqrt{(166.5)^2 + (578)^2} = 601.5 \text{ MPa}$$

$$\sigma_2 = 166.5 - 601.5 = -435 \text{ MPa}$$

$$\sigma_1 = 166.5 + 601.5 = 768 \text{ MPa}$$

13-18

$$F_x = 135 \pi \text{ N} \rightarrow, F_y = 180 \pi \text{ N} \downarrow$$

$$I = \frac{\pi (0.025)^4}{4} = 4.77 \times 10^{-8} \pi \text{ m}^4$$

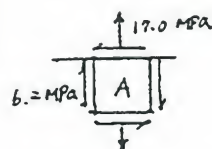
$$J = \frac{\pi (0.025)^4}{2} = 1.95 \times 10^{-7} \pi \text{ m}^4$$

$$A = \pi (0.025)^2 = 6.25 \times 10^{-4} \pi \text{ m}^2$$

Point A

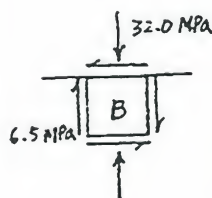
$$\sigma_A = \frac{F_y}{A} + \frac{M_{xx} \cdot c}{I} = -\frac{180 \pi}{6.25 \times 10^{-4} \pi} + \frac{(135 \pi \times 0.375 \times 0.05)}{4.77 \times 10^{-8} \pi} = 17.0 \text{ MPa}$$

$$\tau_A = \frac{TC}{J} + \frac{VQ}{It} = -\frac{135 \pi \times 0.375 (0.025)}{1.95 \times 10^{-7} \pi} + \frac{135 \pi (6.25 \times 10^{-4} \pi / 2) (\frac{4 \times 0.025}{3 \pi})}{9.77 \times 10^{-8} \pi (0.05)} = -6.2 \text{ MPa}$$



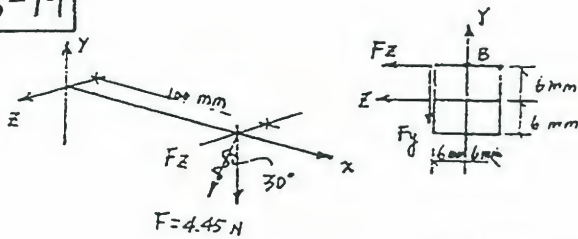
Point B

$$\sigma_B = \frac{F_y}{A} + \frac{M_{zz} \cdot x}{I} = -\frac{180 \pi}{6.25 \times 10^{-4} \pi} - \frac{(135 \pi \times 0.50 \times 0.025)}{4.77 \times 10^{-8} \pi} = -32.0 \text{ MPa}$$



$$\tau_B = \frac{TC}{J} = \frac{-135 \pi \times 0.375 (0.025)}{1.95 \times 10^{-7} \pi} = -6.5 \text{ MPa}$$

13-19



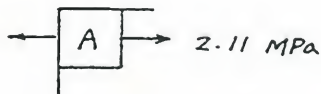
$$F_z = 4.45 \sin 45^\circ = 3.12 \text{ N}$$

$$F_y = 4.45 \cos 45^\circ = 3.12 \text{ N}$$

Point A

$$\sigma_A = \frac{M_{yy} C}{I} + \frac{M_{zz} C}{I} = \frac{(F_z + F_y) 100 \times 6}{1728} = 2.11 \text{ MPa}$$

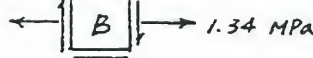
$$\tau_A = 0$$



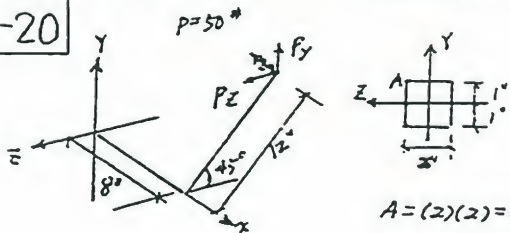
Point B

$$\sigma_B = \frac{M_{zz} C}{I} = \frac{(F_y \times 100) \times 6}{1728} = 1.34 \text{ MPa}$$

$$\tau_B = \frac{3V}{2A} + \frac{T}{\alpha b c^3} = -\frac{3F_z}{2(144)} - \frac{(F_y + F_z) \times 6}{0.208(12)(12)^3} = -0.023 - 0.009 = -0.032 \text{ MPa}$$



13-20



$$A = (2)(2) = 4 \text{ in}^2$$

$$I = \frac{(2)(2)^3}{12} = \frac{4}{3} \text{ in}^4$$

$$\sigma_A = \frac{M_{yy} C}{I} = -\frac{(P_z \times 8)(1)}{4/3} = -\frac{(50 \cos 45^\circ \times 8) \times 3}{4} = -212.1 \text{ psi}$$

$$\tau_A = \frac{3V}{2A} + \frac{T}{\alpha b c^3} = \frac{3P_y}{2A} - \frac{P(12)}{0.208 \times 2 \times (2)^3} = \frac{3(50 \sin 45^\circ)}{2(4)} - \frac{50(12)}{(0.208)(16)} = -167.0 \text{ psi}$$

13-21

$$A = 1.704 \text{ in}^2, I = 1.530 \text{ in}^4$$

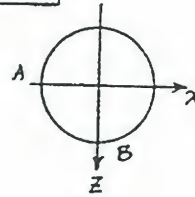
$$P = 400 \text{ lb} (-)$$

$$V = 90 \text{ lb}$$

$$M_{zz} = 400 \times 3 = 1200 \text{ in-lb}$$

$$M_{xx} = 90 \times 10 = 900 \text{ in-lb}$$

$$T = 90 \times 3 = 270 \text{ in-lb}$$



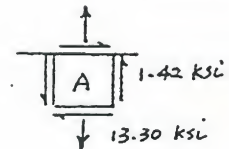
Point A

$$\sigma_A = \frac{P}{A} + \frac{M_{zz} C}{I} = -\frac{400}{1.704} + \frac{1200 \times 12 \left( \frac{2.875}{2} \right)}{1.530} = +13,295 \text{ psi}$$

$$\tau_A = \frac{VQ}{It} + \frac{TC}{J}$$

$$= -\frac{90 \left( \frac{1.704}{2} \times \frac{2}{\pi} \times 1.33 \right)}{1.530 \times 0.406} + \frac{270 \times 12 \times (1.438)}{2 \times 1.530}$$

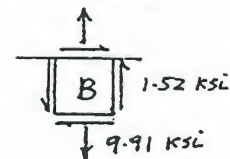
$$= 1,418 \text{ psi}$$



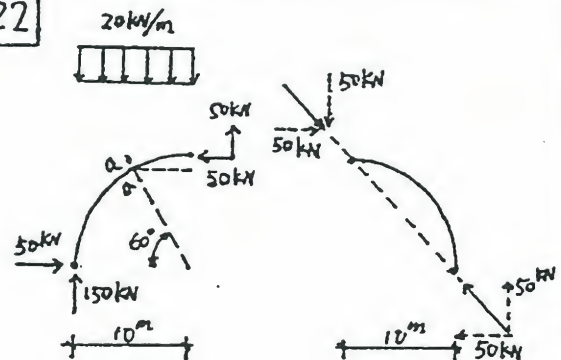
Point B

$$\sigma_B = \frac{P}{A} + \frac{M_{xx} C}{I} = -\frac{400}{1.704} + \frac{900 \times 12 \left( \frac{2.875}{2} \right)}{1.530} = +9,912 \text{ psi}$$

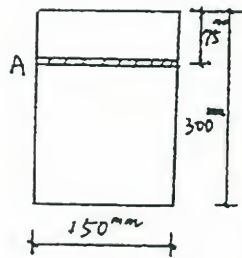
$$\tau_B = \frac{TC}{J} = \frac{270 \times 12 \times (1.438)}{2 \times 1.530} = 1,523 \text{ psi}$$



13-22





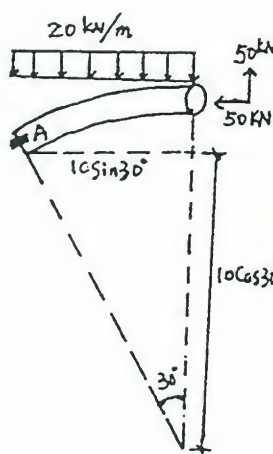


section a-a

$$A = 0.3 \times 0.15 = 0.045 \text{ m}^2$$

$$I = \frac{1}{12} \times 0.15 \times 0.3^3 = 3.375 \times 10^{-4} \text{ m}^4$$

$$Q = 0.075 \times 0.15 \times 0.1125 = 1.266 \times 10^{-3} \text{ m}^3$$



$$M = \frac{1}{2} \times 20 \times (10 \sin 30^\circ)^2 - 50 \times 10 \sin 30^\circ - 50 (10 - 10 \cos 30^\circ) = -67 \text{ kN-m}$$

$$P = -(50 \sin 30^\circ + 50 \sin 60^\circ) = -68.3 \text{ kN}$$

$$V = 18.3 \text{ kN}$$

$$\sigma = \frac{-68.3}{0.045} - \frac{67 \times 0.075}{3.375 \times 10^{-4}}$$

$$= -16.42 \text{ MPa}$$



$$\tau = \frac{VQ}{It} = \frac{18.3 \times 1.266 \times 10^{-3}}{3.375 \times 10^{-4} \times 0.15} = 0.44 \text{ MPa}$$

$$\sigma_2 = \frac{-16.42}{2} - \sqrt{\left(\frac{-16.42}{2}\right)^2 + 0.44^2} = -16.43 \text{ MPa}$$

$$\sigma_1 = -\frac{16.42}{2} + \sqrt{\left(\frac{-16.42}{2}\right)^2 + 0.44^2} = 0.01 \text{ MPa}$$

13-23

$$V = 9.22 \text{ k} \downarrow, M = 9.22 \times 10 = 92.2 \text{ k-in}$$

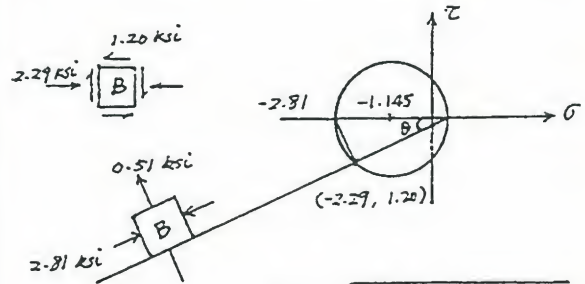
Point A

$$\sigma_A = \frac{Mc}{I} = \frac{92.2 \times 6}{221} = 2.50 \text{ ksi}$$

Point B

$$\sigma_B = \frac{5.50 \times 2.50}{6} = 2.29 \text{ ksi}$$

$$\tau_B = \frac{VQ}{It} = \frac{9.22 \left(\frac{1}{2} \times 5\right) 5.75}{221 (0.5)} = 1.20 \text{ ksi}$$



$$r = \sqrt{(1.145)^2 + (1.20)^2}$$

$$= 1.66$$

$$\theta = \tan^{-1}\left(\frac{1.20}{2.80}\right) = 23.20^\circ$$

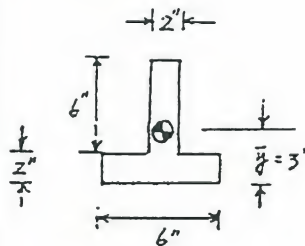
Point C  $\sigma_c = 0$

$$\tau_c = \frac{9.22 \left(\frac{1}{2} \times 5 \times 5.75 + \frac{1}{2} \times 5.5 \times 2.75\right)}{221 \times 0.5}$$

$$= 1.83 \text{ ksi}$$



13-24



$$\bar{y} = \frac{6 \times 2 \times 5 + 2 \times 6 \times 1}{6 \times 2 + 2 \times 6} = 3"$$

$$A = 24 \text{ in}^2$$

$$I = \frac{1}{3} \times 2 \times 5^3 + \frac{1}{3} \times 2 \times 1^3 + \frac{1}{3} \times 6 \times 2^3 + 6 \times 2 \times 2^2 = 100 \text{ in}^4$$

$$Q_A = 2 \times 2 \times 4 = 16 \text{ in}^3$$

$$\left(\frac{4}{5}P \times 10 - \frac{3}{5}P \times 3\right) \cdot 3 - \frac{\frac{3}{5}P}{A}$$

$$= E \times 20 \times 10^{-6} = 600$$

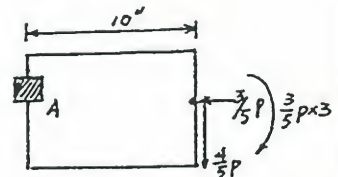
$$(a) \Rightarrow P = 3.73 \text{ K} = 3730 \text{ lb}$$

$$(b) V_A = \frac{4}{5}P$$

$$Q_A = 16$$

$$\tau = \frac{V_A Q_A}{It} = \frac{\frac{4}{5}P \times 16}{100 \times 2}$$

$$= 238.72 \text{ psi}$$





ref Appendix.

$$G = 12 \times 10^6 \text{ psi}$$

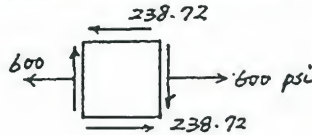
$$\therefore \frac{\tau}{2} = \frac{\tau}{2G}$$

$$= 9.95 \times 10^{-6}$$

Max. principal strain  $\epsilon_1$

$$= \frac{1}{2}(20 \times 10^{-6} + 0) + \sqrt{\left(\frac{20 \times 10^{-6}}{2}\right)^2 + (9.95 \times 10^{-6})^2}$$

$$= 24.1 \times 10^{-6}$$



element A :

$$\sigma_1 = \sigma_y = \frac{-80}{0.05} = -1.6 \times 10^3 \text{ kPa}$$

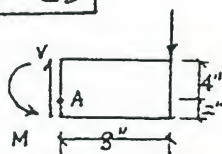
$$= -1.6 \text{ MPa}$$

$$\sigma_2 = \sigma_x = -\frac{M \cdot \frac{h}{2}}{I}$$

$$= \frac{-20 \times 0.06}{7.2 \times 10^{-6}} = -166.7 \text{ MPa}$$

x-y direction is the principal stress and direction.

13-25



$$I = \frac{bh^3}{12} = \frac{2(6)^3}{12} = 36 \text{ in}^4$$

$$V = P$$

$$M = 8P$$

$$\sigma_x = \frac{My}{I} = \frac{8P(1)}{36} = \frac{2P}{9}$$

$$\tau = \frac{VQ}{Ib} = \frac{P(2 \times 2 \times 2)}{36 \times 2} = \frac{P}{9}$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + (\tau)^2}$$

$$120 = \frac{P}{9} \sqrt{(1)^2 + (1)^2} = \frac{1.414P}{9}$$

$$P = \frac{120 \times 9}{1.414}$$

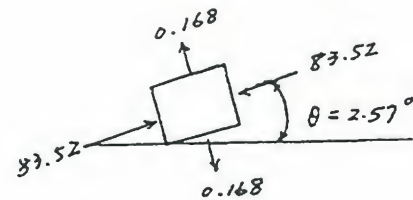
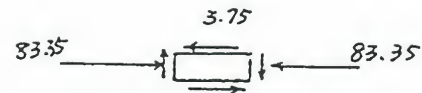
$$P = 764 \text{ lb.}$$

element B :

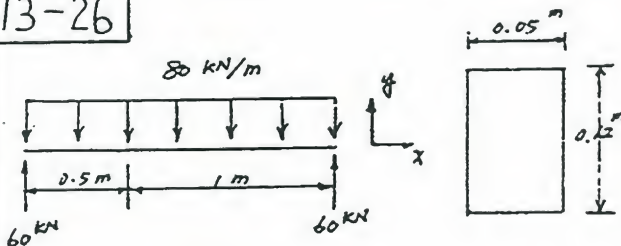
$$\sigma_x = -\frac{M \cdot y}{I} = -166.7 \times 0.5 = -83.35 \text{ MPa}$$

$$\tau_{xy} = \frac{VQ}{I \cdot t} = \frac{20 \times 0.03 \times 0.05 \times 0.045}{7.2 \times 10^{-6} \times 0.05}$$

$$= 3.75 \text{ MPa}$$



13-26



$$I = \frac{1}{12} \times 0.05 \times 0.12^3 = 7.2 \times 10^{-6} \text{ m}^4$$

$$V = 60 - 80 \times 0.5 = 20 \text{ kN}$$

$$M = 60 \times 0.5 - \frac{1}{2} \times 80 \times 0.5^2 = 20 \text{ kN} \cdot \text{m}$$

$$\sigma_1 = \frac{0 + (-83.35)}{2} + \sqrt{\left(\frac{83.35}{2}\right)^2 + (3.75)^2}$$

$$= 0.168 \text{ MPa}$$

$$\sigma_2 = \frac{-83.35}{2} - \sqrt{\left(\frac{83.35}{2}\right)^2 + (3.75)^2}$$

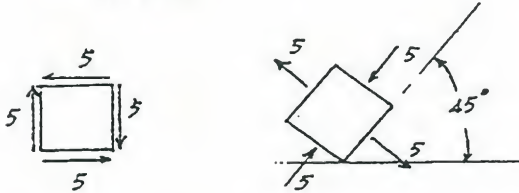
$$= -83.52 \text{ MPa}$$

$$\theta = \frac{1}{2} \tan^{-1} \frac{3.75 \times 2}{0 - (-83.35)} = 2.57^\circ$$

element C:

$$\tau_{xy} = \frac{VQ}{It} = \frac{20 \times 0.05 \times 0.06 \times 0.03}{7.2 \times 10^{-6} \times 0.05}$$

$$= 5 \text{ MPa}$$

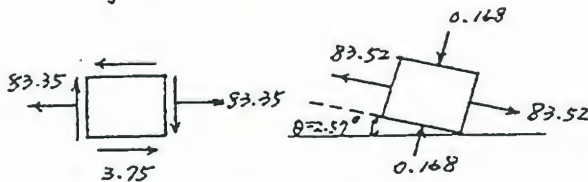


$$\tau_{xy} = 5 \text{ MPa} \quad \sigma_1 = 5 \text{ MPa}, \quad \sigma_2 = -5 \text{ MPa}$$

element D:

$$\sigma_x = 83.35 \text{ MPa}$$

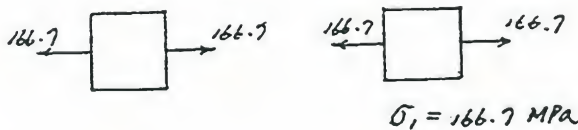
$$\tau_{xy} = 3.75 \text{ MPa}$$



$$\sigma_1 = 83.52 \text{ MPa}, \quad \sigma_2 = -0.168 \text{ MPa}$$

element E:

$$\sigma_x = \frac{M \cdot \frac{h}{2}}{I} = 166.7 \text{ MPa}$$



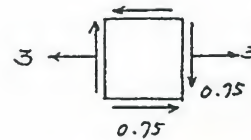
element A:

$$V_A = \frac{80}{3} \text{ kN}, \quad M_A = 16 \text{ kN-m}$$

$$\sigma_x = \frac{M_A \cdot y_A}{I} = \frac{16 \times 0.1}{5.33 \times 10^{-4}} = 3 \text{ MPa}$$

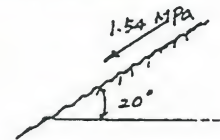
$$\tau_{xy} = \frac{V_A Q_A}{It} = \frac{\frac{80}{3} \times 0.1 \times 0.1 \times 0.15}{5.33 \times 10^{-4} \times 0.1}$$

$$= 0.75 \text{ MPa}$$



$$\tau_{xy}' = -\left(\frac{\sigma_x - \sigma_y}{2}\right) \sin(2 \times 70^\circ) + \tau_{xy} \cos(2 \times 70^\circ)$$

$$= -1.54 \text{ MPa}$$



element B:

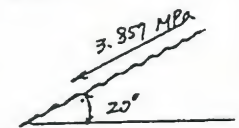
$$M_B = \frac{80}{3} \times 1.2 = 32 \text{ kN-m}, \quad Q_B = 0$$

$$\sigma_x = \frac{M_B \cdot y_B}{I} = \frac{32 \times 0.2}{5.33 \times 10^{-4}} = 12 \text{ MPa}$$



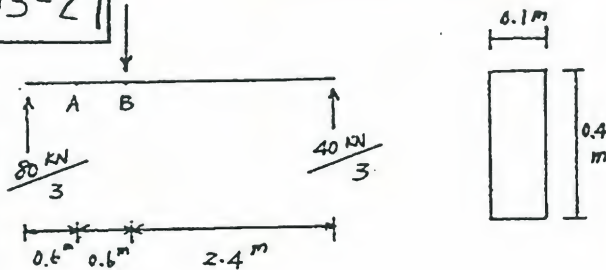
$$\tau_{xy}' = -\left(\frac{\sigma_x - \sigma_y}{2}\right) \sin(2 \times 70^\circ)$$

$$= -3.857 \text{ MPa}$$

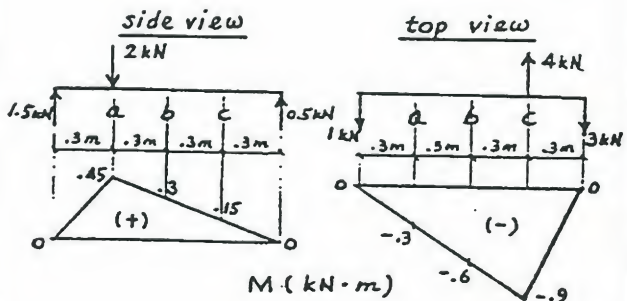


13-28

13-27



$$I = \frac{1}{12} \times 0.1 \times 0.4^3 = 5.33 \times 10^{-4} \text{ m}^4$$



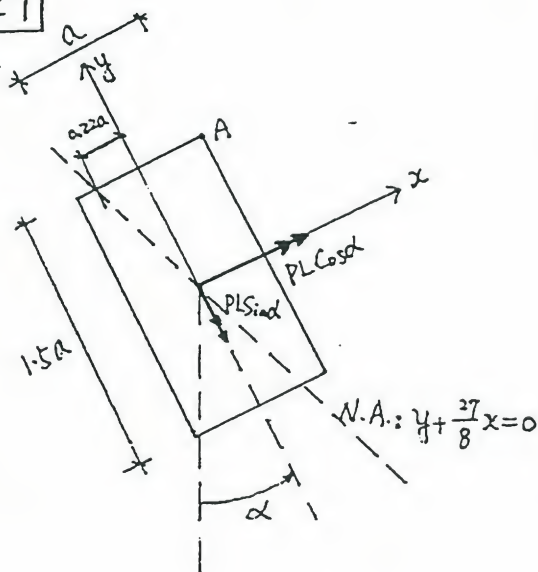
$$\sigma_a = \pm \frac{0.45}{\frac{(1)(.15)^2}{6}} \pm \frac{0.3}{\frac{(1.15)(.1)^2}{6}} = \pm 1200 \pm 1200 = \pm 2400 \text{ kPa}$$

$\sigma_b < \sigma_c$  (by inspection)

$$\sigma_c = \pm \frac{0.15}{\frac{(1)(.1)(.1)}{6}} \pm \frac{0.9}{\frac{(1.1)(.1)}{6}} = \pm 900 \pm 5400$$

$$\therefore \sigma_{\max} = \sigma_c = \pm 6300 \text{ kPa}$$

13-29



$$\sigma_A(\alpha) = \frac{PL \cos \alpha \cdot (\frac{1.5a}{2})}{\frac{1}{12} a \cdot (1.5a)^3} + \frac{PL \sin \alpha (\frac{a}{2})}{\frac{1}{12} 1.5a(a)^3}$$

$$\frac{d\sigma_A(\alpha)}{d\alpha} = 0 \Rightarrow \tan \alpha = \frac{3}{2} \Rightarrow \alpha = \tan^{-1} \frac{3}{2} = 56.3^\circ$$

Neutral Axis:

$$\frac{PL \cos \alpha \cdot y}{\frac{1}{12} a (1.5a)^3} + \frac{PL \sin \alpha \cdot x}{\frac{1}{12} (1.5a) \cdot a^3} = 0$$

$$\frac{4}{9} y + \tan \alpha \cdot x = 0$$

$$\frac{4}{9} y + \frac{3}{2} x = 0$$

$$y + \frac{27}{8} x = 0$$

13-30

$$(\sigma_x)_{\max} = \frac{Mr}{I} = \frac{Mr}{\pi (r^4/4)}$$

$$M = \frac{\pi r^3}{4} (\sigma_x)_{\max} = \frac{\pi (25)^3}{4} \times \frac{120}{10^6} = 1.47 \text{ kN-m}$$

$$(\sigma_1)_{\max} = \frac{-(\sigma_x)_{\max}}{2} + \sqrt{\left[\frac{(\sigma_x)_{\max}}{2}\right]^2 + \left(\frac{Tr}{J}\right)^2}$$

$$T = \frac{\pi r^3}{2} \sqrt{\left[(\sigma_1)_{\max} - \frac{-(\sigma_x)_{\max}}{2}\right]^2 - \left[\frac{(\sigma_x)_{\max}}{2}\right]^2}$$

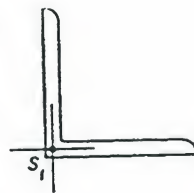
$$= \frac{\pi (25)^3}{2 \times 10^6} \sqrt{(160 - 60)^2 - 60^2} = 1.96 \text{ kN-m}$$

13-31

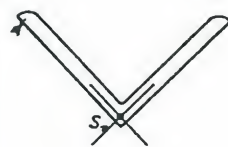
from Table  $S_{xx} = 3.26 \times 10^4 \text{ mm}^3$

$$A = 2.42 \times 10^3 \text{ mm}^2$$

$$r_{zz} = 19.9 \text{ mm}$$



$$M_1 = \sigma_y S_1 = 3.26 \times 10^4 \sigma_y$$



$$M_2 = \sigma_y S_2$$

$$= \sigma_y \frac{I_2}{c_2}$$

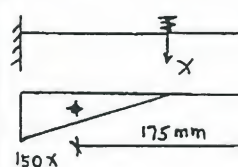
$$= \sigma_y \frac{(2.42 \times 10^3)(19.9)^2}{(30.2 \times \sqrt{2})}$$

$$= 2.24 \times 10^4 \sigma_y$$

$$\frac{M_1}{M_2} = \frac{3.26 \times 10^4 \sigma_y}{2.24 \times 10^4 \sigma_y} = 1.46$$

13-32

$$\Delta_T = (20 \times 10^{-6})(40)(4.5 \times 10^3) = 3.6 \text{ mm}$$



$$\frac{\frac{1}{2}(150)(150 \times)(175)}{20 \times 10^6} = 2$$

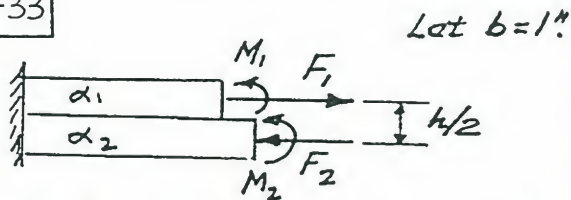
$$x = 20.317 \text{ N}$$

$$\Delta = \Delta_T + \Delta_x = 3.6 + \frac{x(4500)}{(6)(10^5)} + \frac{x}{20} + \frac{\frac{1}{2}(150)(150 \times)(106)}{20 \times 10^6} = 5.91 \text{ mm}$$

$$\text{number of turns} = 5.91$$



13-33



If free to expand ends do not match. Need  $F_1$  &  $F_2$ , which cannot exist, however, without  $M_1$  &  $M_2$  as  $\sum M_3 = 0$ .

$$\sum F_x = 0 : F_1 = F_2 = F$$

Both bars curve alike;

$$\frac{1}{\rho} = \frac{M_1}{EI} = \frac{M_2}{EI}$$

hence

$$M_1 = M_2 = M = \frac{EI}{\rho} \quad \text{where } I \text{ for one bar.}$$

$$\sum M_3 = 0 : 2M = Fh/2, \text{ and}$$

$$F = \frac{4M}{h} = \frac{4EI}{h\rho} \quad \text{where } \rho \text{ on}$$

EQUATING THE STRAIN AT THE BOTTOM OF THE TOP BAR TO THE STRAIN AT THE TOP OF THE BOTTOM BAR, ONE HAS

$$\begin{aligned} \alpha_1 \delta T + \frac{F}{\frac{1}{2}hE} + \frac{h/4}{\rho} &= \\ &= \alpha_2 \delta T - \frac{F}{\frac{1}{2}hE} - \frac{h/4}{\rho} \end{aligned}$$

OR

$$\frac{4F}{hE} + \frac{h}{2\rho} = (\alpha_2 - \alpha_1) \delta T$$

$$\frac{16EI}{h^2 E \rho} + \frac{h}{2\rho} = (\alpha_2 - \alpha_1) \delta T$$

$$\text{But } I = \frac{1}{12} \times 1'' \times \left(\frac{h}{2}\right)^3, \text{ hence}$$

$$\frac{1}{\rho} = \frac{3}{2h} (\alpha_2 - \alpha_1) \delta T$$

$$\Delta = \frac{L^2}{2\rho} = \frac{3(\alpha_2 - \alpha_1) \delta T L^2}{4h}$$

13-36

$$\tau_{max} = \sqrt{\left(\frac{\sigma_B}{2}\right)^2 + \tau_T^2}$$

$$80 \times 10^3 = \sqrt{\left(\frac{M \cdot r}{2I}\right)^2 + \left(\frac{T \cdot r}{J}\right)^2}$$

$$= \sqrt{\left(\frac{4 \cdot r}{2 \cdot \frac{\pi r^4}{4}}\right)^2 + \left(\frac{6 \cdot r}{\frac{\pi r^4}{2}}\right)^2}$$

$$80 \times 10^3 = \frac{1}{\pi r^3} \sqrt{8^2 + 12^2}$$

$$r = 3.86 \text{ cm}$$

$$d = 2 \times 3.86 = 7.72 \text{ cm}$$

13-35

$$\frac{r-t}{r+t} = 0.8 \Rightarrow r = 4.5t \quad \text{--- ①}$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_B}{2}\right)^2 + (\tau_T)^2}$$

$$80 \times 10^3 = \sqrt{\left(\frac{Mr}{2I}\right)^2 + \left(\frac{T \cdot r}{J}\right)^2}$$

$$80 \times 10^3 = \sqrt{\left(\frac{4 \cdot r}{2 \cdot \pi r^3 \cdot t}\right)^2 + \left(\frac{6 \cdot r}{2 \pi r^3 t}\right)^2}$$

$$80 \times 10^3 = \frac{1}{\pi r^2 t} \sqrt{2^2 + 3^2}$$

$$r^2 t = 14.346 \text{ cm}^3 \quad \text{--- ②}$$

$$\text{from ①, ②} \Rightarrow r = 4 \text{ cm},$$

$$t = 0.89 \text{ cm}$$

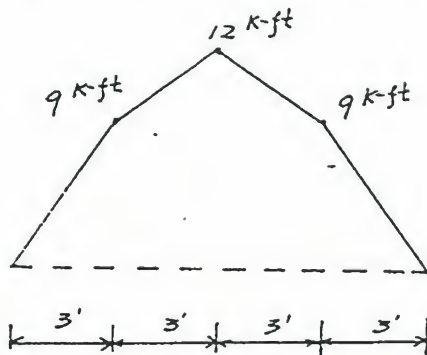
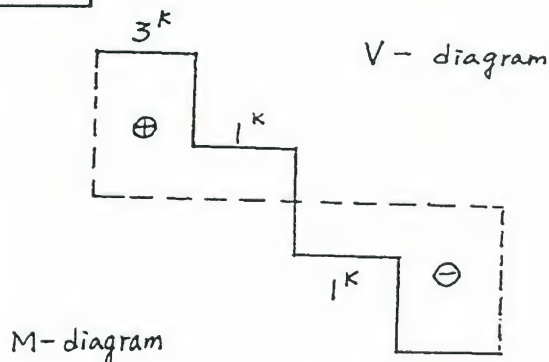
$$\text{inside diameter} = 2(r - \frac{t}{2})$$

$$= 7.11 \text{ cm}$$

$$\text{outside diameter} = 2(r + \frac{t}{2})$$

$$= 8.89 \text{ cm}$$

13-36



$$S_1 = \frac{M_{\max}}{\sigma_{\text{AWB}}} = \frac{12 \times 10^3 \times 12}{1250} = 115.2 \text{ in}^3$$

$$A_1 = \frac{3}{2} \frac{V_{\max}}{\tau_{\text{aw}}} = 1.5 \times \frac{3 \times 10^3}{95} = 47.37 \text{ in}^2$$

choose standard dressed size

5  $\frac{1}{2}$ "  $\times$  11  $\frac{1}{2}$ " (ref table 10)

$$W = 17.5 \text{ lb/ft}$$

$$S = 121 \text{ in}^3, A = 63.3 \text{ in}^2$$

$$S = 121 \text{ in}^3 > S_1 + \frac{\frac{1}{8} W l^2}{\sigma_{\text{AWB}}}$$

$$= 115.2 + \frac{\frac{1}{8} \times 17.5 \times 12^2 \times 12}{1250}$$

$$= 118.22 \text{ in}^3 \quad \text{O.K.}$$

$$A = 63.3 \text{ in}^2 > A_1 + \frac{3}{2} \frac{\frac{1}{8} W l}{\tau_{\text{aw}}}$$

$$= 47.37 + 1.5 \frac{0.5 \times 17.5 \times 12}{95}$$

$$= 49.03 \text{ in}^2 \quad \text{O.K.}$$

$$l_{\min} = \frac{2 \times 10^3}{625 \times 5.5} = 0.582 \text{ in}$$

$\therefore$  choose 5.5"  $\times$  0.6" bearing plates under the concentrated forces.

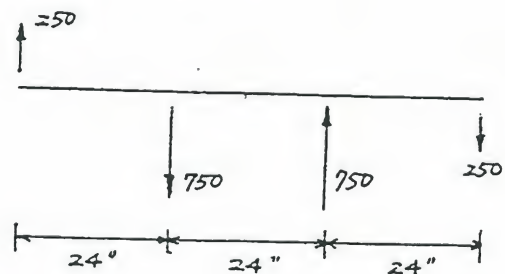
the minimum support length

$$= \frac{3 \times 1000 + 17.5 \times 12 \times 0.5}{625 \times 5.5}$$

$$= 0.903 \text{ in}$$

13-37

$$p = 250 \text{ lb}$$



$$M_{\max} = 250 \times 24 = 6000 \text{ lb-in}$$

$$V_{\max} = 500 \text{ lb} \text{ can use Table 10}$$

choose Nominal Size 3"  $\times$  4"

$$S = 5.10 \text{ in}^3, A = 8.75 \text{ in}^2$$

$$\sigma_{\max} = \frac{M_{\max}}{S} = 1176.47 \text{ psi} <$$

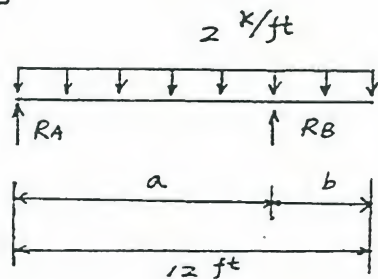
$$\sigma_{\text{AWB}} = 1250 \text{ psi}$$

$$\tau_{\max} = \frac{3 V_{\max}}{2 A} = 1.5 \times \frac{500}{8.75}$$

$$= 85.7 \text{ psi} < \tau_{\text{aw}} = 95 \text{ psi}$$

O.K.

13-38



$$(M_{\max})_{\text{left}} = (M_{\max})_{\text{right}}$$

$$\begin{aligned} (wl - \frac{1}{2}wl^2/a)(l - \frac{l^2}{2a}) - \frac{1}{2}w(l - \frac{l^2}{2a}) \\ = \frac{1}{2}w(l-a)^2 \frac{1}{2}(l - \frac{l^2}{2a})^2 \\ = \frac{1}{2}(l-a)^2 \end{aligned}$$

$$4a^4 - 8a^3l + 4al^3 - l^4 = 0 \quad (l=12)$$

$$4a^4 - 96a^3 + 6912a - 20736 = 0$$

$$(a) \text{ solve } a = 8.485 \text{ ft}, b = 3.515 \text{ ft}$$

$$M_{\max} = \frac{1}{2} \times 2 \times (3.515)^2 = 12.355 \text{ K-ft}$$

$$\begin{aligned} V_{\max} &= R_B - 2 \times 3.515 = 16.97 - 2 \times 3.515 \\ &= 9.94 \text{ K} \end{aligned}$$

$$(b) S_{\text{req}} = \frac{M_{\max}}{\sigma_{\text{aw}}} = \frac{12.355 \times 10^3 \times 12}{1250}$$

$$= 118.6 \text{ in}^3$$

$$A_{\text{req}} = \frac{3}{2} \frac{V_{\max}}{\tau_{\text{aw}}} = 1.5 \times \frac{9.94 \times 10^3}{95}$$

$$= 157 \text{ in}^2$$

choose Nominal size  $12 \text{ in} \times 14 \text{ in}$

$$A = 155 \text{ in}^2, S = 349 \text{ in}^3$$

approximately O.K. (ref table 10)

$$R_A = 7.03 \text{ K}, R_B = 16.97 \text{ K}$$

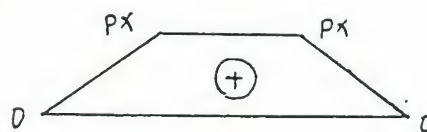
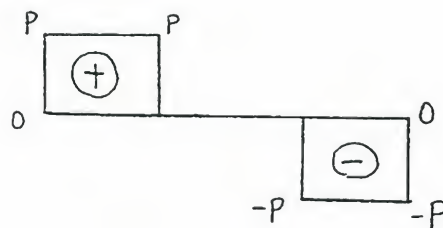
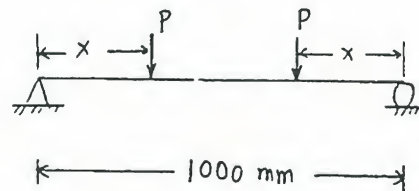
min support length at A

$$l_A = \frac{R_A}{\sigma_{\text{aw}} \times 12} = \frac{7.03 \times 10^3}{12 \times 625} = 0.94 \text{ in}$$

min support length at B

$$l_B = \frac{R_B}{\sigma_{\text{aw}} \times 12} = \frac{16.97 \times 10^3}{12 \times 625} = 2.26 \text{ in}$$

13-39



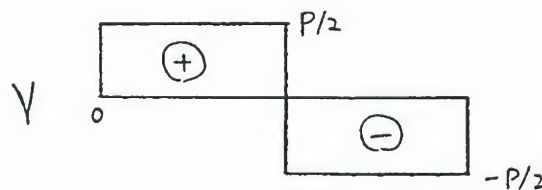
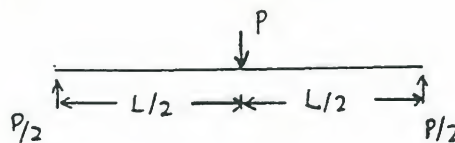
$$\tau_{\max} = \frac{3V}{2A} = \frac{3P}{2(15000)} = 2.5$$

$$P = 25000 \text{ N}$$

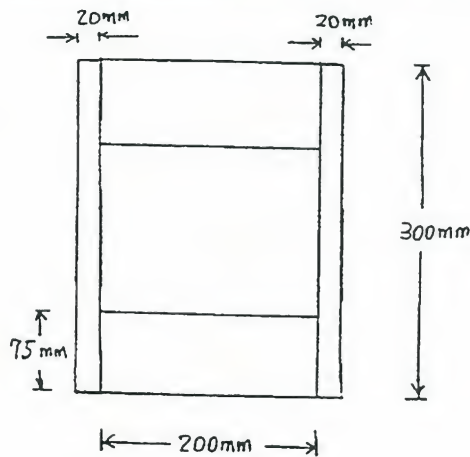
$$\sigma_{\max} = \frac{Mc}{I} = \frac{Px(75)}{\frac{1}{12}(100)(150)^3} = 10$$

$$x = 150 \text{ mm}$$

13-40







$$I = \frac{1}{12} [(240)(300)^3 - (200)(150)^3]$$

$$= 4.8375 \times 10^8 \text{ mm}^4$$

(a) shear in glue

$$V = \frac{T_{aw} I t}{Q} = \frac{(0.4)(4.8375 \times 10^8)(2 \times 75)}{(200 \times 75)(150 - 37.5)}$$

$$= 15600 \text{ N}$$

$$P = 2V = 31.2 \text{ kN}$$

shear in plywood

$$V = \frac{T_{aw} I t}{Q} = \frac{(0.8)(4.8375 \times 10^8)(2 \times 20)}{(200 \times 75 \times 112.5) + (2 \times 20 \times 150 \times 75)}$$

$$= 6568.4 \text{ N}$$

$$P = 2V = 13.14 \text{ kN (control)}$$

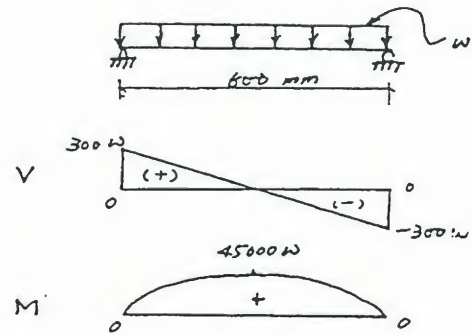
$$(b) \sigma_{aw} = \frac{M_{max} C}{I} = \frac{(PL/4) C}{I} = 8$$

$$L = \frac{32 I}{P C} = \frac{32 (4.8375 \times 10^8)}{(13140)(150)} = 7856 \text{ mm}$$

$$(c) A = \frac{P}{\sigma_{aw}} = \frac{13140}{4.3} = 3055 \text{ mm}^2$$

Use 3055 mm<sup>2</sup> bearing plate

13-41



T Section :

$$\bar{y} = \frac{(1200 \times 10) + (1200 \times 50)}{(1200 \times 2)} = 30 \text{ mm from top}$$

$$I = \frac{60(20)^3}{12} + (1200)(20)^2 + \frac{(20)(60)^3}{12} + (1200)(20)^2$$

$$= 1.36 \times 10^6 \text{ mm}^4$$

$$\sigma_{all} = \frac{MC}{I} = \frac{45000 \text{ W} \times 50}{1.36 \times 10^6} = 4 \times 10^{-3}$$

$$W = 2.42 \times 10^{-3} \text{ kN/mm}$$

$$\tau_{glue} = \frac{VQ}{It} = \frac{300W \times 1200(20)}{(1.36 \times 10^6) \times 20}$$

$$= 400 \times 10^{-6}$$

$$W = 1.51 \times 10^{-3} \text{ kN/mm}$$

$$\tau_{pl.} = \frac{VQ}{It} = \frac{300W \times (50 \times 20) \times 25}{(1.36 \times 10^6) \times 20}$$

$$= 600 \times 10^{-6}$$

$$W = 2.18 \times 10^{-3} \text{ kN/mm}$$

Rectangular section :

$$I = \frac{bh^3}{12} = \frac{(40)(60)^3}{12} = 7.2 \times 10^5 \text{ mm}^4$$

$$A = (60 \times 20) \times 2 = 2400 \text{ mm}^2$$

$$\sigma_{all} = \frac{MC}{I} = \frac{45000 \text{ W}(30)}{7.2 \times 10^5} = 4 \times 10^{-3}$$

$$W = 2.13 \times 10^{-3} \text{ KN/mm}$$

$$Z_{pl.} = \frac{3V}{2A} = \frac{3(300W)}{2 \times 2400} = 600 \times 10^{-6}$$

$$W = 3.2 \times 10^{-3} \text{ KN/mm}$$

$\therefore$  use rectangular section  
with  $W = 2.13 \text{ KN/m}$

13-42

$$W_L = 1 \text{ k/ft} \quad l = 24 \text{ ft}$$

$$M_L = \frac{1}{8} W_L l^2 = \frac{1}{8} \times 1 \times 24 \times 1000 \times 12 = 864000 \text{ lb-in}$$

$$V_L = \frac{1}{2} W_L l = \frac{1}{2} \times 1 \times 24 = 12 \times 1000 = 12000 \text{ lb}$$

Steel

$$S_1 = \frac{M_L}{\sigma_{AWB}} = \frac{864000}{24000} = 36 \text{ in}^3$$

$$A_1 = \frac{3V_L}{2\tau_{AW}} = 1.5 \times \frac{12000}{14400} = 1.25 \text{ in}^2$$

(a) Choose  $W \ 10 \times 39$

$$W_D = 39 \text{ lb/ft} \quad A = 11.5 \text{ in}^2$$

$$S = 42.1 \text{ in}^3$$

$$S = 42.1 \text{ in}^3 > S_1 + \frac{M_D}{\sigma_{AWB}}$$

$$= 36 + \frac{\frac{1}{2} \times 39 \times 24^2 \times 12}{24000}$$

$$= 41.62 \text{ in}^3 \quad \text{O.K.}$$

$$A = 11.5 \text{ in}^2 > A_1 + \frac{3}{2} \frac{V_D}{\tau_{AW}}$$

$$= 1.25 + \frac{1.5 \times \frac{1}{2} \times 39 \times 24}{14400}$$

$$= 1.3 \text{ in}^2 \quad \text{O.K.}$$

$$(b) \frac{W_D}{W_T} = \frac{W_D}{W_D + W_L} = \frac{0.039}{1} = 3.9\%$$

Wood

$$S_2 = \frac{M_L}{\sigma_{AWB}} = \frac{864000}{1250} = 691.2 \text{ in}^3$$

$$A_2 = \frac{3}{2} \frac{V_L}{\tau_{AW}} = 1.5 \times \frac{12000}{95} = 189.47 \text{ in}^2$$

(a) Choose Nominal Size  $12" \times 22"$

$$W_D = 68.7 \text{ lb/ft}, \quad A = 247 \text{ in}^2$$

$$S = 886 \text{ in}^3$$

$$S = 886 \text{ in}^3 > S_2 + \frac{M_D}{\sigma_{AWB}} = 691.2 + \frac{\frac{1}{2} \times 68.7 \times 24^2 \times 12}{1250}$$

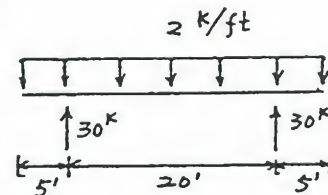
$$= 881.14 \text{ in}^3 \quad \text{O.K.}$$

$$A = 247 \text{ in}^2 > A_2 + \frac{3}{2} \frac{V_D}{\tau_{AW}} = 189.47 + \frac{3}{2} \times \frac{\frac{1}{2} \times 68.7 \times 24}{95}$$

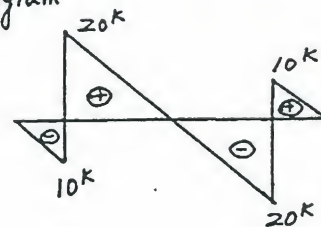
$$= 202.49 \text{ in}^2$$

$$(b) \frac{W_D}{W_T} = \frac{W_D}{W_D + W_L} = \frac{0.0687}{1} = 6.87\%$$

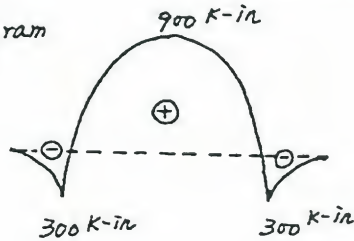
13-43



V-diagram



M-diagram



$$M_{max} = 900 \text{ K-in}$$

$$V_{max} = 20 \text{ K}$$

$$S_{req} = \frac{M_{max}}{\sigma_{awB}} = \frac{900}{24} = 37.5 \text{ in}^3$$

$$A_{req} = \frac{3}{2} \frac{V_{max}}{\tau_{aw}} = 1.5 \times \frac{20}{14.4} = 2.08 \text{ in}^2$$

Choose W 12 x 30

$$A = 8.79 \text{ in}^2 > A_{req} = 2.08 \text{ in}^2$$

$$S = 38.6 \text{ in}^3 > S_{req} = 37.5 \text{ in}^3$$

13-44

$$q_L = 75 \text{ lb/ft}^2, q_D = 25 \text{ lb/ft}^2$$

(a) design for wooden joists

$$W_L = 75 \times \frac{16}{12} = 100 \text{ lb/ft}$$

$$W_D = 25 \times \frac{16}{12} = 33.33 \text{ lb/ft}$$

$$W_T = W_L + W_D = 133.33 \text{ lb/ft}$$

$$M_{max} = \frac{1}{8} W_T l^2 = \frac{1}{2} \times 133.33 \times (12)^2 \times 12$$

$$= 28800 \text{ lb-in}$$

$$V_{max} = \frac{1}{2} W_T l = \frac{1}{2} \times 133.33 \times 12$$

$$= 800 \text{ lb}$$

Choose Nominal Size 2" x 12"

$$S = 31.6 \text{ in}^3 > S_{req} = \frac{M_{max}}{\sigma_{awB}} = \frac{28800}{1200}$$

$$= 24 \text{ in}^3$$

$$A = 16.9 \text{ in}^2 > A_{req} = \frac{3}{2} \frac{V_{max}}{\tau_{aw}}$$

$$= 1.5 \times \frac{800}{100}$$

$$= 12 \text{ in}^2 \text{ O.K.}$$

(b) design for steel beam

$$W_T = (q_D + q_L) \times 12$$

$$= (100) \times 12 = 1200 \text{ lb/ft}$$

$$M_{max} = \frac{1}{8} W_T l^2 = \frac{1}{8} \times 1200 \times (20)^2 \times 12$$

$$= 720 \text{ K-in}$$

$$V_{max} = \frac{1}{2} W_T \cdot l = \frac{1}{2} \times 1200 \times 20$$

$$= 12 \text{ K}$$

Choose W 12 x 26

$$V_D = \frac{1}{2} \times 26 \times 20 = 0.26 \text{ K}$$

$$M_D = \frac{1}{8} \times 26 \times (20)^2 \times 12$$

$$= 15.6 \text{ K-in}$$

$$S = 33.4 \text{ in}^3 > S_{req} = \frac{M_{max} + M_D}{\sigma_{awB}}$$

$$= \frac{720 + 15.6}{24}$$

$$= 30.65 \text{ in}^3$$

$$A = 7.65 \text{ in}^2 > A_{req} = \frac{3}{2} \frac{(V_{max} + V_D)}{\tau_{aw}}$$

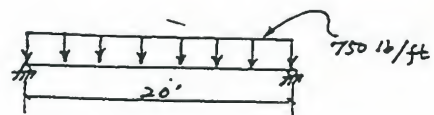
$$= 1.5 \times \frac{12 + 0.26}{14.4}$$

$$= 1.277 \text{ in}^2 \text{ O.K.}$$

13-45

$$\text{Design Load} = 75 + 75 = 150 \text{ lb/ft}^2$$

$$W = 150 \times 5 = 750 \text{ lb/ft}$$





$$M_{max} = \frac{Wl^2}{8} = \frac{750(20)^2}{8} = 37500 \text{ ft-lb}$$

$$V_{max} = \frac{Wl}{2} = \frac{750(20)}{2} = 7500 \text{ lb}$$

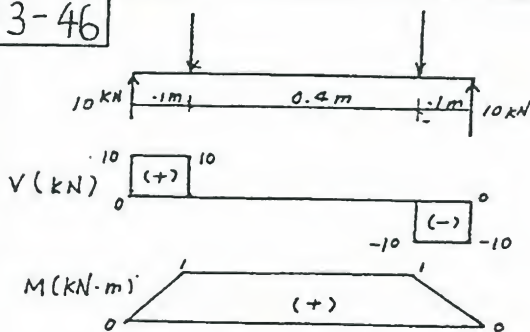
$$S = \frac{M}{\sigma_{all}} = \frac{37500 \times 12}{20,000} = 22.50 \text{ in}^3$$

use S 10 x 25.4 beams

$$\tau_{max} = \frac{V}{A_{web}} = \frac{7500}{10 \times 0.31} = 2420 \text{ psi} < \tau_{all}$$

$\therefore$  O.K.

13-46



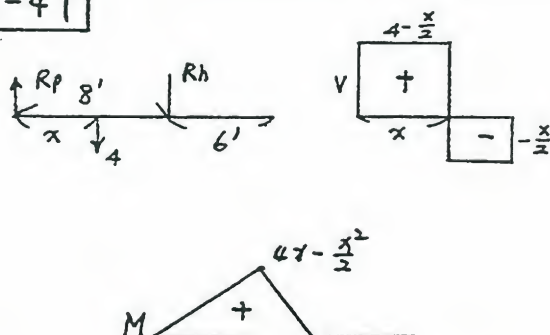
$$\sigma_{all} = \frac{M}{S} = \frac{10^6}{\frac{\pi R^3}{4}} = 80, \quad R = 25.15 \text{ mm}, \quad D = 50.31 \text{ mm}$$

use 51 mm diameter shaft

$$\tau_{max} = \frac{4V}{3A} = \frac{4 \times 10^4}{3 \left( \frac{\pi \times 51^3}{4} \right)} = 6.53 \text{ MPa} < \tau_{all}$$

$\therefore$  O.K.

13-47



$$R \times 8 = 4 \times x$$

$$R = \frac{1}{2} x$$

$$x = 14, \quad R_{max} = 7 \text{ ton}$$

$$M = -42 \text{ ton-ft}$$

$$16 = \frac{42 \times 2.24 \times 12 \times C}{I} \Rightarrow \frac{I}{C} = 70.56 \text{ in}^3$$

choose S 18 x 54.7

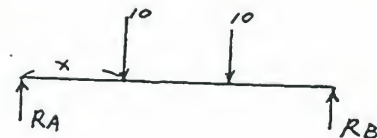
check shear strength

$$V_{max} = 7 \text{ ton} = 7 \times 2.24 = 15.68 \text{ k}$$

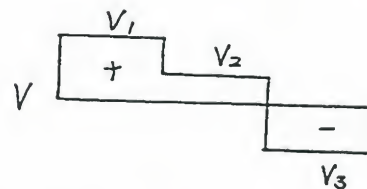
$$\tau_{max} = \frac{15.68 \times A \times C}{I t} \Rightarrow \tau_{max} = 6.1 \text{ ksi} < 9.6 \text{ ksi}$$

O.K.

13-48

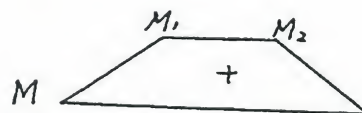


$$R_A = \frac{50}{3} - \frac{2x}{900}, \quad R_B = \frac{2x}{900} + \frac{10}{3}$$



$$V_1 = \frac{50}{3} - \frac{2x}{900}, \quad V_2 = \frac{20}{3} - \frac{2x}{900}$$

$$V_3 = -\frac{10}{3} - \frac{2x}{900}$$



$$M_1 = \frac{50}{3}x - \frac{2}{900}x^2, \quad M_2 = \frac{70}{3}x - \frac{4x^2}{900}$$

$$(a) \frac{\partial M_1}{\partial x} = \frac{50}{3} - \frac{4x}{900} = 0, \quad x = 2625$$

$$M_{1max} = 30625$$

$$\frac{\partial M_2}{\partial x} = \frac{140}{3} - \frac{16x}{900} = 0, \quad x = 3750$$

$$M_{2max} = 31250$$

$$SO, \quad x = 3750 \text{ mm}, \quad M_{max} = 31.25 \text{ kNm}$$

$$(b) \quad x=0, \quad V_1 = \frac{50}{3}, \quad V_2 = \frac{20}{3}, \quad V_3 = -\frac{10}{3}$$

$$x=6000, \quad V_1 = \frac{10}{3}, \quad V_2 = -\frac{20}{3}, \quad V_3 = -\frac{50}{3}$$

$$SO, \quad x=0 \text{ or } 6000 \text{ mm},$$

$$|V_{max}| = \frac{50}{3}$$

$$(c) \quad 14 \times 10^6 = \frac{31.25 \times 10^3 \times 6}{0.1 h^2}, \quad h = 0.366 \text{ m}$$

$$\tau = \frac{\frac{50}{3} \times 10^3 \times 0.366^2 \times 0.1 \times 0.5}{\frac{0.1}{12} \times 0.366^3 \times 0.1}$$

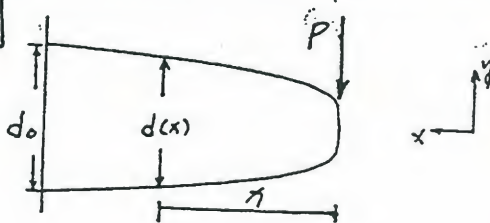
$$= \frac{1 \times 10^6}{0.366}$$

$$\tau = 2.73 \times 10^6 \text{ Pa} > 1.0 \times 10^6 \text{ Pa}$$

$$\text{Choose } h = 1.0, \quad \tau = 1 \times 10^6 \text{ Pa o.k.}$$

$$\text{number of boards} = \frac{1}{0.04} = 25$$

13-49



for any  $y = c$

$$\sigma_x = \frac{Pxc}{I(x)} = \frac{Pxc}{\frac{\pi}{32} d(x)^4} = \text{constant}$$

$$\therefore d(x) \propto x^{\frac{1}{4}} \Rightarrow d(x) = Ax^{\frac{1}{4}}$$

$$d(l) = Al^{\frac{1}{4}} = d_0$$

$$\Rightarrow A = d_0 \cdot l^{-\frac{1}{4}}$$

$$\therefore d(x) = \left(\frac{x}{l}\right)^{\frac{1}{4}} \cdot d_0$$

plan view is the same as elevation view

13-50

existing 4" standard pipe:

$$S = \frac{I}{c} = \frac{7.233}{2.25} = 3.21 \text{ in}^3$$

$$\text{pipe a: } S_a = 1 \times (1.55) = 4.82 \text{ in}^3$$

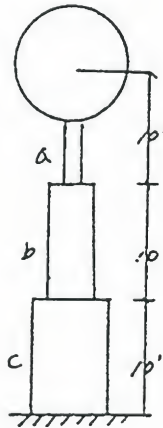
$$\text{use 5" pipe } (S = 5.45 \text{ in}^3)$$

$$\text{pipe b: } S_b = 2 \times (1.55) = 9.64 \text{ in}^3$$

$$\text{use 8" pipe } (S = 16.81 \text{ in}^3)$$

$$\text{pipe c: } S_c = 3 \times (1.55) = 14.47 \text{ in}^3$$

$$\text{use 10" pipe } (S = 29.9 \text{ in}^3)$$



13-51

$$(a) \quad M_1 = \sigma_y S_1 = (160 \times 10^3)(1440 \times 10^{-6}) = 230.4 \text{ kN-m}$$

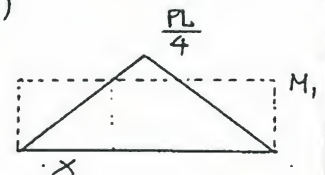
$$\frac{P(5000)}{4} = \frac{(160 \times 10^3)[222 \times 10^6 + 2 \times 200 \times 10 \times (159)^2]}{(154 + 10)}$$

$$P = 252.2 \text{ kN}$$

$$x = \frac{2M_1}{P} = 1.827 \text{ m}$$

length of coverplate

$$= 5 - 2 \times 1.827 = 1.346 \text{ m} \rightarrow \text{use } (1.346 + 0.016) = 1.362 \text{ m}$$



$$\frac{W_0(5^2)}{8} = \frac{(252.2)(5)}{4} \rightarrow W_0 = 100.88 \text{ kN/m}$$

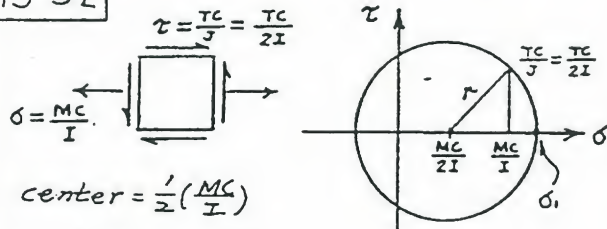
$$\frac{(100.88)(5)}{2} x - \frac{(100.88)}{2} x^2 = 230.4$$

$$x = 1.203$$

length of coverplate

$$= 5 - 2 \times 1.203 = 2.594 \text{ m} \rightarrow \text{Use } (2.594 + 0.016) = 2.61 \text{ m}$$

13-52



$$r = \sqrt{\left(\frac{MC}{2I}\right)^2 + \left(\frac{TC}{2I}\right)^2}$$

$$(a) \sigma_1 = \frac{MC}{2I} + \sqrt{\left(\frac{MC}{2I}\right)^2 + \left(\frac{TC}{2I}\right)^2}$$

$$= \frac{C}{2I} (M + \sqrt{M^2 + T^2}) = \frac{C}{J} (M + \sqrt{M^2 + T^2})$$

$$(b) J = \pi d^4/32, \quad J/C = \pi d^3/16$$

$$\sigma_{all} = \frac{16}{\pi d^3} (M + \sqrt{M^2 + T^2})$$

$$d = \left( \frac{16}{\pi \sigma_{all}} (M + \sqrt{M^2 + T^2}) \right)^{1/3}$$

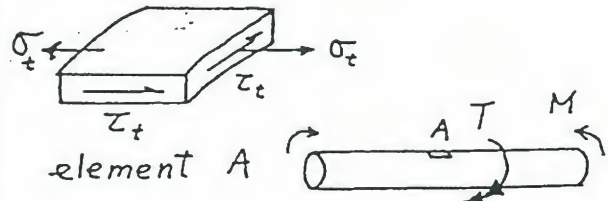
13-53

use equation (13-3)

$$d = \sqrt[3]{\frac{16}{\pi \sigma_{all}} (M^2 + T^2)^{1/2}}$$

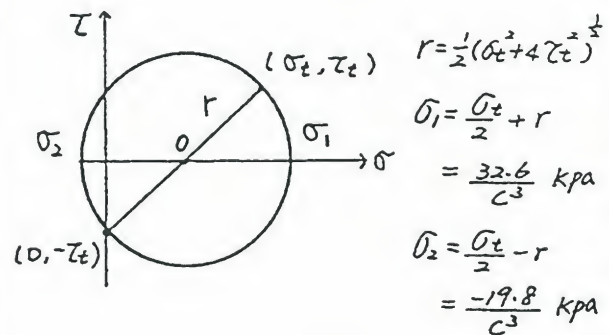
$$= \sqrt[3]{\frac{16}{\pi (50000)} (10^2 + 40^2)^{1/2}} = 0.161 \text{ m}$$

13-54



solid shaft

use Mohr's circle to estimate  $\sigma_1, \sigma_2$



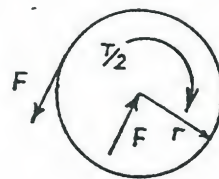
$$\left( \frac{32.6}{50 \times 10^3 \times C^3} \right)^2 - \left( \frac{32.6}{50 \times 10^3 \times C^3} \right) \left( \frac{-19.8}{50 \times 10^3 \times C^3} \right)$$

$$+ \left( \frac{-19.8}{50 \times 10^3 \times C^3} \right)^2 = 1$$

$$\therefore C = 0.097 \text{ m}, \quad d = 0.194 \text{ m}$$

13-55

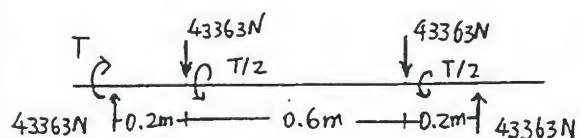
$$T = \frac{159 \times 45}{11/60} = 39027 \text{ N} \cdot \text{m}$$



$$F = T/2r$$

$$= 39027 / (2 \times 0.45)$$

$$= 43363 \text{ N}$$



$$V_{max} = 43363 \text{ N}$$

$$M_{max} = 8672.6 \text{ N} \cdot \text{m}$$



use equation (13-3)

$$d = \sqrt[3]{\frac{16}{\pi(40 \times 10^6)} (43363^2 + 8672.6^2)^{\frac{1}{2}}}$$

$$= 0.178 \text{ m}$$

use standard procedure

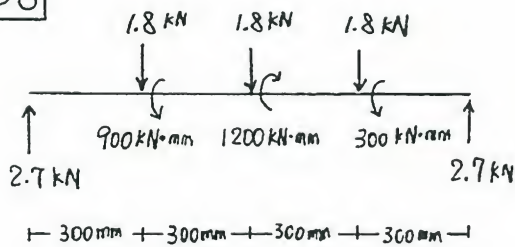
$$T_{all} = \frac{T_c}{J} + \frac{4V}{3A} = \frac{39027 \times 16}{\pi d^3} + \frac{4 \times 43363 \times 4}{3\pi d^3}$$

$$40 \times 10^6 = \frac{198763 + 73615d}{d^3}$$

$$d = 0.174 \text{ m}$$

use 18 cm diameter shaft

13-56



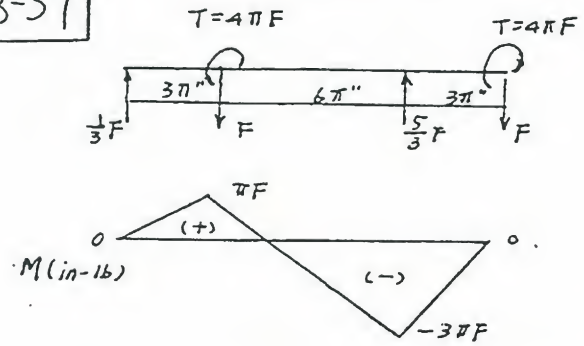
$$M_{max} = 1080 \text{ kN}\cdot\text{mm}$$

$$T_{max} = 900 \text{ kN}\cdot\text{mm}$$

use equation (13-3)

$$d = \sqrt[3]{\frac{16}{\pi(0.4)} (1080^2 + 900^2)^{\frac{1}{2}}} = 26.16 \text{ mm}$$

13-57

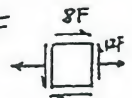


$$T_{max} = 4\pi F$$

$$M_{max} = -3\pi F$$

$$\tau = \frac{T_c}{J} = \frac{4\pi F \times 1}{\frac{\pi(1)^4}{2}} = 8F$$

$$\sigma = \frac{MC}{I} = \frac{3\pi F \times 1}{\frac{\pi(1)^4}{4}} = 12F$$



$$\tau_{max} = \sqrt{\left(\frac{12}{2}\right)^2 + (8)^2} \quad F = 5000$$

$$F = 500 \text{ lb}$$

13-58

$$\sigma = k_1 \frac{M_c}{I} = 1.6 \frac{(0.3P)(0.0375)}{\frac{\pi(0.0375)^4}{4}}$$

$$= 11589P$$

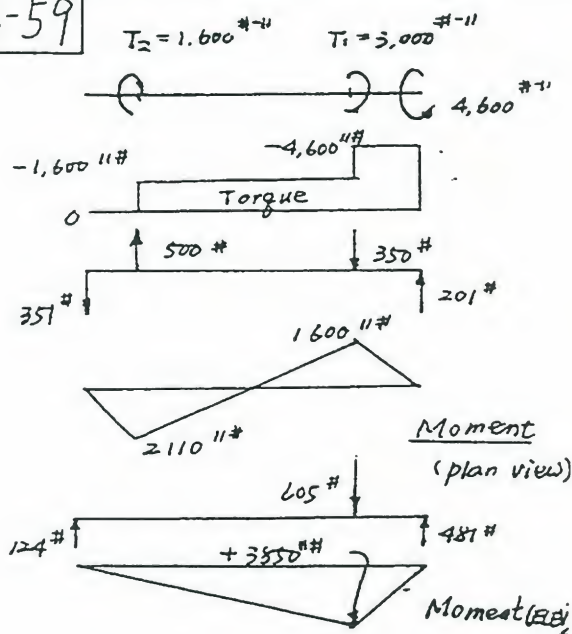
$$\tau = k_2 \frac{T_c}{J} = 1.2 \frac{(0.6P)(0.0375)}{\frac{\pi(0.0375)^4}{2}}$$

$$= 8692P$$

$$\tau_{max} = \sqrt{\left(\frac{11589P}{2}\right)^2 + (8692P)^2} = 45 \times 10^6$$

$$P = 4308 \text{ N}$$

13-59



$$d = \sqrt[3]{\frac{16 \times 1.5}{\pi \times 6000} \times 1000 \sqrt{4.6^2 + 1.6^2 + 3.85^2}} = 2''$$

13-60

(a) from Fig. 2-25 for  $10^7$  cycles

$$S = 190 \text{ MPa}$$

$$\frac{\sigma}{\text{F.S.}} = \frac{Mc}{I}$$

$$\frac{190}{2.5} = \frac{\frac{P(800)^2}{8} (100)}{\frac{\pi (100)^4}{4}}$$

$$\rightarrow P = 746 \text{ N/mm}$$

$$(b) \tau_{all} = \frac{VQ}{It}$$

$$\text{where } \tau_{all} = \frac{\sigma_{all}}{2} = \frac{(190/2.5)}{2} = 38 \text{ MPa}$$

$$Q = \frac{\pi d^2}{2} \frac{4d}{3\pi} = \frac{2d^3}{3}$$

$$I = \frac{\pi d^4}{4}, \quad t = 2d$$

$$\frac{VQ}{It} = \frac{V \left( \frac{2d^3}{3} \right)}{\left( \frac{\pi d^4}{4} \right) (2d)} = \frac{4V}{3\pi d^2} = 38 \times 10^6$$

$$\frac{4(746 \times 400)}{3\pi d^2} = 38 \times 10^6$$

$$d = 5.8 \times 10^{-2} \text{ m}$$

$$= 58 \text{ mm}$$